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ACTUARIAL SOCIETY OF AMERICA

"The work of science is to substitute facts for appearances and demonstra ions for impressions."—RUSKIN.

TRANSACTIONS

1915

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NOTICE

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October, 1915.

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NOTATION

Life Symbols.

As Revised October 11th, 1911.

Resolved, that in the presentation of papers the symbols of the Text-Book of the Institute of Actuaries, with the additions noted below, be followed when convenient, and that, in any event, the use of such symbols in other senses be avoided; all letters used as symbols to be printed in italics except "a" when used to denote an annuity of 1 first payment immediate, *i. e.* $(1+a)$; that N_x and S_x be used in accordance with the practice of the Institute of Actuaries, *i. e.*,

$$N_x = N_{x-1} = D_x + D_{x+1} + \cdots$$
$$S_x = S_{x-1} = N_x + N_{x+1} + \cdots$$

Additional symbols

$$k_x = \frac{d_x}{l_{x+1}} = \frac{C_x}{D_{x+1}}.$$

$$u_x = \frac{l_x}{l_{x+1}}(1+i) = \frac{D_x}{D_{x+1}}.$$

Disability Symbols.

As Adopted May 16th, 1912.

l_x^{aa} = the number of active lives at age x.
l_x^{ii} = the number of invalid lives at age x,

then $l_x^{aa} + l_x^{ii} = l_x$.

l_x^{i} = the survivors at age x in a mortality table based on invalid lives (not to be confused with l_x^{ii} amongst whom there are additions each year from those just disabled).

d_x^{aa} = the number dying as active lives between ages x and $x+1$.

d_x^{ii} = the number dying as invalid lives between ages x and $x+1$.

i_x = the number of active lives becoming invalid between ages x and $x+1$.

p_x^a = the probability of an active life aged x being alive one year hence, whether then active or invalid.

q_x^a = the probability of an active life aged x dying within a year, whether still active or after becoming invalid,

$$\text{then } p_x^a + q_x^a = 1.$$

p_x^i = the probability of an invalid life aged x being alive one year hence.

q_x^i = the probability of an invalid life aged x dying within a year,

$$\text{then } p_x^i + p_x^i = 1.$$

p_x^{aa} = the probability of an active life aged x being alive and active one year hence.

p_x^{ai} = the probability of an active life aged x being alive but invalid one year hence.

q_x^{aa} = the probability of an active life aged x dying while still active within a year.

q_x^{ai} = the probability of an active life aged x becoming invalid and dying within a year.

r_x = the probability of an active life aged x becoming invalid within a year,

then $p_x^{aa} + p_x^{ai} = p_x^a$; $q_x^{aa} + q_x^{ai} = q_x^a$; $p_x^{ai} + q_x^{ai} = r_x$; $p_x^{aa} + q_x^{aa} = 1 - r_x$; $p_x^{aa} + p_x^{ai} + q_x^{aa} + q_x^{ai} = 1$.

r'_x = the absolute annual rate of invalidity,

$$\text{then } r'_x = \frac{i_x}{l_x^{aa} - \frac{1}{2}d_x^{aa}} = \frac{r_x}{1 - \frac{1}{2}q_x^{aa}}.$$

$D_x^{aa} = v^x l_x^{aa}$.

$D_x^{ii} = v^x l_x^{ii}$.

$D_x^i = v^x l_x^i$.

a_x^{ai} = value of an annuity payable at the end of each year

provided an active life now aged x is then alive but invalid.

a_x^a = value of an annuity-due on an active life payable during survival, active or invalid.

a_x^{aa} = value of an annuity-due on an active life payable during activity.

a_x^i = value of an annuity-due on an invalid life.

${}_{n|}a_{xt|}^{a(\overline{in}|)}$ = value of a deferred temporary annuity payable at the beginning of each of t years deferred n years, provided the disability occur during the n years.

$a_{x:\overline{n}|:\overline{y-x}|}^{aa}$ = value of an annuity-due on an active life for n years or for $y-x$ years, whichever is the shorter, during activity.

Vol. XVI, Part I. No. 53.

TRANSACTIONS

MAY 20TH AND 21ST, 1915.

Address of the President, James M. Craig.

Taxation of Life Insurance Companies.

The question of taxation in life insurance has been handled by so many competent men that it would seem almost superfluous to refer to it on this occasion, but recent events presenting it from opposite points of view, warrant bringing it to your attention.

It is no wonder that with pressing demands to meet, the ever increasing expenses of the state and the aversion of all political parties to levy a direct tax, the funds of life insurance companies, which are always like an open book, should be levied on.

In these days it is hardly necessary for every state to have any specific tax rate on life insurance premiums in order to collect taxes, because under the reciprocal laws which seem to be uniform throughout most of the country the same rate of tax is imposed on companies of other states as the laws of those states impose.

Although the amount of tax increases from year to year without any increase in the rate of tax, we never hear of any decrease in the rate.

As an illustration of this it may be stated that one per cent. of premiums received in the state of New York in 1913 would exceed one and one half per cent. of the premiums received in 1903, so that without any increase in the rate of taxation the amount of taxes increased more than fifty per cent. in ten years.

The practice of taxing the funds of life insurance companies is now so general that whenever the subject is brought up for legislative action, it is simply a question of how much to increase the tax.

The state, if it considers the policyholders at all when dealing with this question, takes consolation in the belief that, at the worst, dividends are slightly reduced on participating policies, and on non-participating policies the insured would just as soon pay a premium with the tax added as without it. In the case of non-participating premiums, however, it would be impossible to increase premiums on existing policies to cover increased taxation. It is difficult to convince legislatures that a tax of one or two or even three per cent. would have any material effect on the prosperity of life insurance companies, and it is so easy to levy these taxes because they are not directly imposed on the people who pay the premiums.

One point of view is the upward trend of the rate of taxation in recent years. Without going too much into detail, I will note in a general way some of the changes which have recently taken place.

Alabama.—Tax increased in 1908 from 1 per cent. to 2 per cent. on premiums received.

Arkansas.—Tax of 2½ per cent. on premiums less losses and commissions changed in 1914 to 1½ per cent. on gross premiums. The net result of this change was an increase.

California.—Tax of 1 per cent. changed in 1911 to 1½ per cent. and again in 1913 to 1¾ per cent.

Delaware.—Tax of 1½ per cent. increased in 1908 to 2 per cent.

New Hampshire.—Changed in 1910 from 1 per cent. on gross premiums to 2 per cent. on premiums less death losses.

South Carolina.—Law was changed in 1909 from ½ per cent. together with county taxes, to 2 per cent. as a state tax with a decreasing rate depending on the amount invested in the state with a minimum rate of 1 per cent. In 1910 this law was amended still making the minimum rate 1 per cent. but requiring a larger investment to secure the benefit of a reduction.

Virginia.—In 1911 the rate was increased from 1 per cent. to 1.4 per cent., and at the beginning of the present year this was increased to 2¼ per cent. The latter amendment made the tax a license tax and payable almost immediately so that the state received the benefit of double taxation this year.

Washington.—The rate in 1908 increased from 2 per cent. on premiums less losses to 2 per cent. on gross premiums and again increased to 2¼ per cent. in 1912.

West Virginia.—A change was made in 1908 from 1½ mills per dollar new business written to 2 per cent. on premiums.

Canada.

Province of Manitoba.—Tax of 1 per cent. on gross premiums was increased in 1915 to a graduated tax running from 1 per cent. on premium receipts of less than $50,000 up to 2 per cent. on $200,000 or more.

Nova Scotia.—Changed in 1913 from registration fee of $50 to a tax of 1 per cent. on gross premiums and again in 1915 to 1¼ per cent.

Ontario.—Rate of 1 per cent. on gross premiums changed in 1914 to 1¾ per cent.

Quebec.—Changed in 1906 from 1 per cent. on gross premiums less bonuses to 1¾ per cent.

In some of the provinces an additional tax is made on investments or the income from investments.

Our friends in Canada are apt scholars on the subject of a premium income tax, for no such tax was known anywhere in the Dominion twenty years ago.

It will thus be seen that where any change is made it is always up and never down.

In order to see how great this charge against the business is in the aggregate, we find that the *Year Book* showing the business of 1913 records the payment of taxes, licences, fees and fines of $12,820,449 by 258 life companies. At age 35, this would pay the annual premium of $28.11 on 456,081 whole life policies for $1,000 each or a total insurance of $456,081,000.

Although the officers of life insurance companies believe, as a matter of principle, that life insurance companies should not be taxed at all, they never have raised any strenuous objection against a moderate tax. We often find it intimated that the officers of life insurance companies having vast sums of money at their disposal for the prosecution of the business are not as economical as they might be, and this intimation is closely allied with the thought that a little more in taxation for the benefit of the state can easily be made good by a slight reduction in those expenses which are under control. Even if we assume for the sake of argument that expenses over which the companies have control can be reduced and that increased taxation is resorted to for the double purpose of (1) enforcing such reduction, and (2) diverting the proceeds of it into the state treasury, we fail to see how such a transaction would be of any possible

benefit to the policyholders. If expenses are too high and cannot be reduced voluntarily, they should be regulated by law and the saving effected revert to policyholders. The remedy certainly does not lie in an increase of taxes, for this makes a fixed charge which is beyond the power of companies to regulate.

The other point of view indicates that perhaps the time is ripe to take up this question in an aggressive manner. This is suggested by laws recently passed by the states of New York, New Jersey and Utah making provision for pensions to worthy widows in need. In New York these pensions range from twenty dollars a month up to sixty dollars depending on the number of children to be cared for.

The Governor in signing the bill issued a statement in which he said:

"The bill tends towards the preservation and protection of the home. A careful trial of the proposed scheme may be made without a serious burden upon the localities. The bill gives opportunity to provide for the maintenance of dependent children in the homes of the widowed mothers, where it would otherwise be necessary to provide for them in institutional homes. I am convinced that the possibility of this accomplishment renders necessary the approval of this bill.

"It is a familiar and well-established rule governing the custody of a child that the welfare of the child is the primary consideration. If the mother of a child is a competent and suitable person to be entrusted with its care and custody, the home of the mother is the proper place for him. Experience has shown that where, because of misfortune, the widowed mother is compelled to give up her home, and her children are provided for by persons who have no natural interest in them, such children are injuriously affected thereby, and they do not become as capable citizens as would have been the case if they had remained under the control of their mother."

The sentiment here expressed is really the foundation of life insurance; for the maintenance of the children in the home of the mother is the motive which prompts many men to lay aside a portion of their income for the creation of a fund through the medium of life insurance which shall be sufficient to continue the solidarity of the home after the bread winners have passed away. Surely when life insurance companies are striving to accomplish the very thing the state commends, and without any expense to the state, the moneys

which they collect ought not to be subject to the call of the tax gatherer. Even the laboring man who takes out a small industrial policy is being educated on the subject of life insurance and from the millions of industrial policyholders will come thousands and hundreds of thousands of applicants for larger amounts of insurance.

Governor Whitman either consciously or unconsciously has rendered life insurance companies a great service in stating so concisely the wisdom of preserving and protecting the home. If it is a good thing for the state to provide for deserving widows through general taxation, it is a better thing to persuade men as individuals to do this on their own account for the love they bear to the members of their own households. From this point of view, there is only one logical conclusion in this matter and that is that the state should not only permit life insurance but that it should offer every possible encouragement to the companies to prosecute the business and not impair their usefulness by undue taxation.

In this connection attention is also called to a bill introduced in the last Pennsylvania legistlature which recited:

"WHEREAS it is to the interest of the commonwealth and the various counties and municipal divisions thereof to encourage its citizens to reasonably provide for their own support during old age, whereby the financial burden of their maintenance may be avoided; therefore.

"*Section 1.* Any policy or contract or insurance or annuity heretofore or hereafter taken out or issued by any insurance company, domestic or foreign, lawfully doing business in this commonwealth, to any solvent citizen thereof, whereof the said insured or purchaser for said annuity shall be the beneficiary or annuitant, not exceeding in income or return therefrom one hundred dollars per month, shall be exempt and free and clear from the claims of all his or her creditors and from all legal and judicial processes of execution, attachment or otherwise whatsoever."

While this bill was limited in its scope and moreover did not become a law, yet it is an indication that the impression prevails in the minds of some lawmakers that life insurance policies should be accorded some measure of relief.

Changing the phraseology of this preamble and bill, so that they would more correctly state the conditions and needs, they would read somewhat as follows:

"WHEREAS, it is to the interest of the commonwealth and the vari-

ous counties and municipal divisions thereof to encourage its citizens to reasonably provide for the future support of their families whereby the financial burden of their maintenance by general taxation may be avoided, therefore be it

"*Enacted, Section 1.* All premiums paid on any policy or contract of insurance or annuity heretofore or hereafter taken out or issued by any insurance company, domestic or foreign, lawfully doing business in this commonwealth, to any solvent citizen thereof, shall be exempt from taxation by the commonwealth and the various counties and municipal divisions thereof, and any such policy or contract of insurance or annuity which produces an income not exceeding $100 per month, shall be exempt and free and clear from claims of creditors and from all legal and judicial processes of execution, attachment or otherwise whatsoever."

We cannot but regard it as unfortunate that no tangible results have been secured from the recommendations made in 1908 by the committee of eight members of the Convention of Insurance Commissioners in favor of uniform tax laws. Through the united action of these commissioners uniform laws exist practically throughout the country on all the important functions of life insurance companies; but this one question of taxation never seems to have reached the point of energetic co-operative action. It is a fact worthy of note that of the eight states represented by the commissioners who signed the report only one has made any change in the tax law and in that state two changes have been made and each one was an increase in the existing tax rate.

The situation as presented to the commissioners in the following paragraph could hardly be improved upon and will serve as the foundation at any time for an aggressive movement.

"Life insurance is not a system of old age pensions adopted by the state as a public charity and supported by a burdensome general tax; nor does it contemplate public orphanages, old ladies' homes, or other public charities maintained at the expense of the state. So far as it goes, it is a substitute for all of these and if universal would obviate the necessity of them all. Hence, the state should encourage it, not hinder it; it should help it, not hurt it."

Let us hope that some day a man of strong convictions will plead for the policyholders in the Commissioners Convention with such force as to secure the united action of that important body for the abrogation of all tax laws applying to life insurance or, as an alternative, a nominal tax which shall be uniform in all the states.

The attitude of the states through the officials whose duty it is to interpret the laws, has been, as a general rule, most exacting against the companies. There are some exceptions to this rule, among which may be noted the State of New York. Allusion is here made particularly to the question of taxing, as premium income, the returns or allowances to policyholders in reduction of premiums. The courts however have taken a more liberal view. The judge of the United States District Court of the District of Columbia decided that the credits allowed in reduction of premiums did not constitute income and therefore were not subject to taxation. The judge of the Court of Common Pleas of Dauphin County, Pennsylvaia, has recently rendered a decision in the cases of the Penn Mutual and Metropolitan holding the same view. So far as known no court decision has been rendered to a contrary effect.

In these decisions the courts have brushed aside the technicalities of the forms on which the companies are required to make their annual statement, and have taken the practical common-sense view of the question thus rendering great service to the companies and the public in the matter of tax on life insurance premiums.

Men are resorting to life insurance for the protection of their homes as never before in the history of the world, and the life insurance companies of America take the lead in both the volume of business written and in force. The mutual companies are vying with each other to reduce the cost by the allowance of annual dividends and companies writing non-participating policies fix the premiums at the lowest rates consistent with security.

Although the good work accomplished through life insurance in enabling many thousands to maintain their homes who otherwise would have been in financial distress, is beyond any mere money calculation, its sphere of usefulness would be greatly enlarged if the different commonwealths would co-operate by lessening the burden of taxation and otherwise encourage the work by helpful legislation.

Should we Prepare a New Mortality Table?

BY

HENRY MOIR.

The American Experience Table is now used more than any other for life insurance purposes all over the United States. It was first made public and received official approval as far back as 1867 when the rates of mortality were quoted and recommended for use by the New York Insurance Department. But the statistics on which the Table was based were compiled about and prior to the year 1860 so that the basis for the Table is now more than fifty years old. The general foundation of the Table was the mortality experience of the Mutual Life office, but it was adjusted arbitrarily in some respects by Mr. Sheppard Homans, and the figures at the older ages must have been to a great extent the result of personal judgment and experience on the part of the compiler. He said that he had used for comparison and for guidance other previously existing tables of mortality.

In using an old table like this the most important question to ask ourselves is with reference to its safety. Are the mortality rates adequate and are policyholders fully protected? The latest tables of a similar nature which are authoritative and drawn from the statistics of insured lives are the $O^{M(5)}$ in Britain, and the Medico-Actuarial in America, both excluding the early years of selection. It is desirable therefore to commence our inquiry by placing the mortality rates of these three tables side by side:

It will be seen from the following figures that the American Experience Table makes an adequate allowance for mortality at all ages, even the highest, the rates being much greater than companies are likely to experience at the younger ages as measured by the later publications, and high enough at the older ages. This means that, irrespective of all other considerations, a sufficient provision is made for the payment of death claims as well as maturities of all other kinds which are guaranteed by life companies as the result of calculations of premiums and reserves by that table.

MORTALITY RATES PER 1,000 BY THREE TABLES.

Age.	American Experience.	$O^{M(5)}$	Medico-Actuarial(4).
20	7.80	6.52	—
25	8.06	6.89	4.70
30	8.43	7.47	4.90
35	8.95	8.37	5.10
40	9.79	9.78	5.70
45	11.16	12.00	7.50
50	13.78	15.45	10.60
55	18.57	20.83	15.80
60	26.69	29.21	24.00
65	40.13	42.21	39.00
70	61.99	62.19	61.70
75	94.37	92.67	91.90
80	144.47	138.50	137.20
85	235.55	205.69	203.70

More than three years ago the question of preparing a new table was submitted to a committee of the Council of this Society and the report of that committee is a valuable document, commencing with the statement that "For several years the opinion has been growing that a new mortality table should be prepared, &c.," and ending with the recommendation that "The time is not propitious for undertaking the task, &c." The growth of the same sentiment has continued during three years since that report was rendered; a second suggestion has been received from the National Convention of Insurance Commissioners; and, since the work of the Medico-Actuarial Investigation is now complete, it becomes a question to consider whether the propitious time has now arrived—a matter directly referred to us by our President in his address last October (*T. A. S. A.*, XV, 234–5).

The consensus of opinion in this country is that a general mortality table for use by companies in varying circumstances and in different parts of the country should be based upon an experience which excludes the early years of insurance, since that period is especially affected by the selection to which the lives are subjected. The latest general experience of this class is the Medico-Actuarial Investigation, above mentioned. Such investigation was made with the primal purpose of determining the *extra hazards* incurred in connection with classified diseases, occupations, and conditions of life, including family history. A mortality experience was deduced and rates of mortality were published (Vol. I, p. 89) for the period which excludes the first four years, as well as rates for

the first four years at selected ages. But that experience, like the entire investigation, was based upon policies effected, not upon amounts—it was prepared for comparative purposes only, not for financial transactions. While therefore the results are useful for comparing lives and for determining whether one class of life is subject to higher or lower mortality rates than the average, the statistics are not satisfactory for general use in this country in connection with financial transactions, because the experience in America has been that an investigation based upon sums insured shows a higher mortality than an investigation based upon lives. In other words, those who take policies for large amounts seem to experience higher mortality rates than those who insure for small amounts. Moreover, as insured lives tend to increase their insurance with advancing years, there is a larger average amount of insurance carried on each life at the older ages, than at the younger ages, and this fact is liable to exaggerate the tendencies which will be hereafter discussed (*T. A. S. A.*, XIV, 19). The objections to the Medico-Actuarial Table for use in financial transactions are serious and cannot be ignored.

While fully alive to these objections, I have nevertheless been making calculations on the basis of that table in the hope that they may serve as a guide towards probable results under modern mortality conditions. Although I would not feel justified in forming an opinion on the basis of that table alone, yet when the Medico-Actuarial Table confirms the general trend of events as indicated by other experiences, and as approved by our general knowledge, we can feel reasonably sure of the *direction* in which we are being carried and may prepare ourselves for future developments.

It is often stated that premium rates are too high, that the people are charged too much for their insurance protection. This statement receives superficial support from an examination of the net premium rates in the following:

TABLE OF NET PREMIUMS 3½ PER CENT. BASIS.

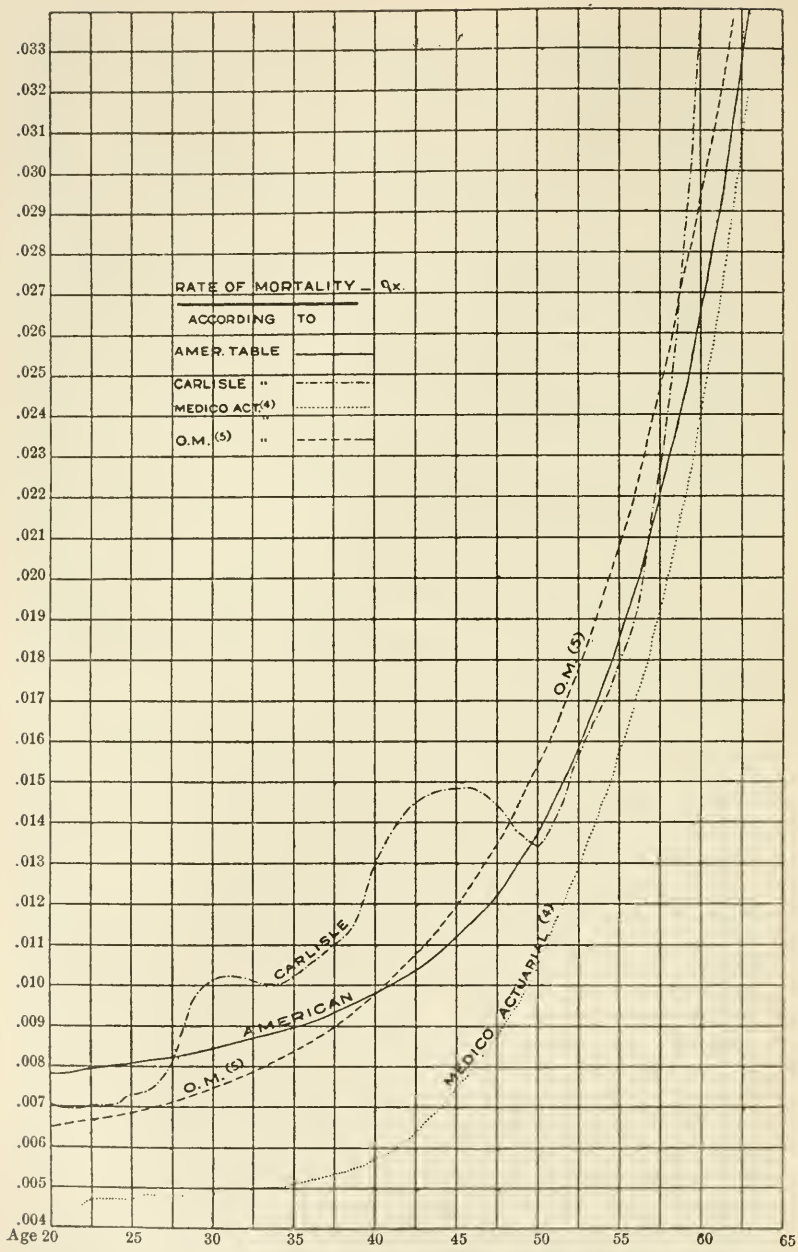
Age.	Ordinary Life.			20 Payment Life.			20 Year Endowment.		
	Am. Exp.	OM(5)	MA(4)	Am. Exp.	OM(5)	MA(4)	Am. Exp.	OM(5)	MA(4)
20	13.48	13.14	10.77	20.72	20.15	17.08	38.90	38.24	36.90
30	17.19	17.35	14.52	24.71	24.77	21.35	39.51	39.24	37.26
40	23.50	24.31	21.11	30.75	31.59	27.98	41.18	41.70	39.00
50	34.99	36.20	33.11	40.82	42.19	38.88	46.46	47.78	44.77

A careless student of life insurance might suppose from the above that policyholders are paying too much for their insurance, but this deduction is not warranted. The above premiums are not the actual contract premiums; they do not represent in any way the cash payments made by policyholders for their insurance. The actual premiums contracted for by policyholders are in nearly all cases higher, as they must include a charge for expenses of management, taxes and other outlays, and although the contract premiums are based upon the American Experience Table, the true premiums paid by policyholders are not so based but are experience premiums. The great bulk of the business of life insurance is on the "participating" form, and a refund is made to each policyholder from the surplus earnings of the company. The net result of this method of insurance is that we get away altogether from the assumed or tabular mortality rates. Each company, as a hard practical fact, finds its surplus larger or smaller according to its own individual experience of mortality, and the payment by a policyholder, as well as the true cost of the insurance in the ultimate analysis, depend upon the surplus distribution based upon developed conditions. The net premiums and reserves by any particular table and interest rate, have only a temporary influence which is ultimately nullified by the actual experience of the company.

It is not necessary to discuss non-participating insurance as a separate question, because the premium rates for that class of policy are determined to a great extent by competitive conditions, and the net result is similar to that attained under participating insurance—namely, that the company's individual experience is the governing principle regulating the rates it can afford to charge.

While, therefore, the American Experience Table is being used as a measure of the company's reserve liability, it is not the basis on which the actual payments by policyholders are controlled, but policyholders in a mutual or participating company receive abatements of premiums which are governed by the mortality experience, interest earnings, &c., of the individual company with which they are associated. Keeping these facts in view, one might be tempted to say that it is of little consequence what mortality table is used as a foundation for premium rates and reserves, but this in my judgment is going to the opposite extreme.

One of the important uses of a standard table is for making reserve calculations which make proper provision for the future and


RATE OF MORTALITY — qx.
ACCORDING TO
AMER. TABLE
CARLISLE "
MEDICO ACT.(4) "
O.M.(5) "
.033
.032
.031
.030
.029
.028
.027
.026
.025
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.019
.018
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.013
.012
.011
.010
.009
.008
.007
.006
.005
.004
Age 20
25
30
35
40
45
50
55
60
65
CARLISLE
AMERICAN
O.M.(5)
MEDICO ACTUARIAL (4)
O.M.(5)

at the same time deal justly with policyholders in the matter of cash and other values allowed to those who terminate their policies. Reserves depend upon the shape of the mortality curve. The mortality curves of the three tables already mentioned are shown in the diagram on page 12; and as a matter of academic interest the old Carlisle Table is added, as it was published about 100 years ago, continued in favor for many years and produced very low reserves as compared with modern requirements.

The important element in reserves is the slope of the curve, and high motality does not necessarily mean high reserves, nor does low mortality result in low reserves. It so happens that the great improvement which has taken place in the mortality of insured lives during the last hundred years has been most noticeable at the younger ages, less apparent in middle life, and scarcely appreciable at all—if indeed there is not some actual deterioration—in old age. Accordingly the effect has been that in spite of the improvement shown by reduced average death rates the mortality curve has been growing steeper and steeper, and each new experience seems to call for higher rather than lower reserves. The strange conclusion therefore is reached that although net premiums have been considerably reduced, reserve values have been noticeably increased. In other words, *out of a smaller net premium a larger amount is in the early years set aside for reserve purposes.* This results from the very low death rates at young ages.

The above reasoning is general, deduced from the diagrams alone; it applies to ordinary life policies. Single premium policies, those by limited payments, as well as endowments, are all subject to other influences as will be readily seen.

It is important to bear in mind that for some policies low mortality necessarily means low reserves; as, for example, paid-up policies, dividend additions, term insurances, etc. These do not constitute any large proportion of the policies of an average company, the older companies having doubtless more business of the paid-up nature than the younger. We may therefore look for another effect of any new table, namely—that the younger companies would be called upon to increase their reserves by a larger percentage than the older companies.

It is well to illustrate such questions by means of actual figures, and this is done in the following tables:

COMPARISON OF RESERVES.

American Experience and Medico-Actuarial Experience 3½ Per Cent.

Ordinary Life Policy.

Year.	Age 25 at Issue.		Age 30 at Issue.		Age 35 at Issue.		Age 40 at Issue.		Age 50 at Issue.	
	Am.	M.A.(4)	Am.	M.A.(4)	Am.	M.A.(4)	Am.	M.A.(4)	Am.	M.A.(4)
1	7.63	*8.18*	9.45	*10.15*	11.76	*12.84*	14.68	*16.26*	22.74	*23.96*
2	15.52	*16.59*	19.22	*20.78*	23.91	*26.19*	29.80	*32.96*	45.87	*48.12*
3	23.70	*25.33*	29.33	*31.75*	36.45	*39.94*	45.39	*50.09*	69.37	*72.48*
5	40.91	*43.77*	50.58	*54.95*	62.73	*68.89*	77.92	*85.36*	117.31	*122.01*
10	89.42	*96.32*	110.14	*120.06*	135.76	*148.37*	166.89	*179.23*	240.96	*248.97*
15	146.54	*158.58*	179.47	*195.17*	219.15	*235.78*	264.62	*279.37*	365.70	*374.18*
20	213.04	*230.40*	258.64	*277.78*	310.75	*329.02*	367.63	*383.57*	485.23	*488.22*
30	372.38	*393.65*	437.28	*457.58*	504.71	*521.33*	571.14	*579.95*	694.75	*679.03*

Percentages of Above Reserves.

Medico-Actuarial to American Experience.					
Year.	Age 25 at Issue.	Age 30 at Issue.	Age 35 at Issue.	Age 40 at Issue.	Age 50 at Issue.
1	107	107	109	111	105
2	107	108	110	111	105
3	107	108	110	110	104
5	107	109	110	110	104
10	108	109	109	107	103
15	108	109	108	106	102
20	108	107	106	104	101
30	106	105	103	102	98

COMPARISON OF RESERVES.

American Experience and Medico-Actuarial Experience 3½ Per Cent.

20-Payment Life.

Year.	Age 25 at Issue.		Age 35 at Issue.		Age 45 at Issue.	
	Am.	M.A.(4)	Am.	M.A.(4)	Am.	M.A.(4)
1	15.37	*15.04*	19.58	*20.11*	25.41	*26.61*
2	31.35	*30.60*	39.90	*40.93*	51.63	*53.90*
3	47.96	*46.72*	60.97	*62.84*	78.67	*81.87*
5	83.17	*81.02*	105.51	*108.07*	135.23	*139.85*
10	184.14	*178.86*	232.19	*235.70*	291.42	*296.78*
15	306.79	*296.54*	384.02	*383.80*	471.96	*475.00*
20	456.00	*436.83*	566.15	*556.25*	688.24	*683.85*

PERCENTAGES OF ABOVE RESERVES.

Medico-Actuarial to American Experience.			
Year.	Age 25 at Issue.	Age 35 at Issue.	Age 45 at Issue.
1	98	103	105
2	98	103	104
3	97	103	104
5	97	102	104
10	97	102	102
15	97	100	101
20	96	98	99

COMPARISON OF RESERVES.

AMERICAN EXPERIENCE AND MEDICO-ACTUARIAL EXPERIENCE 3½ PER CENT.

20-Year Endowments.

Year.	Age 35 at Issue.		Percentage of Medico-Actuarial to American Experience.
	Am.	M.A.(4)	
1	32.86	*34.21*	104
2	67.06	*69.72*	104
3	102.63	*106.58*	104
5	178.18	*184.58*	104
10	395.99	*406.27*	103
15	664.10	*672.61*	101
20	Matures	—	100

The above tables speak for themselves and point to a probable increase in needful reserves which may exceed 5 per cent. in the case of some companies, and a little less for others, depending on the class of business written, the average duration, and also the relationship which any new table might have to the Medico-Actuarial used in this comparison.

If a new table were introduced and applied only to new business, the influence upon companies would depend upon the nature of the new business written, a large increase in reserve being necessary for ordinary life business, a smaller increase for limited payment and endowment policies, and doubtless a decrease in the reserves of short term policies.

I have made some calculations on the assumption that a company writes 30 per cent. of its new business on the ordinary life plan; 45 per cent. on the 20 payment life plan; and 25 per cent. on the 20 year endowment plan, also that the business would lapse and terminate in a normal way. With reference to ages at issue the

assumption is that $1,000,000 of business would be written each year at the ages of 25, 35 and 45. These are rough general assumptions, and the large volume of business at the young age of 25 has the effect of minimizing the reserves in relation to those by the American Experience Table. The results brought out indicate that in the first year the new reserves would be 103.7 per cent. of those by the American Table, and in subsequent years a diminishing percentage as shown in the following table:

RATIO OF RESERVES ON GENERAL BUSINESS AT 3½ PER CENT. INTEREST.

By M.A.[4] *Table to American Experience Table.*

Year of duration 1	103.7
Year of duration 2	103.6
Year of duration 5	103.4
Year of duration 7	103.2
Year of duration 10	102.9
Year of duration 15	102.3
Year of duration 20	101.7

The above table is only illustrative in a very general way because it involves assumptions both as to the classification of business and as to the ages at issue. The effect in some cases might be considerably greater than the above table would indicate, or in other cases might be less.

When public confidence is lost in any of our practices life insurance will immediately suffer. Our business is peculiarly sensitive, and all that we do should be open, above-board, and should invite the most rigid inquiry on the part of the intelligent public. Although we cannot explain our science in its more intricate detail, we should be able to make an intelligent man understand its guiding principles, and should meet and satisfy intelligent suggestions from the outside. There have been prominent statements in the press, made by well known writers in recent years, which indicate a lack of confidence in the American Experience Table. It is our duty to restore confidence in the basis of our calculations or change the basis. The suggestion which has twice been made by insurance commissioners is one which should be carefully heeded as it points in the same direction.

But in addition to this fundamental argument, convincing in itself, we shall doubtless find that the trend will be as above indicated and that the reserves we are now carrying by the American

Experience Table are not too high as newspaper writers indicate, but on the contrary will have to be increased if a new table is generally adopted. It behooves us for the benefit of the business to have premiums and reserves placed for the future on a more accurate basis—more in accord with experience. Moreover, while dividends in general are based upon the individual experience of each company, and the whole amount apportioned is determined as a result of such individual experience, nevertheless the detail of the apportionment at different ages may be modified if there be a change of basis on which premium rates, loadings, and mortality charges are computed. If net premiums do not run regularly, in harmony with experience, there is probably injustice as between one policyholder and another at different ages and under different forms of policies; and while the company on the whole may be charging ample premiums, and may be allowing abatements to its policyholders in dividends which bring the average net cost down to a proper experience basis, nevertheless there may be unfairness, caused by the fact that the mortality curve as experienced now is of a different shape from the mortality curve represented by the American Experience Table.

The foregoing reasons are sufficient in my judgment to make us reach the conclusion that our bases of operation should be periodically reviewed, and modified if found necessary, that one of the reviewing points has been reached, and that in all probability we now need a new mortality table. Admitting such need, what are the requiremeqnts for such new table? Some companies insist that all the business accepted shall be on lives equal to or better than the average; others frankly avow their object to be to do the "greatest good to the greatest number," and with this object adopt a lower standard. Again there is believed to be a marked difference in mortality between such states as Mississippi and Minnesota. A general mortality table should be broad enough to make provision for all such conditions. The basis for any new table, therefore, becomes a matter for careful discussion on the part of a committee of actuaries, yet individual views may not be amiss, and I shall express my own ideas briefly, leaving others to fill in reasons or to raise objections. Our needs may be expressed as:

1. A table prepared on broad liberal lines which may be used from Cape Cod to the Golden Gate, from the Canadian Border to the Gulf of Mexico.

2. A table which will be *safe* in computing premiums, not only for companies which are strict in selection of risks and companies in the East and healthy Middle West, but for all sections of the country; also safe for companies inexperienced in the selection of risks or who take the broad view that the average active business man is entitled to insurance, although not a first-class risk.
3. A table based upon sums insured and giving due effect to the influence of the amount of insurance carried at different ages.
4. A table whose curve of mortality follows the modern line of experience, so that premium rates, reserves, surrender values, and surplus distribution may all be computed in a manner fair and just to policyholders of all classes, of all ages, and of all durations; conditions which do not completely hold at the present time.

Starting out with these requirements, the next and final points to consider relate to the manner of attaining such results:

1. By giving due weight to different sections of the country and providing that the healthy sections shall not overbalance those which are less healthy, also that business strictly selected shall not have undue weight as measured by that secured under more lax conditions.
2. By excluding the early years of selection from the investigation.
3. By tracing the mortality experience only from 1900 to 1915, bringing forward a sufficiency of old risks so that a proper basis of experience may be secured at the older ages.
4. By following liberal lines in the matter of occupation and hazardous risks, allowing for the tendency of modern practice to broaden the basis of acceptance without rating. In this direction indeed we should strive to get ahead of practice, rather than follow after.

In securing broad general results as above indicated there is not in my judgment any need for an immensely large experience. We want rather a broad experience representative of the country as a whole, and in the writer's judgment 100,000 at risk at each age is ample for practical purposes, properly distributed, as regards selec-

tion, locality, etc. It is important to secure an adequate representation from the South as well as from the East and West, also that a sufficient number of the older lives be introduced into the investigation so that the experience at ages above 60 may be trustworthy and reliable.

There is another point already mentioned which will bear repetition and emphasis, namely that contracts now in force on the American Experience basis need not, and should not be disturbed; but should be left to work themselves out as they will do, fairly and equitably on the average, according to the experience of companies.

Abstract of the Discussion.

Mr. Arthur Hunter: As one of the Committee of Seven who were asked to make a report to the Council on the advisability of the Society's undertaking to prepare a new mortality table or tables, I should like to place before you my reasons for being in favor of preparing the plans for such tables as soon as convenient.

The American Table was based on a comparatively few statistics, was practically the experience of a single company, and is over 50 years old. It is considered an ultimate table of mortality, but does not reflect the present experience of American companies, excluding the first five policy years. I doubt whether it represents the present "ultimate" experience if a varying number of policy years were excluded, depending on age at entry. As an actuary, I feel that we are in a weak scientific position in having to defend reserves based on a table which is so old and which does not show our present mortality. It is probable that under the level premium method of valuation, larger aggregate reserves would be required on an ultimate mortality table based on recent experience; but that under the preliminary term methods a lower reserve in the early policy years may be expected. If distinctly larger reserves are required, it is not likely that the superintendents of insurance would recommend any hasty action with regard to making the new tables the reserve basis for their states. There are other difficulties which would arise on account of the likelihood that the net premiums would be lower than those on the American Table. It is held by some persons that this would result in a demand by the public for lower premiums, but is not the gross premium largely the result of competition in the "stock" companies, and is not the gross premium the nominal premium in companies issuing annual dividend policies? Again, it has been mentioned that in one state a maximum loading is provided, and if the new table were adopted by that state it would automatically call for lower gross premiums. I do not believe that the state officials would take any arbitrary action in the matter. It is also urged that if the new table were adopted in the State of New York, it would result in lower margins for expenses; but I believe that the state officials would assist in making the necessary adjustments so as not to injure the companies. There are other difficulties which I can foresee for the officials of the various insurance companies and for the insurance departments of the various states, but I do not know any reason why they cannot be successfully met.

The apparent public demand for new mortality tables had most influence with me in determining my attitude. On three occasions, commencing with 1912, the Convention of Insurance Commissioners have asked this Society to prepare new mortality tables, and these

commissioners represent the public at large. In the insurance press and in correspondence with policyholders, we continually see references to the advisability of preparing new mortality tables.

The Committee has carefully avoided reference to the various technical problems which would arise in connection with the preparation of a new mortality table. The first draft of the Committee's report drew attention to more than a dozen of such problems. For example, should provision be made for separating the data for the United States and Canada, and should the Committee be put in the position of preparing separate tables for different sections of these countries? Should tables be based on the experience among men and women combined? Should the experience under deferred dividend policies be excluded in view of the fact that the greater part of the business in this country is now being written on the annual dividend plan? It seemed to the Committee that it was not advisable to raise such questions at the present time because any committee appointed by the Actuarial Society would be competent to handle such matters and would not take any positive steps before consulting the representatives of the companies, and before the plans had been submitted to the Council.

I believe that the time is ripe to prepare plans for the new mortality experience. Before the tentative plans can be laid before the life insurance companies, the insurance commissioners, and the Actuarial Society, and before the detailed plans for collecting the data can be sent to the insurance companies, a year will probably elapse. It will then be necessary to wait until the companies can furnish the requisite statistics from their books, and this will involve the engaging of a special force for several months. The data from the various companies must thereafter be combined according to the classes or groups selected (habitat, plan of insurance, sex, etc.), the mortality experience tabulated, and the proper deductions made. If the work is done carefully and thoroughly, I question whether the new mortality tables would be available for at least two years, and it would take some time longer before reserve tables based thereon could be furnished. It does not follow that because new mortality tables are prepared that such tables will become the bases for net premiums and reserves in the United States or Canada.

In conclusion, I should like to say that we need more light on this subject of the mortality of the companies as a whole, and it is the duty of this Society to furnish such information to the public. As a Society we have never been afraid of more light on any subject, even though it added materially to our difficulties.

Mr. John K. Gore: I have an opinion, Mr. President. It seems to me however, that the logical order would be for those who are opposed to the preparation of the new table to present their arguments first. As the case stands, we have the paper of Mr. Moir, and

if no one says any thing in opposition, the case will rest. Mr. Moir's paper is before us for discussion, and naturally the next step would be for those who believe Mr. Moir was not right in favoring the new table, to say so, and tell us why they believe he was not right.

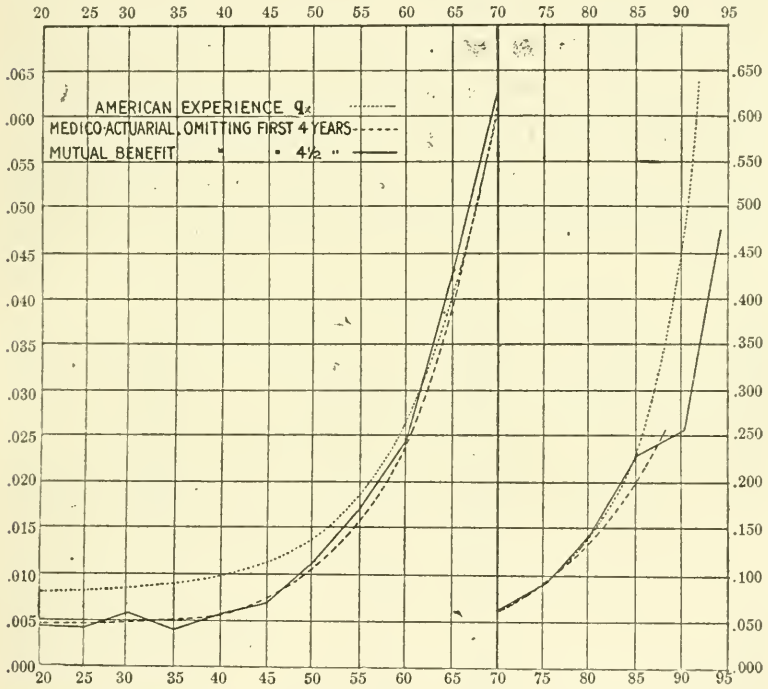
As long as I am on my feet, and you have called upon me, I should like to say that, as an actuarial society, as the oldest and largest actuarial body in this country, in view of the fact that there has been no mortality experience made up by the companies as a class during the last fifty years, and in view of the fact that we all know, from such data as have been given to us, that there has been in many respects a decided decrease in the death rate—I say in view of those facts, even though the insurance commissioners had not brought the matter up, the duty of this Society is to get together the data and to prepare a mortality table. There may be fears expressed as to what may happen if we have a new mortality table. Surely we all want to get at the truth, and, as a scientific body, knowing as we do that there have been important changes in the rate—if, as a scientific body, we simply hold off and say that some actuary of some company somewhere in the country might be deceived by the new table, and might make some mistake that would be fatal for his company—that is not the position for us, as a scientific body of men. We want the truth. Fifty or sixty years is a long enough period for any country to go without having a new mortality table for insured lives.

Mr. Edward E. Rhodes: The title of Mr. Moir's paper is "Should We Prepare a New Mortality Table," and I propose to confine myself to that question rather than to the question of whether we shall prepare a new mortality table which I expect will be answered in the affirmative.

I have no objection to the preparation of a new table based on homogeneous statistics which will show the mortality of insured lives at the present time. There is no disposition to shut out light. We are constantly sifting and analyzing our experience, comparing it with the fundamental bases of the business.

After years of careful and painstaking study I am firmly of the opinion that we do not need a new mortality table for premiums and reserves. The American Table has gradually displaced the tables which were formerly used and it has become established solely on its merits. Mr. Moir has prepared a diagram showing the mortality curves of the American and Medico-Actuarial (4) Tables, for ages 20 to 65 inclusive. I would like to present a diagram showing these mortality curves and also an ungraduated curve, showing the Mutual Benefit's experience during the years 1900–1909, inclusive, on all its outstanding business, excluding the first four and one-half policy years, for attained ages 20 to 94 inclusive. Up to age 45, the Mutual Benefit curve follows very closely that of the Medico-Actuarial Table. From age 45 to age 60, the Mutual Benefit curve

is above that of the Medico-Actuarial. Between ages 60 to 70, the Mutual Benefit curve is above that of both the American and Medico-Actuarial Tables. Between ages 70 and 85, it follows very closely that of the American Table but is above that of the Medico-Actuarial Table. For ages above 85, the mortality rates of the Mutual Benefit would agree more closely with those of the Medico-Actuarial Table, if the latter were extended. Except at the younger ages and the extremely old ages, therefore, the mortality rates of the Mutual Benefit are more closely in accord with those of the American Table than they are with those of the Medico-Actuarial. Eliminat-



ing the question of selection, which has very little bearing if any, on the above comparisons, it seems very clear that if the experience of the Mutual Benefit can be looked upon as standard, there is no need of a new mortality table as a correct measure of present day experience, except possibly for the young ages. I will refer to this later. Mr. Moir, however, bases his conclusion that a new mortality table should be prepared on different grounds, and I desire to review these briefly.

Mr. Moir says, very correctly, that one of the important uses of a standard table is for making reserve calculations which make proper provision for the future and at the same time deal justly with

policyholders in the matter of cash and other values allowed to those who terminate their policies. The severest critic of the business has not claimed that reserves were too low, but that they were too high, ignorantly assuming that lower death rates necessarily mean lower reserves. As a standard of solvency the American Table has been well nigh ideal. It has produced sufficient but not exorbitant reserves. There is nothing in the experience of the business which calls for higher reserves such as will undoubtedly follow the adoption of a new table which approximates the Medico-Actuarial Table. Higher reserve requirements will certainly add to the difficulties of younger companies. The old and established companies with a large proportion of outstanding business can meet practically any reserve requirements, but this is not true of the newer companies, for whom legislatures have established modified reserve requirements. With a new table the select and ultimate method would probably not be available judging from the Medico-Actuarial Table.

The following table compares the mean reserve for ordinary life policies of $1,000 according to the Medico-Actuarial Table and 3 per cent. interest on the select, the ultimate, and the select and ultimate basis, and according to the American Table with 3 per cent. interest.

ORDINARY LIFE MEAN RESERVES, 3 PER CENT.

Age.	Yr.	Basis.	Med.-Act.	Amer.	Age.	Yr.	Med.-Act.	Amer.
25	1	Select.	11.95		35	1	16.87	
		Ultimate.	11.36	12.36			16.35	16.98
		S. and U.	10.46	4.51			15.50	8.26
	2	S.	22.00			2	31.81	
		U.	20.71	21.09			30.67	30.05
		S. and U.	20.53	16.39			30.46	24.80
	3	S.	31.75			3	46.64	
		U.	30.34	30.10			45.38	43.49
		S. and U.	30.30	27.63			45.32	40.72
	4	S.	41.75			4	61.82	
		U.	40.31	39.38			60.52	57.31
		S. and U.	40.31	38.44			60.52	56.23
45	1	S.	25.17		55	1	38.24	
		U.	24.34	24.64			36.81	37.21
		S. and U.	23.08	13.65			33.77	18.62
	2	S.	47.39			2	70.24	
		U.	45.58	44.47			66.69	66.16
		S. and U.	45.35	37.79			65.92	54.66
	3	S.	69.11			3	100.81	
		U.	67.17	64.72			96.72	95.27
		S. and U.	67.11	61.17			96.13	89.07
	4	S.	91.01			4	130.88	
		U.	89.07	85.39			126.85	124.48
		S. and U.	89.07	83.99			126.85	122.01

The calculations were made upon a 3 per cent. basis before it was known that Mr. Moir was preparing his paper. It will be noticed that the Medico-Actuarial select and ultimate reserves afford practically no relief.

It has not been shown that the surrender values now generally used deal unjustly with policyholders. If the contrary were true, it is not clear that surrender values based on a new table would operate more justly. This is a question for each company to determine for itself according to its own experience. Dealing, as life insurance companies do, with many thousand groups of policies, exact justice cannot be rendered to each individual group without an expenditure which, if it were not prohibitive, would be out of all comparison with the benefits gained therefrom. Substantial justice is all that is required and all that is possible.

The difference in the mortality actually experienced at the younger ages and that expected by the American Table, and the lack of such difference at the older ages, is easily adjusted in the dividends. My experience has convinced me that the American Table readily lends itself to an equitable apportionment of mortality gains, and that there is no need of a new table on account of the difference referred to.

It has been unthinkingly assumed that the adoption of a new table would lead to a reduction in the cost of insurance. This of course can only be accomplished by a reduction in the actual death rate, or in expenses, or by an increase in the returns from invested funds. As to the gross premium rates which would be adopted by the companies if a new mortality table were prepared, it must be remembered that in determining the present scale of premium loading the companies have taken into account the probable mortality margins arising from the use of the American Table. If, as may be expected, similar mortality margins are not available under a new table, the companies may deem it prudent to have approximately the same total margin as they have at the present time. It is unlikely that the adoption of a new table will effect any material change in the gross premiums unless it shall appear that the total margin has been excessive. It will not be forgotten that a mutual company has no means of recouping untoward losses except by its redundant premiums.

The adoption of a new table might do more than affect premiums and reserves. It is not at all improbable that it would be followed by a stricter standard of selection, and some people who are safely acceptable under the present table would be deprived of insurance protection.

I, of course, agree with Mr. Moir that contracts issued under the American Table and now in force, will work themselves out fairly and equitably, but I think that his position here is illogical with that taken by him elsewhere in his paper. If outstanding business will work out fairly and equitably, new business will also. I do not

believe that public confidence in the American Table is lost. There is undoubtedly a misconception in some minds regarding the functions of that table and of the results obtained by its use. This misconception will not be remedied by discarding the table and preparing another. My fear is that the announcement that such a table is to be prepared will result in much more serious misunderstandings than those which now prevail, and that it will lead to expectations which will be impossible of fulfillment.

I shall not attempt at this time to review Mr. Moir's ideas of the questions that must be considered in the preparation of a new table. In addition to those named by him, there will be others which will arise, and all of these questions can be best considered by the committee which will have charge of the work. If the Society undertakes the work, the experience of the Mutual Benefit will be placed freely at its disposal.

It is unfortunate that the agitation for a new table should arise at this time. The mortality of the recent past has probably been affected to some extent by deaths in battle and by disease due to the Spanish-American War, and also by the abnormal voluntary cessations following the insurance investigations, and by the recent and present periods of financial depression. The first cause increased the death rate. The second may have decreased it. We must also consider probable changes in the standards of selection which will follow the publication of the results of the Medico-Actuarial Investigation. Experience only can show whether a table prepared now will indicate the mortality likely to be experienced in the future. There are enough disturbing causes to make such a result doubtful.

Mr. William A. Hutcheson (called on by the President): I did not intend to discuss Mr. Moir's paper, but as I have been called upon I should like to speak on the general question of the effect of preparing a new mortality table.

In the first place, there is little use in preparing a new mortality table from our experience unless this new table is ging to be a correct guide to the future. Now the question arises, is the experience of the companies in the past going to be a good guide to the mortality experience amongst insured lives in the future? Upon the passage of the so-called Armstrong laws in New York in 1906 a great change took place in conducting the life-insurance business. First year's expenses were greatly reduced, the standard of selection was raised, and the business written by many of the companies since then has shown a lower mortality rate than the business issued before that time. The experience of these companies would form a considerable portion of the aggregate experience, and the question therefore arises as to whether or not a sufficient time has elapsed since 1906 to give experience enough to form a useful new mortality table.

If we decide to prepare a new mortality table, the next step will be its applicability as the basis for premiums, reserves and provisions for expenses.

The question of net premiums and reserves has already been mentioned. The net premiums by any new table based on the mortality experience of recent years would doubtless be smaller than those of the American Table, especially for ordinary life policies issued at the younger ages, and the reserves would doubtless be greater than those the laws now require us to hold.

The point to which I wish to refer is the effect of the adoption of a new mortality table on the legal provisions regarding the amount which may be expended in procuring new business. The New York law limits this amount to the loadings and the present value of the assumed mortality gains, assuming that the gains will be 50 per cent. of the cost of insurance by the American Table in the first year, 35 per cent. in the second year, and so on. These assumptions as to mortality are by no means excessive, and the amounts which may be expended in getting new business are reasonable. The companies regulated their expenses and adjusted their agency affairs to meet the requirements of the present New York law with excellent results to the policyholder.

Assuming for the moment that the Medico-Actuarial Mortality Table would represent approximately the relative select as well as ultimate mortality derived from the contemplated investigation, let us see how the use of this table would affect the "assumed mortality gains" on this basis. The rate of mortality in the first year by the Medico-Actuarial Table is between 75 per cent. and 80 per cent. of the ultimate rate, and we would not be justified in assuming that the mortality gains during the first few years would be greater than actually shown by the table. I had the present value of these gains, on a 3½ per cent. basis, calculated, and they are as follows:

ORDINARY LIFE PER $1,000.

Present Value of Mortality Gains.

Ages.	Medico-Actuarial.	Select and Ultimate.
25	$1.45	$ 9.61
35	1.36	10.67
45	3.00	13.39
55	4.58	22.49

The assumed mortality gains on the present "select and ultimate" American basis are also given, and from a comparison it will be seen that unless the loadings are very much increased above the present loadings, the expense provision for new business on the existing basis—loadings plus assumed mortality gains—would be altogether insufficient.

While I shall be greatly surprised if the Medico-Actuarial Table correctly represents the relative select and ultimate mortality rates which would be derived from a new investigation if it be carried out a few years hence when a sufficient time has elapsed since 1906, I wish to call attention to the present values of the assumed mor-

tality gains on the bases mentioned in case the Medico-Actuarial Table turns out to be a fair indication of what the new table will be.

This is a practical question which should be very seriously considered at this time, for if a new table is prepared, it will doubtless be adopted in time as a standard for premiums and reserves, and it may also be used as a basis for legislation in connection with the amount which may be expended in procuring new business.

Mr. Archibald A. Welch: I have never enjoyed a fight as much as I have that waged over this subject in the special committee that you appointed, because every point has been contested by the two sides and with the greatest good humor. At the end I think we are all of one mind on the principal point at issue, namely; the necessity for a table at some time and that the Society should have the direction of forming it; the only question on which we did not agree was the time when we should undertake the work.

Personally I believe, as does Mr. Gore, that all the extraneous matters, the difficulties, and dangers—and there will be many—that will result from the preparation of a new mortality table we can eliminate from the discussion. It is the truth that we want, and as scientific men if we are not able to see the truth and use it to our advantage, we do not deserve our membership in this Society. The only question is, when is the time, and the proper time, to find the truth in regard to the mortality that these companies are now experiencing and to form a table that will be of value for a longer period than five or six years?

When I consider that a new mortality table will without doubt require increased reserves, I am forced to consider what increased reserves mean, for the higher the reserve required the more difficult the conduct of the business, the more difficult the starting of a new company, the more difficult the equitable apportionment of surplus in a mutual company.

I feel, that there exists no reason at the present time why we should have a new table, for if one were today before us which represented the mortality of the companies at the present time, it would never be used as a standard for reserves. What has been will be, and there is no company that is twenty years of age at the present time but what has passed through some trial that has resulted for a time in a mortality much higher than that which is being experienced today. If the mortality table now in use represents very closely the rate of mortality which may be expected on business fifteen or twenty years old in our present companies, a table that will show a mortality much lower than that will be a dangerous one to adopt, unless the companies can be guaranteed a successful management and a free working of all the laws of mortality and interest, such as we have experienced during the past ten years.

But the question comes back to the one propounded by Mr. Gore: As a scientific body are we ready to get the truth? It seems to me

that the truth could not be gotten at the present time. Ten years ago (1905) was the date which marked the change in the underwriting in this country, a change so marked that it will be seen in the morality, if any of you carry out the experience of your company, dividing the business prior to 1905 from that issued subsequent to that date. It was a change in the underwriting at the home office as well as a change in the underwriting conducted by the field. If, then, we are to include in the experience from which the new table is formed business prior to 1905, this new table will show a mortality which will never be experienced again by a like body of insurants fifteeen or twenty years old. It will be impossible to confine our experience to the business written since 1905 for reasons only too obvious; and if we include the business written prior to that date our new table will be in need of correction within a very few years, if it is simply the truth that we are after. In other words, the cost of a new table at the present time will have been thrown away.

For all these reasons it seems to me that we should postpone the date for making up our new table. If, however, the majority of this Society feel that the time is now ripe for the formation of a new table, and others who have a right to advise us in regard to the business request that a new table be now made, the little that I can do to further this end will be gladly done. I fear no results from the table except a mass of work and expense that will result in little good, and I can see no advantage from the adoption of such a new table except to the larger companies. The stronger companies could easily adopt it; the weaker ones would have much more trouble.

It is for these reasons, that I cannot see any good from the formation of a new mortality table at the present time. The rate of mortality seen in the M.A. Table just adopted gives us all the truth that can be secured at the present time.

Mr. Miles M. Dawson: The first and most obvious remark concerning the proposal to prepare a new mortality table is: Cui bono?

Most of those who propose it suppose that it will cause life insurance to be supplied at lower cost to policyholders.

But will it do so? And, indeed, how can it do so? The cost to policyholders is, in reality, determined by the net actual cost to mutual companies, i. e., the actual mortality as well as the actual expenses and actual interest returns. This is plainly true as regards holders of participating policies whose dividends reduce the cost to them to the actual cost to the company. It is also true as to non-participating policies, the rates for which are determined primarily by the competition of the participating policies, "dividends off." It is true that an extra reserve is charged if the gross premium is less than the net premium by the reserve standard, to cover the deficiency in premium, but this net premium is lower than the non-participating premium of any company at present— though this has

not always been true—and the extra reserve would not be prohibitory for most of the established companies.

It is because a new mortality table will not *per se* yield lower cost to policyholders that many actuaries, among them the most distinguished and highly honored, have been disposed to oppose the introduction of a new table.

For more than a decade I have personally favored it, as a means of shattering illusions concerning the true nature of premiums, loading, etc., in the minds not only of policyholders and of many managers of life companies but even of not a few actuaries.

The nature of these illusions will presently be discussed. There is, however, one thing— and perhaps one only—by reason of which the adoption of a less redundant mortality table as a basis for rates will tend to reduce the cost of insurance to policyholders. It is this: commissions, both first year and renewal, are paid upon the full premium, for instance, say 55 per cent. the first year upon the participating premium of an ordinary life policy and $7\frac{1}{2}$ per cent. thereafter for nine years. At the usual excess of full-participating life premiums over non-participating at the same ages, i. e., fully 25 per cent., this means an excess of expense, expressed in percentage of the non-participating premium, as follows:

$$(.55 + 9 \times .075)1.25 - (.50 + 9 \times .075)1.00$$
$$= (.55 + .675)1.25 - (.50 + .675)$$
$$= (1.225)1.25 - 1.175 = 1.531 - 1.175 = .366.$$

As this is during 10 years, it averages .0366 of the non-participating premium per annum during that period, which is a very considerable handicap—quite enough to pay a good profit to stockholders.

Now if these participating rates were reduced, say 20 per cent., it is true that the cost as measured by mortality, interest and management expense would be the same, and if these were all that enter into the composition of dividends, the cost "less dividend" to policyholders would be the same. But these are not the sole elements of cost; instead acquisition and renewal provisions and expenses are also components. Therefore if gross premiums were reduced, at least a corresponding reduction in these costs would be realized, i. e., in case the business can be had at the lower cost.

An objection to this course has been that *percentage of dividends*, regardless of net cost after deducting dividends, has counted in competition, and, therefore, to collect a larger premium and return something over 20 per cent. brought greater prestige than to reduce 20 per cent. and pay a small dividend, even though the latter yielded a lower net cost to the policyholder. But that was one of the delusions, already happily dispelled by publication of facts concerning net costs, after deducting dividends.

Another objection has been that premiums so reduced do not yield a loading over the net premiums, sufficient to provide for expenses.

This also is enforced by the limitation of total expenses of a participating company to aggregate loadings, plus the select and ultimate mortality margins, by the laws of New York.

But that the true loading is the excess of the gross premium over the net premium by the American Experience Table, is precisely the delusion which the preparation of a new mortality table should shatter. The true loading is, instead, the excess of the gross premium over the net premium (suitably adjusted to accommodate the incidence of expenses) computed by a mortality table fairly conforming to actual experience.

The provisions of the New York law really require the consecration of a part of this true loading to surplus and, by indirection, to dividends. The same idea, in a direct form, was embraced in principle by the one section of the bill proposed by the Armstrong Committee, which, on account of failure of actuaries to appreciate its merit and value, was withdrawn, viz.: the proposed limitation of expense loading which expressly excepted loading for dividends.

In Great Britain, where premiums have for several years past been computed upon mortality assumptions pretty fairly conforming to the actual mortality, the formula for a participating premium is on the basis of assumption regarding interest and expense, also approximating the actual, and likewise such as provides for the desired rate of "bonus" or dividend, all the assumptions being conservative but none heavily "redundant."

May we not expect this also to result from our preparing a new mortality table representing average experience and not heavily redundant?

Something akin to this I have already made use of in arranging dividend scales, using either a special mortality table arrived at by suitably modifying the rates of the American Experience Table, or the M.A. Select Table. The necessary gross premium, taking interest at the rate used in valuation, was computed by a formula, like this:

$$P'_x = \frac{(F + \phi a_{[x]\overline{9}|} + f\mathrm{a}_{[x]})P'_x + A_{[x]}}{\mathrm{a}_{[x]}},$$

in which $a_{[x]}$, $\mathrm{A}_{[x]}$ and $\mathrm{a}_{[x]}$ are all taken according to the table assumed to represent actual experience, $F =$ rate of initial expense, $\phi =$ rate of renewal commissions and $f =$ rate of expense of management, all on a basis approximating the actual. Then $P'_x - P_x$ is a constant factor of the dividend, to be increased by excess interest on the reserve.

My friend, Mr. Philip Burnet, president of the Continental Life Insurance Company, suggests the short-cut of making this computation by the American Table, adding $3.00 per $1,000 of insurance to the constant, to cover saving in mortality on the basis of the M.A. Select Table, and correcting for having allowed this saving upon an amount equal to the reserve, by reducing the interest factor 3/10 of 1 per cent. This is very neat and convenient.

Suitable alterations of the formula to accommodate premiums for other forms of policies will suggest themselves.

That the new mortality table, when prepared, shall not be a perilous one for such uses, it will be necessary that it be not so low that it will not provide for the death-claims of every reasonably well-managed company.

It may be objected that this would obviate preparing a table, since not many years ago three New York companies were reporting a mortality experience of 100 per cent. of the expected by the American Experience Table. True, and also true that there could be found companies nowadays whose plight is no better; but that is a matter to be dealt with in connection with the reserve question. As regards computation of premium rates, except for reserve purposes, it scarcely need arise since such anomalies are mere instances of poor selection of lives or of merited adverse selection.

Yet even for rate computation surely a mortality table which exhibits death rates fully equal to what the company should reasonably expect, should be employed. Moreover, unless there is to be specialization, a number of tables being employed, respectively suited for valuing lives exposed to special risks of locality, environment, etc., it must be a table reasonably and conservatively representative of general mortality experience throughout the country.

This could conceivably be secured by either of two methods, viz.:

1. Selection of material, using data supplied by some of the older companies only—as, for instance, those which contributed their data for the Medico-Actuarial Investigation—and combining, as the judgment of the actuaries in charge determines, certain portions of the data to form the new table.

2. Inclusion of the experience of all the companies East, West and South, which have had a considerable number of lives insured, so as to secure virtually an aggregate of the insurance company experience of the entire country, thus assuring an average.

The former could, perhaps, be done with approximate accuracy, sufficient for all practical purposes; and the processes could be set forth, with such amplitude of facts that all competent persons could assure themselves of the fairness of the work and probable correctness of the result.

But it would never be understood by the public including many insurance men, state commissioners and others; and, if its use involved annoying alterations of method, the table would surely be viewed with suspicion and the good faith of its framers would be questioned.

On the other hand, if the experience upon which it is founded is inclusive, that will be conclusive as to its representative character and a mere comparison of the graduated and ungraduated death rates, not to speak of a comparison of the actual deaths with the expected, computed by applying table death rates to the original exposures, will silence all cavil.

This is important also because it would enlist the co-operation of scores of other companies which would be affected by this table and which will, most naturally, resent any inconvenience far more if they have no part in it and perhaps on that very account will refuse to tolerate it.

When such a table has been prepared, should it be used as the standard for reserves?

Undoubtedly that is the expectation of many, perhaps of most; and, along with it, is the expectation that it will result in reduced rates and, indeed, in reduced costs to policyholders. It has already been shown that, except as regards a possible saving in commission costs, the latter cannot result from it. If employed in valuation, how would rates and costs to policyholders be affected, otherwise?

As applied to whole life policies, a mortality table, exhibiting lower rates at the younger and middle ages, involves higher reserves. The reserve formula is:

$$A_{x+n} - P_x a_{x+n}.$$

As n increases, the values A_{x+n} and a_{x+n} approach the same values when computed by the table exhibiting higher rates at the younger and middle ages, but P_x is constant at a materially lower figure than when computed by the latter table. Therefore a larger reserve must be accumulated.

But if a larger reserve is required, the current net cost to policyholders, dividends deducted, must be increased by so much in the early years of the insurance especially. In other words, so far as this is effective, the adoption of the new table will increase current cost to policyholders instead of diminishing it. To cover this, premiums upon some forms of non-participating premiums will also be increased. Such a result would undoubtedly foster popular suspicion of the good faith of those in charge, if the table is in any degree based on "judgment"; and would in any case render its use unpopular.

All of which indicates the question, whether or not the new table should be used for reserves or at least as a compulsory standard for valuation, generally or perhaps universally applicable.

If used for reserves, the table must exhibit rates of mortality sufficient to cover the highest experience that is reasonably expected to be encountered, so that solvency may thereby be assured. Solvency is not assured necessarily by high reserves; it may even be endangered by requiring them. It is assured by sufficiency of premiums and reserves, taken together.

Assuming that the net premium method of valuation—by which is merely meant not "gross premium" and is not meant "net level premium"—will be continued, whether upon an ultimate or a select and ultimate basis, the worth of a sufficient and even a redundant table of mortality is in *the emphasis upon the sufficiency of the net premium* to be valued. It is that, accompanied by a reserve big

enough to supplement it, which has made and kept our companies really, and not merely theoretically, solvent.

Valuation takes into account merely guarantees and not expectations in regard to dividend returns. Therefore, although the premium were in fact constructed, using factors (including mortality table) approximating actual experience, and by the formula:

$$P'_x = \frac{(F + \phi a_{[x]\overline{9}|} + f\mathrm{a}_{[x]})P'_x + A_{[x]}}{\mathrm{a}_{[x]}}$$

as suggested, the formula for valuation would be:

$$_nV_x = A_x - P_x\mathrm{a}_{x+n},$$

$$_nV'_x = A_{[x]} - P_x\mathrm{a}_{[x]+n}$$

or

$$_nV = A_{x+n} - P_{x+1}\mathrm{a}_{x+n},$$

according as net level premium, select and ultimate or preliminary term is desired, computing P_x (or P_{x+1}) by the ultimate table and paying no further attention to the expense or dividend matter.

In other words, in effect this is a limitation of the minimum gross premium to P_x or P_{x+1} computed by the valuation table or the ultimate section of it. That this is true, is indicated further by the fact that, if the gross premium is smaller than this valuation premium, an extra reserve will be charged, equal to the present value of the deficiencies in premiums as at present.

The usefulness of our present standards of valuation, then, in preserving solvency has not been that the reserves brought out were very high, but that the valuation premiums were sufficient and, in consequence, that non-participating insurance was not offered at impossible and ruinous premiums.

If, then, the use of the new table were confined to computation of gross or office premiums, the American Experience Table being retained for valuation, this would be to limit, in effect, the minimum *gross* non-participating premium to the net valuation premium, computed by the American Experience Table. That seems in many ways most desirable, in order really to assure solvency by the use of a generally applicable valuation table; for, valuing similarly on a net basis by a true average experience table, the valuation premium would be too small for half the companies—for that is the meaning of "average"—and, if their gross premiums were too close to that delusive "minimum," they would fail, despite the apparent solvency.

But, if it is desirable to continue the American Experience Table as the standard for valuation, it is proper to inquire: Why a new table at all? And why will not the M.A. Table answer?

A new table is desirable in order to dispel illusions, to induce closer and more scientific computation of premiums and provision for dividends and generally to encourage more prudent and accurate

conduct of the business, including *voluntary* maintenance of higher reserves in order to assure the desired financial results—as in Europe special reserves are accumulated to maintain bonus or dividend scales.

The M.A. Table will not do, because its origin is too mysterious or at least will surely be so to the uninformed. It will not command confidence.

A new table, if useful for the purposes named, will on that very account be unsuitable to displace a "safe" table, proven so by long experience, to determine minimum premiums and a minimum standard of solvency.

Mr. Thomas B. Macaulay: I think in order to get a proper understanding of this question, we should ask ourselves just what are the objects to be gained by this new table. Do we wish more information, which we can use in the conduct of our business? For this purpose, such a new table would be quite useless. It would not answer any of the questions which we as managers have to face. The Medico-Actuarial Tables were of immense value to us; they did answer hundreds of questions. But this new table would be an aggregate compilation, while the practical questions that arise for us relate to isolated cases, or to special groups. What benefit then could we obtain? We simply get a new standard for the calculation of premiums and reserves; a standard which we are told could be used from Canada in the north to Mexico in the south; a national standard—if you will, an international standard. Would this proposed table be really suitable for even that purpose? It is assumed that it will give us the truth, as it is called; in other words, that by this table, we will know what mortality we can reasonably expect hereafter for all parts of the country and for all companies in the country.

Now, will a table of this kind be suitable for this purpose? Bear in mind, as I have said, that what is proposed is an aggregate table. In other words, it is an average table. Now, an average is a mean between things that are higher and things that are lower. We all know that most companies that are prosperous have had, and no doubt will have a very favorable mortality; a mortality for the individual company or companies lower than this average brought out by the aggregate table. But remember this, that for every company in that experience which has had a lower experience than the average, there must be another company of equal size with an experience correspondingly in excess. Therefore an aggregate table of mortality would not and could not represent the mortality of *each* of the companies in that experience, but merely the average of the whole of them. A successful company will no doubt have a mortality well within that based on such an aggregate table, and could therefore safely use it, but how about one that is in difficulties, and has had a heavy lapse ratio?

Then again, a company whose business is confined to the North will have a better mortality than that of the aggregate table, but, gentlemen, that means that just to the extent that the average table is lowered by the Northern business included in it, there is Southern business that is higher than the average. We are told that this is to be a table for use throughout the country, from North to South. Is a table which contains much Northern business suitable for a company in the South? If the Northern statistics were left out and the Southern alone taken, the mortality thus found would almost certainly be higher than that of the proposed average table. Great mortality differences also exist between endowment, whole life and short term assurances; also between participating and non-participating. If we had a table representing the real, true average of the whole country, and of all the companies, and of all plans of assurance, the fact that it was an average, would make it unsuitable for universal use, for the calculation of premiums at least. It would be unsuitable for the calculation of premiums in the Southern states; it would be unsuitable for the calculation of term premiums, and even for non-participating premiums of companies doing both classes of business.

Even the ordinary departments of industrial companies, I understand, usually show a different mortality from other companies. I speak with much diffidence, but I doubt if a table like this would correctly represent the mortality of the ordinary branch of an industrial company.

Consider, furthermore, the experience of the British and Canadian companies, as this war progresses. Should not any table we may use have a margin sufficient to provide for contingencies such as this? The first essential in a standard or legal table of mortality is that it should be safe. It should be safe to use by any ordinary company throughout the country. It should represent not the average, but the maximum mortality likely to be experienced by any company managed with moderate ability and success. It should be a little heavier than the mortality that will probably be actually experienced, in order to be safe.

If the table to be used were based on the experience on whole life assurances only, it might perhaps not be so unsuitable, but that is a different question.

The reason why this question of a mortality table is of prime importance is because of the practical results which will follow from it. A table brought out under such auspices will almost certainly become in no long time the legal standard of most of the states of the Union. From this, two results will follow. The first and most important is, that as at the younger ages the net premiums for most assurances issued will in all probability be lower than the present, the tendency will be towards a reduction in the premiums charged by the companies. To maintain present premiums, the loadings would have to be increased. It is very unlikely that all

companies would do this, and if one or two companies reduced premiums, competition would almost compel the other companies to follow their example.

Well, people may say, that means that the public will get cheaper insurance. But this is not so. The curious fact is, that although the net premiums may be reduced the reserves will, on the whole, be materially increased.

Mr. Moir's paper shows that by the Medico-Actuarial Table, the reserves will on the average be three or four per cent. over those by the American Table. We all know that a reduction in mortality does not necessarily mean a reduction in average reserves. A low mortality in the early years of life may steepen the curve and increase the reserves, and that will probably be so in this case. Is that a desirable result? We all believe that the legal standard for reserves is high enough now. We use net premiums only and ignore the loadings, although we know that any normal company can reassure its policy obligations for a less sum than its net reserves. An increase in the legal reserves is quite unnecessary.

And now comes the next point. To the extent that you increase the reserves of the companies, you lessen their surplus, and lessen the amount they can pay as profits to their policyholders. The three or four per cent. increase in reserves which they must set aside as a liability reduces their profit-paying power by just that amount. Instead of reducing the cost to policyholders, the change in valuation basis will probably increase the cost.

This question is one which affects primarily the United States, and I really must apologize, as a Canadian, for expressing views at such length. But I am speaking as a member of the Society, not as a Canadian. In Canada we have as our basis the $O^{M(5)}$ Table, with which we are quite satisfied. It has only been our legal basis since 1910, and I think I can say safely that neither the Canadian government, the Insurance Department nor the companies wish to make a change. As far as Canada is concerned, it is most unlikely that we will adopt the new table, even if it be adopted on this side of the line. We do not feel the need of it. At the same time, I will say this: If you gentlemen decide that it is wise or necessary for you to proceed with the preparation of the table, in view of the circumstances which face you, our company, and I think I can speak also for most of the Canadian companies, will at once fall into line. You can count upon our most hearty cooperation. We are loyal members of the Society, and if you want our cooperation and our statistics you will get them wholeheartedly.

There is one suggestion I would make. If you decide to compile the table, we would like, if possible, to have the statistics on American and Canadian lives (not companies) kept sufficiently separate to enable us to see how Canadian and American experiences compare. As a matter of curiosity we would like to know just how the purely Canadian mortality turns out.

Again let me say, if you decide to go on with this work, and that is a question for you not for us, we Canadians will wholeheartedly cooperate with you.

The question has been asked why a few years ago the $O^{M(5)}$ Table was adopted in Canada. The answer is that it was largely a question of national preference. The $O^{M(5)}$ Table was satisfactory, the premiums were not unduly low, while the reserves were not unduly high, and our Canadian parliament quite naturally preferred a British table, just as I would expect an American legislature to prefer an American table.

Mr. William Young: There was once a man who borrowed a pot, and when he returned the pot, it was cracked. There was a lawsuit about the pot, and the borrower made his own defence. He proved conclusively to the court: First, that he had never borrowed the pot; second, that when he borrowed it it was cracked; and third, that when he returned it it was in perfect condition.

He lost his case because he proved too much. It seems to me that the arguments that have been advanced against preparing a new mortality table in the same way prove too much. It has been shown to us that this proposed table would be useless and that it would be used erroneously; that because of low mortality since 1906 the rates of mortality in the new table would be too low and that because of high mortality previous to 1906 the rates of the new mortality table would be too high; that if we decide that we should not construct a new table, then it is quite plain that if it were constructed it would have no practical effect because the policyholders are already getting their insurance at net cost, and that if we do construct it it will make all the difference in the world—it will change premium rates and reserves, it will change the amounts available under the law for expenses, and it will ultimately radically change the laws.

We are discussing the question "Shall we prepare a new mortality table now?" but the arguments that have been advanced against its preparation now would equally apply if the question were "Shall we ever prepare a new mortality table?" In the consideration of the question "Shall we prepare a new mortality table now?" we do not need to decide in advance what effect a possible new mortality table would have on net premiums; what effect it would have on reserves; what effect it would ultimately have on the law. It does not seem reasonable to attempt to answer all these questions in advance, and then to discuss, on the basis of what we suppose the answers to be, the question "Shall we now prepare a mortality table?" In preparing a new mortality table, all we are doing is scheduling what has happened in the past. We might schedule in a proper way what has happened in the last ten, fifteen, or twenty-five years. It does not seem to matter very much when the scheduling of facts is done, but how it is done. If it were done by the Actuarial Society, it

would be done properly. It would be a very good plan if we were to do it regularly, as in England, about every thirty years, and now is as good a time as any to begin to regularly schedule the mortality experience of the past. Having scheduled the mortality experience of the past, we shall then be in a position to decide whether one, or two, or several mortality tables should be deduced from that experience, and to decide for what purposes the mortality table or tables are suitable. Whatever the mortality experience has been, it will not alter the tests that are applicable to determine whether any mortality table that is deduced from the experience is a suitable one for calculating premiums or reserves.

The principal test of the suitability of a mortality table is whether it is a safe and conservative basis for forecasting future experience. If it is a safe and conservative basis for that purpose, it is suitable for premiums and reserves provided it is properly applied. If it is not a safe and conservative basis, it is unsuitable for these practical purposes of the life insurance business.

Mr. Ray D. Murphy: There is one point that was touched on by Mr. Macauley which I want to emphasize. In several recent discussions it seems to me that certain of our laws have obstructed clear thinking. I refer particularly to the method of net valuation. Laymen, and possibly the Insurance Commissioners, have some justification to think that if, as is sometimes contended, the gross premiums will not be materially changed, and the death losses to be expected in the future will be less, the reserves will be less. And I wish to point out that possibly one means of attaining the object apparently in the public mind, and also aiding generally, I think, toward a clearer statement of insurance assets and liabilities, would be a change to the valuation method used in Great Britain, which takes into account not only the death losses to be expected, but the actual gross premiums to be received.

Mr. Edward B. Morris: Mr. Moir has drawn certain conclusions in reference to what might be expected in the way of a new table on American lives from data obtained from the Medico-Actuarial experience.

It may be of interest to the Society at this time to review the net premiums and reserves obtained from the experience of an American company based strictly upon American lives. Last year the experience of The Travelers Insurance Company was thrown into proper shape and commutation columns obtained for the acquiring of such premiums and reserves for certain purposes. The experience of the Travelers is upon standard policies mostly non-participating and involving insurances in practically all of the settled states and provinces of the United States and Canada. There is quite a considerable volume of Southern business included. The exposure is determined from all the company's standard policies in force from

1897 to 1913 inclusive, *but excluding the first five years of insurance.* The table includes some business written in the earliest days of the company—1866—although, of course, the volume of business is mostly from issues of more recent years.

The experience has been worked up in two ways—first, on the basis of policies; second, on the basis of amounts. Interest at 3½ per cent. has been used.

The data on the basis of policies have been here given for the purpose of comparing the results with those obtained from amounts but based upon the same business. For the purpose of determining premiums and reserves, the table by amounts is, of course, much more reliable.

The table by amounts runs into the Makehamized American at age 62, the experience of the company justifying such a course, although the Makehamized American mortality at the higher ages is slightly in excess of that of the Travelers. The experience by policies follows the Travelers' experience practically throughout.

The experience by numbers produces lower net premiums than the experience by amounts throughout and generally smaller reserves, with the exception of the endowment forms.

The main interest, however, in the results of this experience at this time will be found in a comparison between the premiums and reserves determined by the Travelers' experience by amounts with the present American Table and with the results furnished by Mr. Moir based upon the M.-A. Experience. It will be noted that the results under the Travelers' experience by amounts and those obtained from the M.-A., both in premiums and reserves, are remarkably close. The M.-A. Table, being based upon policies, shows somewhat lower premiums as a rule than does the Travelers' experience by amounts, which partly emphasizes the warnings noted by the Medico-Actuarial Committee in their report that the table is not a suitable one for the calculation of premiums.

Tables are added showing not only the premiums and reserves compared with similar data on the American Experience 3½ per cent. interest, but also ratios indicating the increased reserves necessary on the Travelers' experience over the legal American 3½ per cent. reserves. It will be noted that almost invariably, except at the higher ages, the Travelers' table calls for a higher reserve. On the ordinary life form at ages under 40, the terminal reserve averages over 10 per cent. greater than that required by the Amrican. The difference is not as large, however, on the 20-payment life nor on the 20-year endowment forms; all of which verifies the statement of Mr. Moir in regard to the expectation that any probable change in the American Table will mean an increase in reserve liability.

I wish to emphasize, however, the idea that the experience of one company is not a safe guide for other companies. The facts in reference to the Travelers' experience have been here presented mainly for the reason that they show in a general way what may be expected

from a remodeled American Table and are presented at this time for the reason that I am not aware that rates and values have been calculated from data composed of strictly American lives and based

COMPARISON OF MORTALITY RATES PER 1,000.

Age.	Amer. Exp.	O^M(5)	Medico Actuarial.	Travelers(5) (Amounts).	Travelers(5) (Policies).
20	7.80	6.52	——	4.35	4.00
25	8.06	6.89	4.70	4.46	4.03
30	8.43	7.47	4.90	4.72	4.23
35	8.95	8.37	5.10	5.22	4.68
40	9.79	9.78	5.70	5.98	5.47
45	11.16	12.00	7.50	7.53	7.08
50	13.78	15.45	10.60	11.04	10.08
55	18.57	20.83	15.80	17.22	15.58
60	26.69	29.21	24.00	26.48	24.83
65	40.13	42.21	39.00	38.88	38.88
70	61.99	62.19	61.70	60.03	57.93
75	94.37	92.67	91.90	94.81	82.33
80	144.47	138.50	137.20	150.81	116.19
85	235.55	205.69	203.70	237.96	176.24

COMPARISON OF NET PREMIUMS AND TERMINAL RESERVES AMERICAN EXPERIENCE, 3½ PER CENT. INTEREST, AND TRAVELERS EXPERIENCE(5) (POLICIES), 3½ PER CENT. INTEREST.

Ordinary Life.

$1,000.

End of Year.	Age 20.		Age 30.		Age 40.		Age 50.		Age 60.	
	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.
	Net Pr. $13.48.	Net Pr. $10.31.	Net Pr. $17.19.	Net Pr. $14.12.	Net Pr. $23.50.	Net Pr. $20.69.	Net Pr. $34.99.	Net Pr. $32.45.	Net Pr. $56.83.	Net Pr. $53.66.
1	6.19	6.70	9.45	10.43	14.68	16.03	22.74	23.75	33.00	31.49
2	12.60	13.65	19.22	21.20	29.80	32.48	45.87	47.75	66.00	62.60
3	19.24	20.87	29.33	32.32	45.39	49.33	69.37	71.95	98.93	93.27
4	26.11	28.36	39.78	43.79	61.43	66.57	93.19	96.32	131.73	123.45
5	33.23	36.14	50.58	55.63	77.92	84.20	117.31	120.76	164.34	153.11
6	40.61	44.23	61.74	67.82	94.87	102.21	141.68	145.25	196.67	182.22
7	48.24	52.61	73.27	80.40	112.25	120.56	166.27	169.73	228.66	210.75
8	56.14	61.29	85.18	93.35	130.06	139.25	191.04	194.14	260.23	238.72
9	64.32	70.27	97.46	106.70	148.29	158.25	215.96	218.41	291.30	266.14
10	72.78	79.56	110.14	120.42	166.89	177.54	240.96	242.51	321.81	293.02
11	81.54	89.16	123.20	134.52	185.83	197.06	266.01	266.36	351.70	319.41
12	90.60	99.08	136.66	148.98	205.10	216.81	291.05	289.94	380.93	345.32
13	99.98	109.32	150.52	163.80	224.68	236.71	316.05	313.15	409.55	370.80
14	109.66	119.87	164.80	178.97	244.52	256.76	340.95	336.01	437.60	395.92
15	119.68	130.77	179.47	194.47	264.62	276.86	365.70	358.48	465.19	420.74
16	130.03	142.00	194.56	210.31	284.92	297.00	390.24	380.55	492.38	445.39
17	140.72	153.57	210.02	226.44	305.41	317.14	414.52	402.16	519.24	469.76
18	151.77	165.50	225.88	242.89	326.04	337.21	438.48	423.33	545.78	494.04
19	163.15	177.78	242.09	259.60	346.80	357.17	462.07	444.11	571.99	518.25
20	174.90	190.41	258.64	276.56	367.63	377.00	485.23	464.46	597.84	542.48

COMPARISON OF NET PREMIUMS AND TERMINAL RESERVES AMERICAN EXPERIENCE, 3½ PER CENT. INTEREST, AND TRAVELERS EXPERIENCE[5] (POLICIES), 3½ PER CENT. INTEREST.

20-Payment Life.

$1,000.

End of Year.	Age 20.		Age 30.		Age 40.		Age 50.		Age 60.	
	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.
	Net Pr. $20.72.	Net Pr. $16.44.	Net Pr. $24.71.	Net Pr. $20.86.	Net Pr. $30.75.	Net Pr. $27.56.	Net Pr. $40.82.	Net Pr. $38.45.	Net Pr. $59.85.	Net Pr. $57.51.
1	13.75	13.06	17.30	17.44	22.25	23.19	28.87	30.02	36.22	35.57
2	28.05	26.63	35.27	35.50	45.30	47.08	58.43	60.60	72.64	71.04
3	42.91	40.73	53.94	54.19	69.17	71.69	88.70	91.70	109.26	106.37
4	58.36	55.37	73.32	73.54	93.88	97.03	119.68	123.32	146.04	141.58
5	74.44	70.57	93.46	93.56	119.46	123.11	151.35	155.41	182.97	176.70
6	91.16	86.37	114.39	114.28	145.93	149.93	183.74	187.99	220.01	211.77
7	108.55	102.78	136.12	135.73	173.31	177.49	216.84	221.06	257.19	246.85
8	126.64	119.79	158.69	157.91	201.62	205.83	250.70	254.60	294.52	282.05
9	145.46	137.44	182.12	180.88	230.88	234.94	285.33	288.65	332.04	317.52
10	165.03	155.74	206.47	204.61	261.10	264.85	320.77	323.26	369.84	353.42
11	185.39	174.71	231.75	229.15	292.31	295.53	357.08	358.43	408.03	390.00
12	206.58	194.38	258.01	254.51	324.55	327.05	394.34	394.28	446.82	427.49
13	228.62	214.75	285.29	280.69	357.85	359.39	432.64	430.87	486.54	466.22
14	251.54	235.85	313.63	307.75	392.27	392.62	472.11	468.38	527.64	506.60
15	275.39	257.73	343.07	335.68	427.87	426.71	512.92	506.98	570.69	549.16
16	300.22	280.40	373.67	364.53	464.74	461.79	555.27	546.94	616.41	594.59
17	326.05	303.88	405.45	394.30	502.94	497.90	599.45	588.47	665.73	643.46
18	352.93	328.20	438.48	425.08	542.61	535.12	645.80	631.94	719.86	696.91
19	380.91	353.42	472.81	456.85	583.89	573.58	694.78	677.84	780.47	756.22
20	410.03	379.54	508.49	489.68	626.92	613.46	746.98	726.68	849.97	823.14

largely upon comparatively recent exposures. The experience, as I have already stated, on which these tables are based is the Travelers' experience without differentiation between the several policy forms. The experience of the Travelers, however, on its ordinary life contract shows a considerably heavier mortality than on its limited payment life and endowment forms—a further point which should not be overlooked in the actual use of this table.

It is hardly necessary to add that the results are somewhat more favorable than were recorded in the Medico-Actuarial experience and presumably more favorable than would be obtained for a new American Table and, consequently, all results must be used with considerable caution.

COMPARISON OF NET PREMIUMS AND TERMINAL RESERVES AMERICAN EXPERIENCE, 3½ PER CENT. INTEREST, AND TRAVELERS EXPERIENCE[5] (POLICIES), 3½ PER CENT. INTEREST.

20-Year Endowment.

$1,000.

End of Year.	Age 20.		Age 30.		Age 40.		Age 50.		Age 60.	
	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.
	Net Pr. $38.90.	Net Pr. $36.54.	Net Pr. $39.51.	Net Pr. $37.01.	Net Pr. $41.18.	Net Pr. $38.80.	Net Pr. $46.46.	Net Pr. $44.71.	Net Pr. $61.65.	Net Pr. $59.93.
1	32.71	33.95	32.74	34.22	33.15	34.88	34.79	36.56	38.13	38.14
2	66.79	69.23	66.84	69.72	67.59	70.96	70.58	73.99	76.60	76.36
3	102.29	105.88	102.35	106.55	103.38	108.26	107.39	112.28	115.41	114.63
4	139.29	143.96	139.32	144.75	140.58	146.82	145.28	151.46	154.56	153.01
5	177.84	183.53	177.83	184.40	179.23	186.70	184.26	191.52	194.05	191.57
6	218.02	224.64	217.95	225.53	219.41	227.93	224.39	232.54	233.91	230.39
7	259.90	267.36	259.74	268.21	261.16	270.56	265.73	274.55	274.17	269.59
8	303.55	311.73	303.29	312.50	304.57	314.67	308.36	317.63	314.93	309.35
9	349.05	357.80	348.67	358.49	349.71	360.31	352.38	361.86	356.29	349.89
10	396.50	405.66	395.98	406.21	396.66	407.57	397.92	407.41	398.42	391.48
11	445.97	455.37	445.29	455.76	445.52	456.50	445.12	454.39	441.55	434.48
12	497.56	507.01	496.74	507.19	496.40	507.25	494.18	503.04	486.03	479.26
13	551.38	560.65	550.41	560.60	549.46	559.93	545.33	553.56	532.36	526.34
14	607.52	616.36	606.42	616.11	604.86	614.71	598.89	606.34	581.22	576.34
15	666.10	674.26	664.91	673.80	662.78	671.68	655.22	661.78	633.47	630.07
16	727.23	734.45	726.02	733.81	723.45	731.16	714.79	720.37	690.22	688.59
17	791.05	796.99	789.89	796.24	787.16	793.39	778.20	782.66	752.90	752.89
18	857.69	862.01	856.71	861.31	854.21	858.65	846.20	849.37	823.44	824.72
19	927.28	929.64	926.67	929.15	925.01	927.34	919.72	921.47	904.54	906.15
20	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00

COMPARISON OF NET PREMIUMS AND TERMINAL RESERVES AMERICAN EXPERIENCE, 3½ PER CENT. INTEREST, AND TRAVELERS EXPERIENCE[5] (POLICIES), 3½ PER CENT. INTEREST.

10-Year Term.

$1,000.

End of Year.	Age 20.		Age 30.		Age 40.		Age 50.		Age 60.	
	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.
	Net Pr. $7.76.	Net Pr. $3.91.	Net Pr. $8.59.	Net Pr. $4.48.	Net Pr. $10.67.	Net Pr. $6.70.	Net Pr. $17.37.	Net Pr $14.42.	Net Pr. $36.57.	Net Pr. $35.00.
1	.24	.05	.47	.41	1.26	1.48	4.26	4.90	11.46	11.68
2	.42	.09	.87	.76	2.36	2.78	7.96	9.16	21.45	21.67
3	.56	.13	1.19	1.05	3.27	3.84	10.98	12.65	29.69	29.70
4	.67	.16	1.42	1.25	3.95	4.64	13.23	15.22	35.86	35.46
5	.72	.19	1.54	1.37	4.34	5.10	14.53	16.69	39.54	38.65
6	.71	.21	1.55	1.38	4.42	5.18	14.72	16.88	40.26	38.86
7	.65	.22	1.42	1.26	4.10	4.79	13.60	15.57	37.45	35.66
8	.51	.19	1.14	1.01	3.33	3.86	10.94	12.51	30.41	28.54
9	.29	.12	.66	.61	2.00	2.30	6.52	7.43	18.27	16.90
10	—	—	—	—	—	—	—	—	—	—

RATIOS OF RESERVES BY "TRAVELERS" EXPERIENCE[5] (POLICIES) TO "AMERICAN," 3½ PER CENT.

Ordinary Life.

End of Year.	Age 20.	Age 30.	Age 40.	Age 50.	Age 60.
1	1.082	1.104	1.092	1.044	.954
2	1.083	1.103	1.090	1.041	.948
3	1.085	1.102	1.087	1.037	.943
4	1.086	1.101	1.084	1.034	.937
5	1.088	1.100	1.081	1.029	.932
6	1.089	1.098	1.077	1.025	.927
7	1.091	1.097	1.074	1.021	.922
8	1.092	1.096	1.071	1.016	.917
9	1.093	1.095	1.067	1.011	.914
10	1.093	1.093	1.064	1.006	.911
11	1.093	1.092	1.060	1.001	.908
12	1.094	1.090	1.057	.996	.907
13	1.093	1.088	1.054	.991	.905
14	1.093	1.086	1.050	.986	.905
15	1.093	1.084	1.046	.980	.904
16	1.092	1.081	1.042	.975	.905
17	1.091	1.078	1.038	.970	.905
18	1.090	1.075	1.034	.965	.905
19	1.090	1.072	1.030	.961	.906
20	1.089	1.069	1.025	.957	.907

RATIOS OF RESERVES BY "TRAVELERS" EXPERIENCE[5] (POLICIES) TO "AMERICAN," 3½ PER CENT.

20-Payment Life.

End of Year.	Age 20.	Age 30.	Age 40.	Age 50.	Age 60.
1	.950	1.008	1.042	1.040	.982
2	.949	1.007	1.039	1.037	.978
3	.949	1.005	1.036	1.034	.974
4	.949	1.003	1.034	1.030	.969
5	.948	1.001	1.031	1.027	.966
6	.947	.999	1.027	1.023	.963
7	.947	.997	1.024	1.019	.960
8	.946	.995	1.021	1.016	.958
9	.945	.993	1.018	1.012	.956
10	.944	.991	1.014	1.008	.956
11	.942	.989	1.011	1.004	.956
12	.941	.986	1.008	1.000	.957
13	.939	.984	1.004	.996	.958
14	.938	.981	1.001	.992	.960
15	.936	.978	.997	.988	.962
16	.934	.976	.994	.985	.965
17	.932	.972	.990	.982	.967
18	.930	.969	.986	.979	.968
19	.928	.966	.982	.976	.969
20	.926	.963	.979	.973	.968

RATIOS OF RESERVES BY "TRAVELERS" EXPERIENCE[5] (POLICIES) TO "AMERICAN," 3½ PER CENT.

20-Year Endowment.

End of Year.	Age 20.	Age 30.	Age 40.	Age 50.	Age 60.
1	1.038	1.045	1.052	1.051	1.000
2	1.037	1.043	1.050	1.048	.997
3	1.035	1.041	1.047	1.046	.993
4	1.034	1.039	1.044	1.043	.990
5	1.032	1.037	1.042	1.039	.987
6	1.030	1.035	1.039	1.036	.985
7	1.029	1.033	1.036	1.033	.983
8	1.027	1.030	1.033	1.030	.982
9	1.025	1.028	1.030	1.027	.982
10	1.023	1.026	1.028	1.024	.983
11	1.021	1.024	1.025	1.021	.984
12	1.019	1.021	1.022	1.018	.986
13	1.017	1.019	1.019	1.015	.989
14	1.015	1.016	1.016	1.012	.992
15	1.012	1.013	1.013	1.010	.995
16	1.010	1.011	1.011	1.008	.998
17	1.008	1.008	1.008	1.006	1.000
18	1.005	1.005	1.005	1.004	1.002
19	1.003	1.003	1.003	1.002	1.002
20	1.000	1.000	1.000	1.000	1.000

RATIOS OF RESERVES BY "TRAVELERS" EXPERIENCE[5] (POLICIES) TO "AMERICAN," 3½ PER CENT.

10-Year Term.

End of Year.	Age 20.	Age 30.	Age 40.	Age 50.	Age 60.
1	.208	.872	1.175	1.150	1.019
2	.214	.874	1.178	1.151	1.010
3	.232	.882	1.174	1.152	1.000
4	.239	.880	1.175	1.150	.989
5	.264	.890	1.175	1.149	.977
6	.296	.890	1.172	1.147	.965
7	.338	.887	1.168	1.145	.952
8	.373	.886	1.159	1.144	.939
9	.414	.924	1.150	1.140	.925

COMPARISON OF NET PREMIUMS AND TERMINAL RESERVES AMERICAN EXPERIENCE, 3½ PER CENT. INTEREST, AND TRAVELERS EXPERIENCE[5] (AMOUNTS), 3½ PER CENT. INTEREST.

Ordinary Life.

$1,000.

End of Year.	Age 20.		Age 30.		Age 40.		Age 50.		Age 60.	
	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.
	Net Pr. $13.48.	Net Pr. $10.83.	Net Pr. $17.19.	Net Pr. $14.79.	Net Pr. $23.50.	Net Pr. $21.59.	Net Pr. $34.99.	Net Pr. $34.02.	Net Pr. $56.83.	Net Pr. $56.33.
1	6.19	6.89	9.45	10.64	14.68	16.46	22.74	24.44	33.00	32.69
2	12.60	14.04	19.22	21.62	29.80	33.41	45.87	49.05	66.00	65.46
3	19.24	21.46	29.33	32.95	45.39	50.81	69.37	73.78	98.93	97.95
4	26.11	29.15	39.78	44.65	61.43	68.66	93.19	98.61	131.73	131.01
5	33.23	37.12	50.58	56.71	77.92	86.93	117.31	123.46	164.34	163.99
6	40.61	45.37	61.74	69.14	94.87	105.58	141.68	148.32	196.67	196.82
7	48.24	53.91	73.27	81.95	112.25	124.59	166.27	173.18	228.66	229.43
8	56.14	62.75	85.18	95.16	130.06	143.89	191.04	197.98	260.23	261.75
9	64.32	71.90	97.46	108.76	148.29	163.46	215.96	222.72	291.30	293.70
10	72.78	81.35	110.14	122.78	166.89	183.26	240.96	247.41	321.81	325.26
11	81.54	91.12	123.20	137.23	185.83	203.21	266.01	272.00	351.70	356.33
12	90.60	101.20	136.66	152.09	205.10	223.32	291.05	296.52	380.93	386.83
13	99.98	111.61	150.52	167.36	224.68	243.51	316.05	320.94	409.55	416.71
14	109.66	122.35	164.80	183.02	244.52	263.81	340.95	345.80	437.60	445.90
15	119.68	133.42	179.47	199.05	264.62	284.10	365.70	370.60	465.19	474.36
16	130.03	144.84	194.56	215.40	284.92	304.41	390.24	395.30	492.38	502.14
17	140.72	156.60	210.02	232.08	305.41	324.72	414.52	419.80	519.24	528.99
18	151.77	168.73	225.88	249.01	326.04	344.96	438.48	444.08	545.78	554.98
19	163.15	181.22	242.09	266.18	346.80	365.18	462.07	468.09	571.99	580.11
20	174.90	194.10	258.64	283.55	367.63	385.35	485.23	491.80	597.84	604.42

COMPARISON OF NET PREMIUMS AND TERMINAL RESERVES AMERICAN EXPERIENCE, 3½ PER CENT. INTEREST, AND TRAVELERS EXPERIENCE[5] (AMOUNTS), 3½ PER CENT. INTEREST.

20-Payment Life.

$1,000.

End of Year.	Age 20.		Age 30.		Age 40.		Age 50.		Age 60.	
	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.
	Net Pr. $20.72.	Net Pr. $17.13.	Net Pr. $24.71.	Net Pr. $21.64.	Net Pr. $30.75.	Net Pr. $28.45.	Net Pr. $40.82.	Net Pr. $39.79.	Net Pr. $59.85.	Net Pr. $59.29.
1	13.75	13.44	17.30	17.77	22.25	23.60	28.87	30.48	36.22	35.84
2	28.05	27.40	35.27	36.16	45.30	47.99	58.43	61.42	72.64	71.79
3	42.91	41.89	53.94	55.20	69.17	73.15	88.70	92.81	109.26	107.87
4	58.36	56.94	73.32	74.92	93.88	99.09	119.68	124.65	146.04	144.79
5	74.44	72.56	93.46	95.32	119.46	125.80	151.35	156.91	182.97	181.97
6	91.16	88.77	114.39	116.45	145.93	153.26	183.74	189.63	220.01	219.37
7	108.55	105.58	136.12	138.31	173.31	181.50	216.84	222.84	257.19	256.99
8	126.64	123.03	158.69	160.94	201.62	210.47	250.70	256.54	294.52	294.86
9	145.46	141.12	182.12	184.37	230.88	240.19	285.33	290.83	332.04	333.01
10	165.03	159.88	206.47	208.62	261.10	270.65	320.77	325.79	369.84	371.55
11	185.39	179.34	231.75	233.74	292.31	301.85	357.08	361.46	408.03	410.56
12	206.58	199.49	258.01	259.74	324.55	333.84	394.34	398.01	446.82	450.19
13	228.62	220.40	285.29	286.63	357.85	366.62	432.64	435.51	486.54	490.67
14	251.54	242.05	313.63	314.45	392.27	400.30	472.11	474.60	527.64	532.32
15	275.39	264.51	343.07	343.19	427.87	434.84	512.92	515.04	570.69	575.62
16	300.22	287.79	373.67	372.86	464.74	470.49	555.27	557.06	616.41	621.31
17	326.05	311.90	405.45	403.53	502.94	507.16	599.45	600.86	665.73	670.05
18	352.93	336.91	438.48	435.15	542.61	544.98	645.80	646.77	719.86	723.25
19	380.91	362.83	472.81	467.82	583.89	584.19	694.78	695.27	780.47	782.71
20	410.03	389.72	508.49	501.55	626.92	624.98	746.98	746.90	849.97	851.10

COMPARISON OF NET PREMIUMS AND TERMINAL RESERVES AMERICAN EXPERIENCE, 3½ PER CENT. INTEREST, AND TRAVELERS EXPERIENCE[5] (AMOUNTS), 3½ PER CENT. INTEREST.

20-Year Endowment.

$1,000.

End of Year.	Age 20.		Age 30.		Age 40.		Age 50.		Age 60.	
	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.
	Net Pr. $38.90.	Net Pr. $36.78.	Net. Pr. $39.51.	Net Pr. $37.31.	Net Pr. $41.18	Net Pr. $39.20.	Net Pr. $46.46.	Net Pr. $45.52.	Net Pr. $61.65.	Net Pr. $61.07.
1	32.71	33.87	32.74	34.06	33.15	34.80	34.79	36.48	38.13	37.73
2	66.79	69.06	66.84	69.39	67.59	70.84	70.58	73.71	76.60	75.70
3	102.29	105.63	102.35	106.07	103.38	108.15	107.39	111.70	115.41	113.95
4	139.29	143.63	139.32	144.12	140.58	146.77	145.28	150.51	154.56	153.21
5	177.84	183.10	177.83	183.61	179.23	186.70	184.26	190.12	194.05	192.91
6	218.02	224.12	217.95	224.62	219.41	227.98	224.39	230.64	233.91	233.07
7	259.90	266.74	259.74	267.16	261.16	270.68	265.73	272.15	274.17	273.72
8	303.55	311.01	303.29	311.35	304.57	314.80	308.36	314.70	314.93	314.94
9	349.05	357.02	348.67	357.22	349.71	360.42	352.38	358.47	356.29	356.84
10	396.50	404.81	395.98	404.89	396.66	407.62	397.92	403.99	398.42	399.59
11	445.97	454.48	445.29	454.41	445.52	456.43	445.12	450.69	441.55	443.40
12	497.56	506.07	496.74	505.87	496.40	507.04	494.18	499.20	486.03	488.53
13	551.38	559.71	550.41	559.34	549.46	559.55	545.33	549.73	532.36	535.42
14	607.52	615.43	606.42	614.96	604.86	614.21	598.89	602.97	581.22	584.59
15	666.10	673.37	664.91	672.77	662.78	671.08	655.22	658.99	633.47	636.85
16	727.23	733.65	726.02	732.88	723.45	730.48	714.79	718.26	690.22	693.37
17	791.05	796.28	789.89	795.54	787.16	792.77	778.20	781.24	752.90	755.34
18	857.69	861.47	856.71	860.75	854.21	858.12	846.20	848.69	823.44	824.98
19	927.28	929.27	926.67	928.84	925.01	927.09	919.72	921.56	904.54	905.18
20	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00

COMPARISON OF NET PREMIUMS AND TERMINAL RESERVES AMERICAN EXPERIENCE, 3½ PER CENT. INTEREST, AND TRAVELERS EXPERIENCE[5] (AMOUNTS), 3½ PER CENT. INTEREST.

10-Year Term.

$1,000.

End of Year.	Age 20.		Age 30.		Age 40.		Age 50.		Age 60.	
	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.	Am.	Trav.
	Net Pr. $7.76.	Net Pr. $4.30.	Net Pr. $8.59.	Net Pr. $4.99.	Net Pr. $10.67.	Net Pr. $7.21.	Net Pr. $17.37.	Net Pr. $15.81.	Net Pr. $36.57.	Net Pr. $35.71.
1	.24	.11	.47	.45	1.26	1.48	4.26	5.38	11.46	10.76
2	.42	.21	.87	.83	2.36	2.83	7.96	10.00	21.45	19.93
3	.56	.29	1.19	1.13	3.27	3.97	10.98	13.73	29.69	27.28
4	.67	.36	1.42	1.36	3.95	4.87	13.23	16.41	35.86	33.30
5	.72	.40	1.54	1.48	4.34	5.44	14.53	17.87	39.54	36.97
6	.71	.41	1.55	1.48	4.42	5.59	14.72	17.95	40.26	37.82
7	.65	.39	1.42	1.34	4.10	5.25	13.60	16.44	37.45	35.31
8	.51	.32	1.14	1.07	3.33	4.30	10.94	13.11	30.41	28.77
9	.29	.20	.66	.63	2.00	2.59	6.52	7.73	18.27	17.32
10	—	—	—	—	—	—	—	—	—	—

RATIOS OF RESERVES BY "TRAVELERS" EXPERIENCE[5] (AMOUNTS) TO "AMERICAN," 3½ PER CENT.

Ordinary Life.

End of Year.	Age 20.	Age 30.	Age 40.	Age 50.	Age 60.
1	1.113	1.126	1.121	1.075	.991
2	1.114	1.125	1.121	1.069	.992
3	1.115	1.123	1.119	1.064	.990
4	1.116	1.122	1.118	1.058	.995
5	1.117	1.121	1.116	1.052	.998
6	1.117	1.120	1.113	1.047	1.001
7	1.118	1.118	1.110	1.042	1.003
8	1.118	1.117	1.106	1.036	1.006
9	1.118	1.116	1.102	1.031	1.008
10	1.118	1.115	1.098	1.027	1.011
11	1.117	1.114	1.094	1.023	1.013
12	1.117	1.113	1.089	1.019	1.015
13	1.116	1.112	1.084	1.015	1.017
14	1.116	1.111	1.079	1.014	1.019
15	1.115	1.109	1.074	1.013	1.020
16	1.114	1.107	1.068	1.013	1.020
17	1.113	1.105	1.063	1.013	1.019
18	1.112	1.102	1.058	1.013	1.017
19	1.111	1.100	1.053	1.013	1.014
20	1.111	1.097	1.048	1.014	1.011

RATIOS OF RESERVES BY "TRAVELERS" EXPERIENCE[5] (AMOUNTS) TO "AMERICAN," 3½ PER CENT.

20-Payment Life.

End of Year.	Age 20.	Age 30.	Age 40.	Age 50.	Age 60.
1	.977	1.027	1.061	1.056	.990
2	.977	1.025	1.059	1.051	.988
3	.976	1.023	1.058	1.046	.987
4	.976	1.022	1.055	1.042	.991
5	.975	1.020	1.053	1.037	.995
6	.974	1.018	1.050	1.032	.997
7	.973	1.016	1.047	1.028	.999
8	.971	1.014	1.044	1.023	1.001
9	.970	1.012	1.040	1.019	1.003
10	.969	1.010	1.037	1.016	1.005
11	.967	1.009	1.033	1.012	1.006
12	.966	1.007	1.029	1.009	1.008
13	.964	1.005	1.025	1.007	1.008
14	.962	1.003	1.020	1.005	1.009
15	.960	1.000	1.016	1.004	1.009
16	.959	.998	1.012	1.003	1.008
17	.957	.995	1.008	1.002	1.006
18	.955	.992	1.004	1.002	1.005
19	.953	.989	1.001	1.001	1.003
20	.950	.986	.997	1.000	1.001

RATIOS OF RESERVES BY "TRAVELERS" EXPERIENCE[5] (AMOUNTS) TO "AMERICAN," 3½ PER CENT.

20-Year Endowment.

End of Year.	Age 20.	Age 30.	Age 40.	Age 50.	Age 60.
1	1.035	1.040	1.050	1.049	.990
2	1.034	1.038	1.048	1.044	.988
3	1.033	1.036	1.046	1.040	.987
4	1.031	1.034	1.044	1.036	.991
5	1.030	1.033	1.042	1.032	.994
6	1.028	1.031	1.039	1.028	.996
7	1.026	1.029	1.036	1.024	.998
8	1.025	1.027	1.034	1.021	1.000
9	1.023	1.025	1.031	1.017	1.002
10	1.021	1.023	1.028	1.015	1.003
11	1.019	1.020	1.024	1.013	1.004
12	1.017	1.018	1.021	1.010	1.005
13	1.015	1.016	1.018	1.008	1.006
14	1.013	1.014	1.015	1.007	1.006
15	1.011	1.012	1.013	1.006	1.005
16	1.009	1.009	1.010	1.005	1.005
17	1.007	1.007	1.007	1.004	1.003
18	1.004	1.005	1.005	1.003	1.002
19	1.002	1.002	1.002	1.002	1.001
20	1.000	1.000	1.000	1.000	1.000

RATIOS OF RESERVES BY "TRAVELERS" EXPERIENCE[5] (AMOUNTS) TO "AMERICAN," 3½ PER CENT.

10-Year Term.

End of Year.	Age 20.	Age 30.	Age 40.	Age 50.	Age 60.
1	.458	.957	1.175	1.263	.939
2	.500	.954	1.199	1.256	.929
3	.518	.950	1.214	1.250	.919
4	.537	.958	1.233	1.240	.929
5	.556	.961	1.253	1.230	.935
6	.577	.955	1.265	1.219	.939
7	.600	.944	1.280	1.209	.943
8	.627	.939	1.291	1.198	.946
9	.690	.955	1.295	1.186	.948

MR. ROBERT HENDERSON: On this question of reserves, there is a point on which I think Mr. Moir's paper is a little misleading to some people, on account of its having been made up on the basis of terminal reserves. If it had been made up on the basis of mean reserves, it would have shown a lower reserve on the M.A. Table for the first two years practically throughout on all forms of policies, and the percentage of increase thereafter would have been smaller.

Of course, as far as it affects the possibility of paying dividends, the terminal reserve to be provided must be set aside by the company before paying dividends, and all its effect on the net cost to the policyholder would be there just the same, because terminal reserves are what would affect that question.

Some few years ago the company with which I am connected made up a table to be used as a guide for comparison of special classes and special localities. And, to get a fairly stringent base line, we based that table on our experience from the policy anniversaries in 1899 to the policy anniversaries in 1909 on all business in force during those periods issued in the Northern United States. That is, we confined it to the United States, and excluded thirteen Southern states. I have recently made a few calculations based on the ultimate part of that table (it was made up in select form, the select running into the ultimate at the end of the fifth year) and I find that, comparing with Mr. Moir's table of net premiums, the net premiums at ages 20 and 30 are a little lower than the M.A. Table, and at the higher ages, a little higher. That would seem to indicate that, as a whole, the reserves, if made on the ultimate part of our table, would be probably a little higher than by the M.A. Table. I think the tendency at extreme old ages would be to work the other way, as I recollect it. But that table was made up on its general lines in the way that a new table would be made up. That is, it was based on the experience of a certain interval of time covering all the policies in force during that time, and not as the M. A.

was made, from the issues of a certain period carried forward to a later period.

I think the point raised by Mr. Hutcheson regarding the effect of a new table on the assumed mortality gains would not be so important as the M.A. Table would seem to indicate, because, while the precentage of mortality in the first year is a fairly high percentage at the extreme young ages under 25 years, and gets up to sixty per cent. at the age of sixty or older ages, it falls for usual entry ages almost as low as forty per cent. of the ultimate table (that ultimate table being as low as the M.A.). This would seem to indicate that there might be a possibility that the assumed mortality gains could be applied to the new table.

Mr. Edward E. Rhodes: I have made some calculations. These are 3 per cent. mean reserves, ordinary life, age twenty-five, fourth year, $39.38 for the American; $40.31 for the Medico-Actuarial ultimate; an increase of over two per cent. Age thirty-five, $57.31 for the American, $60.52 for the Medico-Actuarial; an increase of over five per cent. Age forty-five, the increase is four per cent. Age fifty-five, two per cent. The maximum increase is at the average age of 35.

Mr. James D. Craig: The general question has been asked, just what a new mortality table would do, and outside of the general answer that it would give us truer information, there are one or two points which I think would be brought out stronger. In the first place, it will emphasize the weaknesses of the present M.A. Table. In the report of the Committee, emphasis was laid upon the fact that that table should not be used too generally, but it seems as if it is gradually coming into use. Nearly all of us have made some calculations upon it, and unless a more correct standard is produced in the near future, that table will probably come into more general use than it has. Another reason is stated in Mr. Moir's paper in regard to dividends, where he states that while the "whole amount apportioned is determined as the result of such individual experience, nevertheless the detail of the apportionment at different ages may be modified if there be a change of basis on which premium rates, loadings and mortality charges are computed." That is only another phase of the statement that truer information will be at our disposal.

The question was raised as to the mortality on non-participating policies issued by industrial companies. I might say that for the year 1914 on the ordinary policies issued by the Metropolitan—and by ordinary policies I mean those policies issued for one thousand dollars or more—and after the first five calendar years were excluded, the mortality followed the general lines of the experience of the Mutual Benefit, in this respect, that for ages up to 40, the mortality was lower than the M.A. While at about age 60, it reached the American Table.

Mr. J. Burnett Gibb: I do not think there is much to add to what previous speakers have said as to the objections which can be raised from a practical standpoint, if a table—a new table—is made which may result in a change of premiums and reserves and possibly re-open the whole expense question, a matter which is not always within the control of the Insurance Commissioners and State Actuaries, and which, if left to the legislatures without proper guidance, might result in impracticable working conditions. In every legislature it is difficult to get laws which are on the books changed simultaneously with changes in conditions.

The difficulty, as I see it, is this, that if we try to construct a table from a scientific, point of view which would be more widely applicable than the Medico-Actuarial, we get into the position that as a practical proposition the new table would be of little real use. It would require a great expenditure of energy to prepare that table, and necessarily it would have to be made up of a mass of material which would not be at all homogeneous, and the actual use of the table would not do substantially any more justice to the general body of policyholders than is possible under present conditions.

So far as the sufficiency of reserves is concerned, I think the American Table has proved that it is amply safe. The adoption of any table necessitating higher reserves would result in making it more difficult for young companies to do business. Their difficulties to-day are great enough, and I do not think they ought to be made any greater than they are.

The preparation of a table involves many different considerations, such as the varying rules of companies for limit of age at entry and the amounts insured, which have a very decided influence on the mortality; and there is one other element that I think ought to be considered. Since 1905, there is no question but that there has been a great difference in the class of business written. The New York law, which affects a large proportion of the business written in this period, has resulted in an increasing tendency towards the writing of new business on the ordinary life plan. If that tendency continues, we are going to have an entirely different type of ultimate experience as compared with the period when a great deal of all companies' business was written either on the deferred dividend or endowment plan. Enough time has not yet elapsed under the new conditions. In summing up the various points there does not seem very much to be gained by the preparation of a new mortality table at this time.

I think the companies themselves are to a great extent to blame in allowing the sentiment that the American Table is not a good standard to grow as far as it has. There has been practically no effort on the part of actuaries, so far as I have seen, to try to counteract the growth of this belief in the public mind. My feeling is that we ought to analyze carefully, as carefully as we can, the causes which have led up to the repeated demands for a new mortal-

ity table, and I believe if that is done, the practical objections can be put in such a strong form to the Commissioners that they will decide that it is not a good thing to undertake the preparation of a new mortality table at the present time.

Mr. Harwood E. Ryan: Mr. President, I would like to say just a word as a member of the Commissioners' Committee on Mortality Table, rather than as an Associate of this Society.

First, by way of explaining the resolution which resulted in the appointment of the Commissioners' Committee. The Committee was arranged for at the Chicago meeting of the Insurance Commissioners' Convention, and with one or two exceptions I think that the members there appointed were not present, and are not familiar with the discussion that led up to their appointment. But, so far as the presence of the Committee, or its representatives, here at this meeting today is concerned, we are simply in the position of trying to get information which will form the basis of a future report to the Insurance Commissioners' Convention.

I am not sure that all of the members quite fully understand the nature of the resolution, and with your permission, I should like to read it for the purpose of getting it into the record of this meeting.

The resolution commences with a preamble, which relates back to a previous report of another committee which had under consideration the same subject. It is as follows:

"Whereas, at the 1914 meeting of this convention at Asheville, N. C., the Committee on Rates of Mortality and Interest submitted a carefully prepared report in which the mortality experience of eighteen American life insurance companies for a period of years was set forth as the basis for the conclusion: 'That the mortality table which is at present used as a standard in the several states, does not represent the true mortality experience of the several companies, and

"Whereas, the result of the protracted labors of a joint committee of medical examiners of life insurance companies and members of the Actuarial Society of America, covering the actual experience of forty companies of the United States and Canada for a period of twenty-four years, and having under observation three millions of lives, conclusively proves that mortality has materially improved since the tables of mortality now in use were constructed;

"*Therefore, Be It Resolved,* that a special committee of five, to be composed of Department Actuaries, be appointed to co-operate with the Actuarial Society of America in the construction of a mortality table which is in keeping with the ascertained experience of American companies, and report thereon at the September meeting of the convention."

I would like to say just a word of appreciation of the fact that the Society has seen fit to admit to the discussion of Mr. Moir's paper

and of the general subject, the committee which has been appointed by the Commissioners. Unfortunately, we were not able to get a full attendance of the Committee, because of the fact that we learned only this afternoon that the usual rule of the Society had been waived, and that persons who are not Associates or Fellows of the Society would be permitted to be present.

Speaking now as an Associate of the Society, I simply want to say,—not by way of discussing the subject, because I think we are still in the position of trying to ascertain the merits and demerits of the proposition,—that I have been a little disappointed to find that there was so much discussion on the probable use to which the new mortality table might be put. Mr. Young has very well stated that the principal subject before the meeting is whether a new mortality table should be prepared or constructed, and it does not necessarily mean, if a new mortality table should be constructed, and if that table should develop that the rates of mortality at the various ages were materially different from the rates of mortality expressed by the American Table, that any serious results might come from the construction of such a table, such as the adoption of the table for premiums and reserves, or as a statutory basis of reserve liability for life insurance companies.

I think that one of the things that has arisen in the past, and which perhaps to some extent explains the reason for the Commissioners' action, is, however, that there has been expressed at the meetings of the Insurance Commissioners, a feeling that companies were holding too large reserves, and too much surplus, and that if we could only get a new mortality table, that it would be possible to reduce the amount of reserve funds, and perhaps distribute large amounts of surplus to the policyholders.

Now, if the Society concludes not to proceed with the formation of a new table, irrespective altogether of what the effect of that table might be, or what it might show, it seems to me that the popular fallacy, if it may be so termed, will prevail and that the Insurance Commissioners (who do not have the benefit of attendance at the meetings of the Actuarial Society and who are not familiar with all the influences which affect the rate of mortality among insured lives) will still be inclined to the opinion that there is something mysterious about the American Experience Table. It is more or less a matter in which there is a burden of proof upon the Actuarial Society to show whether the American Experience Table is a representative table for practical insurance use and whether, if a new mortality table should be adopted, the effect of such a table would be the one which it is popularly supposed ought to result. The reserves would perhaps increase—undoubtedly would increase, from the discussion I have heard this afternoon, and from the nature of the problem. While the Commissioners' Committee is not prepared to advocate that a new table be adopted, it would seem in the last

analysis that something rests upon the Actuarial Society by way of showing the facts.

Mr. M. Albert Linton: We seem to be so thoroughly in a net premium atmosphere this afternoon that I feel like saying a few words along the line which was opened up by Mr. Murphy.

It seems to me that it would be unfortunate if the public should infer from this discussion of the American Table that the reserves now being held against old business were inadequate. For obviously from the point of view of security this is a false inference. The actual premiums on outstanding policies are fixed by the contract, and in viewing the situation from the point of view of safety the premium that is actually going to be received, and not a much smaller one which has been arrived at on the basis of lower mortality, should be taken into account.

To obtain an idea of the effect upon reserves if the liability for future death claims should be that shown by the M.A. Table, and if at the same time American net premiums should be received, I constructed a model company on the principles employed by Mr. Moir, and made a valuation. If the company were 20 years old, I found that for every $1000 of reserve on the American Table $1014 would be held on the net M.A. basis, whereas only $921 would be held on the assumption that American net premiums would be received and M.A. mortality experienced (all negative values were counted as zero).

It is especially upon my mind to emphasize the necessity that in this discussion we do not give expression to unclear thoughts regarding the sufficiency of present reserves. From the point of view of safety the aggregate reserve on outstanding business produced by the American Table is becoming more and more safe as the rate of mortality thereunder is improved and the liability to pay death claims thereby reduced.

Mr. Percy C. H. Papps: I presume it is right to say that one factor in fixing what we have considered to be proper to hold as a contingency reserve has been the fact that we have had a mortality table which, at the normal insuring ages, gave us a considerable margin of safety. It would seem reasonable therefore if we get out this new table, which reflects the mortality which we are liable to experience in the near future, then our margin of safety has very nearly disappeared. If that is the case, the natural conclusion seems to be that we should increase our margin of safety. I therefore have in mind that not only would it result in holding a larger surplus, but it might raise another question, as to whether it would not be possible to remove some of the restrictions as to the amount of surplus which a company should hold.

Mr. Alexander T. Maclean: The question "Should we prepare a new Mortality Table?" presupposes that we have the necessary

data upon which to construct such a table, and I think that whether or no we have such data deserves consideration because it were futile to discuss the preparation of a table without being sure of the material necessary.

The material available at this time upon which to base a mortality table is made up of (1) non-participating business; (2) deferred dividend business; and (3) annual dividend business.

In the following remarks I have left out of consideration the non-participating business for the reason that it constitutes more or less a special class, and the writer of the opening paper has shown the relation that must exist between the non-participating and participating companies.

It is common knowledge that prior to the year 1907 a very large proportion of the business written by American life companies was on the deferred dividend plan. Apart from the fact that the conditions under which business is done nowadays are very different from those prevailing when most of the existing deferred dividend business was written, there is the greater objection that on the maturity of the dividend period an undoubted selection is exercised by the deferred dividend policyholder, at least once in twenty years in the large majority of cases, and in many cases as often as four times within that period. Such selection is, of course, bound to have its effect on any general mortality experience (*T. A. S. A.*, XIV, 43), and for that reason, apart from any other, I do not believe that a mortality table formed from deferred dividend business would be a true indication of what might be expected to prevail under present conditions.

Turning, then, to the annual dividend business, it is to be noted in the first place that prior to 1907 the proportion of annual dividend business was very small as compared with the deferred dividend business, and although several companies did write a large amount of such business, nevertheless, on the whole, the proportion to the total business written was small. It may be suggested, however, that we are not concerned with the proportion of business written but with the amount, and that if a sufficient amount be obtained that is all that is necessary; but here again business conditions must be considered. Circumstances were such that in most cases it paid the agent to sell a deferred dividend policy in preference to an annual dividend policy; larger cash values—in some cases, at least—were given to deferred dividend policyholders; and, in fact, in many companies those who did get annual dividend policies might almost be said to have insisted upon having them in spite of the agent's perfectly legitimate desire to sell a deferred dividend policy. This again would have its effect on the mortality of this class, and for these and other minor reasons it seems to me that this section of the business also should be omitted.

We are therefore practically limited to experience derived from the years 1907–1914, and I think it will be agreed that the data that

could be obtained from this period would be of far too recent origin and the policies of far too short a duration to give any indication of what ultimate mortality might be.

It appears to be admitted that, if we were to prepare a new table, it would be desirable to exclude the early years of selection.

Assuming that selection is evidenced for a period of 5 years, this would mean that from the already meager data it would be necessary to exclude data derived from years from 1910–1914, thus leaving us with the experience of three years only.

In view of the great change in methods of writing business, already referred to, and of conditions relative to the conduct of the life insurance business, I think it will be admitted that those three years (which one might almost call years of experiment) would form anything but an ideal period from which to derive a mortality table.

Another thought on this subject suggests itself, and that is that, in view of the general opinion as to the differences in the mortality experienced under various classes of policies, it would surely be desirable in any new experience to investigate separately at least three different kinds of policies, i. e., life, endowment and term contracts. This investigation would, of course, further reduce the data under each subdivision, and I think it would prove to be the case that especially at the younger and older ages the experience would be very meager. It might possibly even be the case that reserves would be materially affected simply through paucity of data at the older ages, and the difficulties of gradation, etc., would certainly be much increased.

The above consideration seems to me to suggest one of the most serious difficulties in the way of the preparation of a new table at this time. There are, of course, many others which have been well expressed today, and, in fact, the opening discussion, although it seems to conclude in favor of the preparation of a new table, is to my mind a very good argument for the retention of the American Experience Table. The remark is made that "it is our duty to restore confidence in the basis of our calculations or change the basis." It seems to me that the way to restore confidence, if indeed such confidence has been lost, is to explain to the policyholder or other critic that, as stated in the opening discussion, when any particular table of mortality is used, that table is not the actual basis upon which payments to policyholders are controlled, but that any overcharges in the original premium are credited through the abatements of such premiums and that these abatements or refunds are governed by the actual mortality experience of the company, the actual interest earnings and the actual cost of running the business.

I believe that any intelligent policyholder could easily be satisfied by a non-technical explanation of our dividend systems and how such systems return to the policyholder the proportion of the premium not required to cover the risk, put up the reserves for future claims and meet the necessary expenses of the business.

In conclusion I would express the opinion that the time is not

yet propitious for the formation of a new table but that we should wait at least until such time as we have sufficient data derived from an experience of annual dividend business.

Mr. Henry Moir: The discussion this afternoon has dealt more with the probable effect of a new mortality table than it should have done; I would have liked to have heard a little more from the members present as to the expediency of preparing such new table. The statement is made publicly by the Insurance Commissioners "that the American Table does not represent the true mortality of the companies." This is a statement that not a single man in this room can refute. It is a true statement. And what is more, there is not a table existing in this country that does represent the true mortality. Consider this fact alone! I might stop there, and I think you would have to agree that a new table should be prepared. The requests of the Insurance Commissioners and that statement are arguments enough. Moreover, there are some laymen who accuse us of withholding the truth from them. I have heard that accusation made and made publicly. Do we want to stand under that accusation, failing to answer it frankly with "Here is the best we can do"? These are the strongest arguments to me.

I tried to express my paper in a judicial frame of mind, stating both sides as fairly as possible. A new table would not alter the cost of insurance to the public to any appreciable extent. The cost of insurance is determined by the companies' individual experience, and that condition will continue; therefore any radical change in cost cannot take place. There may be trifling changes at one age or at another—but no general change. On the whole, the American Table is doing good work, is effecting its purpose reasonably, and outstanding contracts based upon it are better left alone than disturbed. Mr. Rhodes says I am illogical in this attitude. But I always view an existing contract differently from a contract on which a company may be about to enter. If I am wrong in taking one view for contracts now existing, and another view in discussing the question of the basis upon which new contracts should be offered, then I may be illogical; but it does not seem to me illogical to differentiate between existing and new contracts. An existing contract is sacred. You cannot make it *less* favorable to the policyholder for the courts would interfere—if you make it *more* favorable, the concession is made at the expense of someone whose interests should be fully considered.

As we have to reach a quick decision, I propose to enter into some details which in general would be answered by a written memorandum. An examination of the chart of the Mutual Benefit recent mortality does not seem to bear out the deduction made by Mr. Rhodes when he says: "Except at the younger ages and the extremely old ages, therefore, the mortality rates of the Mutual Benefit are more closely in accord with those of the American Table than

they are with those of the Medico-Actuarial." If the Mutual Benefit Experience were graduated, the effect would be that at the younger ages (meaning all ages up to fifty) the mortality would be *lower* than the Medico-Actuarial; whereas you remember that the American Table is tremendously higher. Then at or about age 50 it would pass through the Medico-Actuarial curve, and run up to and through the American curve about age 62 or 63. It would join them both again about 75 (the M.A.$^{(4)}$ and the American being from 65 to 75 very close together). Thereafter the Mutual Benefit curve would run close to the Medico-Actuarial curve to age 90. Accordingly, the Mutual Benefit Experience is very much closer to the Medico-Actuarial Experience than to the American. This would probably bring out higher reserves on an ordinary life policy—even higher than the M.A.$^{(4)}$.

Mr. Hutcheson spoke almost entirely on the assumption that any new table, if prepared, would be adopted. Now of course, there is quite a chance that such new table would be adopted; but there is going to be another big discussion before such adoption. If we prepare a new table, we may get opposition from all sorts of sources on the question as to whether or not it should be adopted. Mr. Welch referred to future mortality, stating that if we adopt a new table now, it will not represent the mortality ten years hence. Perhaps not; but it would represent it more nearly than the American Experience Table.

Mr. Macaulay defended the American Table, yet nearly all of his remarks would apply as criticisms of the American Table. What we should have is a more correct curve of mortality, and one more truthful to experience. The curve of the American Table is entirely out of accord with present-day facts. We surely can get a curve that represents modern mortality, and get away from the rates which may have been experienced from 1840 to 1860, such as we are now using. I am not prepared to subscribe to Mr. Macaulay's statement "We all believe that the legal standard for reserves is high enough now." The Canadian standard, adopted a few years ago (and if my impression is correct adopted with Mr. Macaulay's advice and approval) is much higher than the American. Moreover, with preliminary term valuation and premium rates which are sometimes cut down to a minimum, I think Mr. Macaulay's further statement "an increase in the legal reserves is quite unnecessary," while it may apply to Canada, certainly does not apply to all conditions in the United States.

Of course the question of separating Canadian from American mortality would be discussed, and thoroughly taken into consideration by any committee put in charge of this work. My own personal view would be that it is better to prepare a table for the United States; possibly, at the same time, for the convenience of Canadian companies, should they wish it, we should investigate the mortality of Canadian business, so that the two could be published separately if desired.

The question of preparing tables for different classes of policies is a detail which would be considered by the committee. Probably it would be well to do so. As to whether or not they should be combined in one table, would be a matter for later discussion.

Replying to specific questions by Mr. Rhodes, Mr. Moir added: "If higher reserves as a test of solvency are called for by the curve which comes out, I should rather see them. I am inclined to think that a curve based on amounts insured would show still greater steepness and higher reserves than the Medico-Actuarial. Of course you know that I do not approve of the full net premium method of valuation as a test of solvency at all."

Supplemental Memorandum by Mr. Moir.

The terminal reserves quoted in the paper give the figures on which cash and other values are predicated, also the sums which must be carried in order to make good the ultimate guarantee and pay the sum insured. Mr. Henderson has pointed out in the discussion that the reserves from year to year by an active company are on a different basis; and, in order to prevent any misconception as to the operation of a new table as applied to the valuation of policies, especially by the younger companies, it seems desirable to illustrate how mid-year values work out. On the whole, they are lower than the terminal values because of the low premium rate which any new table will doubtless show.

On the ordinary life and 20-payment life plans, 3½ per cent. interest, the following figures are applicable at age 35:

Ordinary Life—Mid-Year Reserves, 3½ Per Cent.

Year.	M.A.$^{(4)}$	Am. Exp.	Percentage of New to Old.
1	15.08	15.83	95
2	28.18	27.79	101
3	41.72	40.13	104
4	55.71	52.87	105
5	70.18	66.02	106
10	148.72	137.99	108
15	235.42	220.39	107
20	328.16	311.29	105

20-Payment Life—Mid-Year Reserves, 3½ Per Cent.

Year.	M.A.$^{(4)}$	Am. Exp.	Percentage of New to Old.
1	22.19	23.49	94
2	42.66	43.44	98
3	64.02	64.13	100
4	86.00	85.60	100
5	108.62	107.87	101
10	234.33	232.29	101
15	380.36	381.43	100
20	549.98	560.23	98

But there is an even more important feature which introduces another modification into this question of reserves—the fact that the great majority of the younger companies are valuing their business on modified preliminary term plans. The low premium rates for one year term insurance by the new table reduce the values to an even greater extent. Modified preliminary term mid-year reserves will therefore still further reduce the above percentages in the early years; and the following tables bear this out:

Ordinary Life—Preliminary Term Mid-Year Reserves, 3½ Per Cent., Age 35.

Year.	M.A.(4)	Am. Exp.	Percentage of New to Old.
1	2.47	4.33	57
2	15.77	16.42	96
3	29.54	28.91	102
4	43.53	41.80	104
5	58.13	55.10	106
10	137.82	127.94	108
15	225.64	211.31	107
20	319.67	303.29	105

20-Payment Life—Preliminary Term Mid-Year Reserves, 3½ Per Cent., Age 35.

Year.	M.A.(4)	Am. Exp.	Percentage of New to Old.
1	2.47	4.33	57
2	23.67	24.95	95
3	45.66	46.35	99
4	68.25	68.55	100
5	91.74	91.59	100
10	222.15	220.32	101
15	373.48	374.78	100
20	550.08	560.22	98

The general effect on the business of a young company by the introduction of a new mortality table would be to reduce the reserves for at least the first five years after the change. If such a company were writing a normal business on standard plans the percentage of the reserves under the new table to those required under the existing American Experience Table would be somewhat as follows:

	Per Cent.
First year	60
Second year	90
Third year	94
Fourth year	97
Fifth year	99

Thereafter the reserves would run up to and doubtless beyond those now being carried under the American Experience Table. Such companies should be duly warned that the low reserves arise from the mode of valuation, and that in years subsequent to the first larger sums have to be set aside to reserve since the ultimate reserves will be greater by a new table than by the American Experience Table. While, therefore, such new table would doubtless under preliminary term valuation show very small reserves in the first year, the building up of the reserve in years subsequent to the first would cause a greater strain on the income of such companies, since ultimately the same sum insured is guaranteed and the larger reserve is necessary so as to provide for such sum insured.

Practical Treatment of Under-average Lives.

BY

ARTHUR HUNTER.

The subject of insuring underaverage or substandard lives possesses a great deal of interest at the present time in view of the publication of the results of the Medico-Actuarial Mortality Investigation. Many companies are doubtless taking advantage of the information obtained through that investigation to remodel their plans for the insurance of lives which are not considered up to the standard. All companies which issue policies to substandard lives have to meet practical problems which are frequently not apparent on the surface. The main purpose of this paper is to discuss some of these, in the hope that there will be called forth a free discussion which will result in improved methods for the treatment of such lives. Few references to this subject appear in the *Transactions of the Actuarial Society,* although many actuaries have had extensive experience in this line, and accordingly students are at a loss to obtain authoritative information. As there are no fixed standards for the insurance of underaverage lives, there must be a marked diversity in the practical treatment of them, and, in order to put before the readers the problems regarding which there are likely to be differences of opinion, it will be necessary to take up questions which appear to be of comparatively small moment to those who are not actively engaged in this branch of the business. It would be impossible to consider at this time the actuarial bases underlying the various systems of handling underaverage lives, or to consider in detail the advantages or disadvantages of the methods in use for the treatment of these lives.

Basic Standard for Rating Risks.

Whatever actuarial system of treating underaverage lives is adopted by a company, a method must first be determined of rating these risks, or of ascertaining approximately their relative mortality from the time the policy is issued. It is hardly necessary to point out that while reference may be made to the relative mor-

tality of an individual life, I am merely taking that as an example of a group of lives whose prospective mortality is under consideration.

Until recent years the appraisal of the case was largely the result of the impressions of the medical director or of a committee of medical directors, actuaries, and other officers of insurance companies. No attempt was made to place a valuation on the risk, such as 150 per cent., 200 per cent., of the American Table, but after a careful consideration of all the factors, a lien or extra premium was decided upon, which, in the judgment of the officers charged with this duty, was deemed adequate to cover the extra mortality expected. This has been referred to as an empirical method based on the personal observation of the officers of the company, but as in most cases it has not resulted in loss, no serious criticism of it can be made, provided substantial justice was done to these policyholders. The methods in use in the past for the treatment of underaverage lives are said to be based on common sense, which may be assumed to be the product of impressions, observations, wide reading and a few statistics. During recent years these methods have not satisfied the officers of companies, and the Specialized Mortality Investigation was the first attempt to develop more scientific bases. While that investigation was considered far-reaching and thorough at the time it was first planned, the more recent Medico-Actuarial Mortality Investigation shows how rapidly we have progressed during the intervening period of less than ten years.

A few years ago there was no way common to actuaries and medical directors of expressing the relative mortality. At the present time the members of both these professions use the percentage of the normal mortality in expressing the mortality that is likely to be experienced. If a statement is made that among certain risks the mortality is 150 per cent., it means that the death rate is 50 per cent. in excess of that among lives accepted as standard risks. In order that such a comparison may not lead to erroneous conclusions, the basic mortality table should represent the experience of the company in question, and it is hardly necessary to point out that such comparisons should not be made with the American Table, but with a suitable modification of that table or of the M. A. Table. In this connection it cannot be too strongly pointed out that there may be a marked difference between the

mortality by policies, by lives, and by amounts insured. The experience of the American and Canadian companies as represented by the M. A. Table came as a surprise to actuaries, on account of the light mortality, the apparently short period of selection, and the very gradual increase in the ultimate rate of mortality. Until a corresponding table based on amounts insured is prepared, we cannot outline the different incidence in this country between the experience by amounts insured and by policies. It is practically certain, however, that the combined experience of the American companies would show a higher rate by amounts than by policies, especially at the older ages of entry.

While many companies have not enough experience to furnish data for a mortality table which would take account of ages at entry and duration of insurance, they are in a position to prepare a standard from their own experience expressed in terms of some existing table. The medical director and the actuary will probably look at this problem from different standpoints. The former is dealing with lives, and the latter principally with amounts insured. In determining the premium rates and surplus from mortality the actuary is not concerned with the number of persons who die, but with the rate of financial loss through death. It is proper, therefore, that the relative mortality for the various types of lives should be based on the company's experience by amounts insured. This need not cause any confusion, because, with few exceptions, the mortality in the different types of lives bears about the same relation to the standard of the company whether the relation be determined by an investigation based on lives, policies, or amounts insured,—i. e., if the relative mortality in a certain group of lives is 150 per cent., the corresponding percentages based on policies or on amounts insured are not likely to differ to a material extent.

Method of Rating Risks.

Since my paper of May, 1911, was delivered before the Actuarial Society several companies have adopted the "numerical" method of rating risks, but the majority of the companies still accept or decline an application for insurance without endeavoring to place a numerical rating on it. Medical directors and other officers generally use the method of weighing in their minds the strong and weak points in each case before deciding upon their recommendation. While these officers may have clearly in their minds the re-

sults of the M. A. Investigation and other published experiences, this method of judging a risk does allow a large factor for personal prejudices and opinions. When a company undertakes to grant policies with an extra premium or a lien, it is difficult to do justice between the various applicants without a numerical method of rating or some similar plan. In the discussion of my paper, already referred to, many good points were made, and the defects of the numerical method were brought out. Freely admitting that the system is not a perfect one, and acquiescing in the opinion of one of my critics that it requires men of unusual experience and unusual judgment to use it, and that there are pitfalls for the unwary and inexperienced, I believe that the adoption of such a method is better than to cling to the old plans. Mr. George King stated that the older methods were largely guess work, although he considered that the experience of actuaries gave them the kind of intuition which enabled them to determine a fair premium for underaverage lives.

In this paper it will be assumed that the rating is based upon a numerical or similar system which enables the actuary to forecast the mortality in relation to the experience of his company on standard lives.

Principal Methods of Treating Underaverage Lives.

There are four principal methods of treating underaverage lives:

First, a lien is charged against the face of the policy, generally reducible by the premiums paid from year to year;

Second, the policy is placed in a special class as to dividends;

Third, an extra premium is charged;

Fourth, the premium is taken at an age in advance of the true age.

There may be combinations of the foregoing types.

Lien System.

The system of placing liens against the insurance has been in use for many years. The regular rate of premium is generally charged, but the amount payable at death in the early insurance years is less than the face value of the policy. The amount of this lien depends upon the estimate of the rating of the risk—the poorer the risk the heavier the lien—and is usually reducible by the annual premiums paid. There is no scientific reason for re-

ducing the lien by the annual premium, but that method is generally adopted in this country on the grounds of simplicity and saleability. In Great Britain the general practice is to impose the lien for a period equal to the expectation of life, reducing it by equal annual payments during that period. In Vol. XXIX of the Journal of the Institute of Actuaries A. W. Sunderland gives formulae and tables for decreasing liens under the British system.

The following is an illustration of a method which may be used to determine what extra mortality can be carried under policies issued with various liens. Let us consider the case of an ordinary life policy of $1,000 issued at age 40 with a lien of $300, decreasing each year by an annual gross non-participating premium of $25.61. By making calculations for the policy with lien, assuming various mortality factors (i. e., 100 per cent., 125 per cent., 150 per cent., etc., of the American q_x), we have a basis for interpolation to obtain approximate net premiums at multiples of the normal mortality.

NET PREMIUMS ON ORDINARY LIFE PLAN; LIEN $300, REDUCIBLE BY $25.61 ANNUALLY.

(1) Multiple of American q_x.	(2) $M_{40} \times 700$.	(3) $R_{40} - R_{51} - 11M_{51}$.	(4) Col. (3) $\times$ $25.61.	(5) $M_{51} \times 300$.	(6) (2)+(4)+(5).	Net Premium Col. (6) $\div N_{40}$.
100%	7,700,282	14069.71	360,325	2,588,258	10,648,865	$23.96
125	7,724,332	16234.51	415,766	2,485,059	10,625,157	26.98
150	7,645,449	17974.72	460,333	2,358,264	10,464,046	29.81
175	7,499,345	19355.81	495,702	2,220,396	10,215,443	32.50

Comparisons of the net premiums on basis of 100 per cent., 105 per cent., 110 per cent., American q_x with the regular American net premium for the policy without lien indicates the percentage of mortality which the policy with lien can carry. Thus the net premiums interpolated from the above table are:

Multiple of American q_x.	Net Premium.
105%	$24.58
110	25.19
115	25.79
120	26.39

The regular American net premium for a life policy is $24.75, and that premium corresponds in this schedule to a percentage of

106. Therefore with a $300 lien on a life policy of $1,000 at age 40, an additional mortality of 6 per cent. will be covered.

The additional mortality which the lien will cover may be seen from the following exhibit, based on the American Table of Mortality with 3 per cent. interest, the lien being reducible by the annual premium which is charged by a large insurance company issuing non-participating insurance only:

	Lien per $1,000.	Percentage of American Mortality Borne by Lien.		
		Age 25.	Age 40.	Age 55.
Ordinary life.....	$300	9%	6%	5%
	500	31	22	17
	700	92	65	49
20-payment life.....	300	9	7	5
	500	28	23	18
	700	78	69	52
20-year endowment....	300	11	9	6
	500	39	33	21
	700	120	99	60

The meaning of the above may be made clear by an example. At age 40, on the ordinary life plan, the company can cover an extra mortality of 22 per cent. of the American Table, if a lien of $500 per $1,000 is placed against the insurance, such lien being reducible by the premiums paid. In considering the comparatively small extra mortality borne by these liens, it should be remembered that a reserve higher than the American 3 per cent. should be provided except at certain ages under endowment insurances.

Where liens are used without the policies being placed in a special class as to dividends, the comparatively small extra mortality which is covered by means of these liens should be kept in mind. It is on this account that the lien system is now little used among the prominent companies. The above table indicates the practical difficulty which lies in the way of inducing an applicant to accept a policy with a large lien when he is not a seriously impaired risk.

In Volume XXXII of the *Journal of the Institute of Actuaries* there appears an article by G. F. Hardy, dealing with a method of computing the temporary deductions to be made from the sums assured, on underaverage lives. He bases his method upon the assumptions (*a*) that the extra mortality is confined to the period

during which the policy is subject to a deduction and that thereafter the policy may be treated for all purposes as having been effected at the ordinary rates of premium; and (*b*) that the risk is so distributed throughout the above period that the reserve value of the policy subject to such temporary deduction is the same as that of an ordinary policy of the same age and duration. While this is a very ingenious way to meet certain of the problems arising in connection with underaverage lives, its usefulness is limited. From a number of examples which he gives it appears that a lien of $600 per $1,000, decreasing by equal parts during the expectation of life, is equivalent to an advance of about 11 years at age 25 at entry, and of 8 years at age 40. Under the former the lien would continue for 38 years, and under the latter for 27 years. The extra mortality which would be borne by the lien at age 25 would be about 60 per cent., and at age 40 about 70 per cent. The large lien, the long period over which it is continued, and the comparatively small additional mortality which is covered, militate against the use of Mr. Hardy's method.

It has been asserted, and my observation seems to confirm the assertion, that the placing of liens on policies results in a favorable selection for the company. It seems that when a policy with a lien is offered to a substandard risk, he is apt to take the policy if he feels that he is a better risk than the company considers him, while the man who knows that he is not a good risk is apt to seek insurance in a company which will grant him a policy with an extra premium.

If a company decides to issue policies with lien, it has two difficult questions to consider. First, what reserve should be carried? Second, what modifications should be made in the non-forfeiture values on account of the lien? With regard to the first, it is hardly necessary to point out that the reserve is different for each form of lien, partly because the higher the lien the greater would be the extra mortality anticipated. If, therefore, a company adopts a system of $200, $300, $400, etc., of initial liens, and assumes that the extra mortality would be a percentage of the standard table throughout the life of the policy, it would have nine different sets of reserves for ordinary life policies, and the same for each of the other plans. For practical purposes the reserves for the average lien (say, $500) could be determined and applied to all policies with lien, or two sets of reserves might be used, one for high lien ($600

to $900) and the other for low lien ($200 to $500). In calculating such average reserves the mortality anticipated at each age by the company must be taken into account. For example, if a $500 lien per $1,000 is imposed on applicants who are rated 150 per cent. at age 25, that ratio of the mortality rate of the standard table should be used in determining the reserve, and not the average assumed mortality (say, 135 per cent.) for all ages of issue with that lien.

With regard to the second question, that is, the modification which should be made in non-forfeiture values, certain states require the companies to allow values equivalent to the reserve under the policy less a specified surrender charge. In general, the maximum surrender charge is $25 per $1,000, but in the state of Louisiana it is 20 per cent. of the reserve. It should be borne in mind that 80 per cent. of the reserve on a regular ordinary life policy would in the early policy years be generally less than is required under the law in that state on a similar policy with a high lien, because the reserve on a contract with such a lien is greater than on a regular policy. This may be seen more clearly by an exhibit. In this example the net premiums for the policies with lien are not the same as the American—in fact, the net premium under a rating of 125 per cent. of the American is $25.06 against the gross non-participating premium of $25.61, indicating that other factors than the loading must take care of the expenses.

ORDINARY LIFE POLICY—AGE 40.

LIEN OF $500 REDUCIBLE BY ANNUAL PREMIUM OF $25.61.

AMERICAN TABLE OF MORTALITY.

End of Year.	Regular Reserve.	Reserve, taking Account of Lien, American Mortality.	Reserve, taking Account of Lien, 125% of American Mortality.
	(*a*)	(*b*)	(*c*)
2d	$ 32.14	$ 36.74	$ 39.62
5th	83.54	94.56	101.86
7th	119.88	134.83	145.13

The foregoing indicates that 80 per cent. of the reserve on a policy with lien is greater than 80 per cent. of the regular reserve, and it might be held that the former should be the minimum value under the Louisiana law. If the prospective mortality of risks accepted with the $500 lien is taken into account, the reserve by that method, (*c*), would be greater than under the method providing for the American mortality, (*b*).

Under 10-, 15- and 20-year endowment insurances the true reserves under various multiples of the American mortality do not differ materially from the corresponding reserves on 100 per cent. of the mortality under that table, in fact the former is frequently less than the latter. The same is true in general of such policies with moderate liens. Under limited-payment life policies the reserves are materially larger at 125 per cent., 150 per cent., 175 per cent., etc., of the American than at 100 per cent. of that table. This raises the question: Should the surrender values be based on the reserve? If, for example, the full reserve were allowed at the end of 20 years on a 20-payment life policy issued at age 30, the cash value on the basis of 200 per cent. of the American mortality with 3 per cent. would be $652.14 against $555.22 on the regular American 3 per cent. table.

The lien system is particularly applicable to risks in which the extra mortality gradually decreases,—such as that due to light weight combined with a history of tuberculosis. It is not entirely satisfactory in connection with a heavy extra mortality which exists for a short period after the issue of the policy—such as that resulting from a history of appendicitis, because of the very high lien required in the early policy years. Theoretically a rapidly decreasing lien would be excellent, but practically the lien is so heavy that agents find great difficulty in inducing applicants to accept such policies. For applicants, therefore, who have had appendicitis, cancer, and similar diseases which are liable to recur within the early years following the attack, an extra premium for a few years is the most practical form of treatment. Liens are of little value and are unsatisfactory when the extra hazard does not occur until the policy has been in existence for many years. Probably the most striking example of such cases to which the lien system is inapplicable is that of persons distinctly overweight at the young ages of entry, as the relative mortality increases from the young to the middle ages (attained) of life.

Special Class as to Dividends.

The device of placing policyholders in a special class as to dividends was used to a considerable extent prior to 1907 under deferred dividend policies. It is an equitable way of treating persons in hazardous occupations. For example, a class might be created of policemen, and another of electricians who handle live

wires. Applicants for insurance in such occupations consider it fair to ascertain the dividends according to the mortality experienced among persons in their occupation, provided they are charged the regular rate of premium.

With regard to lives substandard for other reasons than occupation, a difficulty arises in determining the boundaries of the classes. It might, for example, be considered that all risks rated up to 115 per cent. of the standard should be put in the regular class, which might be called standard; those above 115 per cent. but not above 140 per cent., in the intermediate class; those over 140 per cent. but not above 180 per cent., in the hazardous class; and those above 180 per cent. but not above 220 per cent., in the extra hazardous class. While a system of this kind does not discriminate with exactness between the different individuals in each class, it gives as much justice as the present practice of granting insurance at the same rates of premium alike to those who are exceedingly good, those who are very good, and those who are average lives. The grade of lives accepted by the various companies at the regular premium scale probably ranges from as low as 60 per cent. to as high as 115 per cent. or 120 per cent.

If the plan outlined above were adopted, it would be necessary to determine in advance the approximate proportion of the dividend which would be used in meeting extra mortality. It would not, for instance, be advisable to have such a highly substandard body of risks in the extra hazardous class that no dividends would be due under policies with 20-year deferred dividend periods. In this connection it will be found that a much higher relative mortality can be carried by using for extra death losses the dividends under a 20-year endowment than those under an ordinary life policy. Before any plan for creating special dividend classes is put into effect, the maximum extra mortality which can be borne by using the surplus to pay extra mortality should be ascertained. It is then a matter of business judgment to determine what proportion of the regular dividends it would be advisable to pay in each of the substandard classes, and thereafter to calculate the range of cases which should be put into each class. Thus, if in the intermediate class it is desired to pay at the end of 20 years three quarters of the surplus on corresponding policies in the standard class, calculations should be made to determine approximately what extra mortality one-quarter of the surplus under such policies would

cover. This calculation may be made in several ways, but probably the most satisfactory is to prepare a model history of a group of 10,000 individuals taking account of the death, lapse, interest and expense rates, also the surrender values allowed. (It is assumed that these factors in the future would not be widely different from the past.) By then changing the rates of mortality to 125 per cent., 150 per cent., etc., of the normal, and by making allowance for the lower surrender values which may be granted on substandard policies, the approximate extra mortality which will eliminate one-quarter of the surplus in the group may be ascertained. From the relative mortality thus determined a deduction should be made to allow for unforeseen contingencies, such as a different rate of retirement, or a different incidence of mortality than was anticipated.

A simple but rough way to determine the maximum additional mortality which can be borne in 20 years is to consider the difference between the non-participating and the corresponding participating premium as an annual extra to meet additional mortality—due allowance being made for difference in expenses.

When special dividend classes are made, the difficulties in connection with the reserves and the surrender values are not serious. A reserve on the basis of the Actuaries' or of the American Table is believed by some actuaries to be sufficient for a class of risks rated from 115 per cent. to 135 per cent. with an average of about 125 per cent., as the actual mortality experienced by the company on such risks will probably be less than under either of the two tables just mentioned. When the mortality anticipated is at least 150 per cent. of the company's normal experience, a higher reserve than the American or Actuaries', except on endowment insurances with periods of not more than 20 years, should be provided for. For endowment insurances with periods of not more than 20 years, the reserves on double the American rates of mortality are not materially higher, indeed are frequently lower than the corresponding reserves on the regular American Table except at the older ages at entry.

Where no extra reserve is carried the loan, cash and paid-up values are customarily the same as under standard policies, but the continued insurance or term insurance is frequently lower. If a multiple of the mortality by the American Table is used, such as 125 per cent., it will be found that the reserve thereunder applied

in purchasing paid-up insurance by the single premium on the basis of 125 per cent. of the American gives practically the same paid-up insurance as if the net American Table were used throughout. This, however, is not generally true of extended insurance. As the latter benefit is usually automatically granted when the insured does not apply for some other form of non-forfeiture value, it behooves the companies to guard against granting under life policies the full period of term extension on the American Table to risks whose mortality in the future may be distinctly heavier than that standard.

Where an extra reserve is carried the question arises of granting higher loan and cash values. A higher cash value than that of the minimum of the state, usually American 3 per cent. or American 3½ per cent. would call for an extra reserve by the company, and, conversely, an extra reserve would in some instances call for a higher cash value than on policies in the regular class as to dividends in the case of companies doing business in the state of Louisiana. Reference has already been made to the problem presented in that State.

Policies with Extra Premium or Advance in Age.

For convenience, the two methods of charging an extra premium will be treated under one heading, although the underlying principles differ in certain particulars. Before adopting either method a company must consider which is best adapted to its needs. It may be said in passing that there is no system for treating underaverage lives which is satisfactory in every particular, and that, accordingly, the method should be adopted which gives justice in general to all policyholders, which does not discriminate markedly between different sets of policyholders, and which is adapted to the conditions under which the company does business. The first question to be considered is whether an extra premium per $1,000, or an advance in age, would be most suitable to the type of business received by the company. If the underaverage business of the company is largely limited to cases in which there is a hazard from occupation, and if this hazard is principally from accident, the system of charging an extra premium is the better one. In general, where the additional mortality is a constant, irrespective of age, the extra premium method is better than the advance in age; but if the additional hazard is expected for many years to be a

percentage of the regular mortality, the advance in age is the better plan. The extra mortality may also be a decreasing percentage of the normal for a longer or shorter period, in which case the extra premium is probably better than the advance in age. As the business of the company is likely to be composed of applications from persons under these and other conditions, neither of the systems under discussion is entirely satisfactory in every particular. Some companies have therefore adopted both the extra premium and the advance in age methods, while others, for the sake of simplicity, have adopted the one which in the main fits the majority of the cases received.

Let us first consider the practical questions which arise in the case of a company which has adopted the extra premium method. If an extra premium is charged on a participating policy, should that extra be entitled to dividends? This question must be met before the extra premium is calculated, because a larger loading should be added if it is to participate in surplus. Should the dividend be the same percentage as of the regular premium? The percentage of dividends increases with the duration of the insurance, and therefore the extra premium should be so loaded as to provide therefor; or if a flat addition is made to provide for dividends on the extra premium, it must be assumed that the mortality hazard decreases with the increasing duration of the policy. What reserve should be carried on policies with an extra premium? If the additional hazard is assumed to be a current one, then one-half the net extra premium is sufficient. A few companies have converted the extra premium into an advance in age, and have carried the reserve at the advanced age. This gives an unnecessarily high reserve, which raises questions regarding larger cash values, especially under limited payment life. The general custom of the companies is to grant the same cash, loan and paid-up values as under similar policies at the regular rate of premium, but the term extension is frequently eliminated. Another question arises: Should commission be paid on the entire premium? There does not seem to be uniformity of practice in this respect, but a majority of companies apparently pay the same percentage of commission. Where an extra premium is charged for a brief period to cover a history of appendicitis, cancer, or other disease where the liability to recurrence disappears in a few years, a commission is not usually allowed. The extra premiums should not of course be fixed until the question of commission is settled.

Some companies prefer to adopt a system of advancing the age in all cases whatever the type of hazard, on account of the simplicity of the system and the ease by which it is understood by both agents and insured. It is customary to allow the cash, loan, paid-up and extended insurance values at the increased age, and also to grant the dividends at that age. This results in great simplification, especially as for valuation purposes the rated up age of the insured may be taken in all cases. The advantage of the system from the standpoint of the insured is that he gets the increased cash values at the advanced age. From the standpoint of the company, however, an additional problem is introduced by that fact. Those who surrender their policies in the early years may get a substantial portion of the extra premium returned through increased surrender values, and the actuary should consider this factor in determining the advance in age. For example, if a policy were applied for at age 30 on the 20-payment life plan, at a premium of $34.76, and if it were issued at a rated up age of 40 at a premium of $42.79, the insured would pay $8.03 per annum extra, or $40.15 in five years; but the surrender values at the end of the fifth year if he were then to terminate the contract would be $25 more than the value at age 30, i. e., more than one-half the extra premiums would be returned through an increased cash surrender value. Under policies terminated for any other cause than death, it is generally held that a provision should be made for this increase in the surrender value, although the principle underlying the advance in age system would permit of such return under certain types of impairments.

It is not my intention to deal with the formulae for calculating the advance in age, as they are familiar to actuaries. In at least one company the question has been handled in the following manner:

In order to determine the advance in age, the extra premium must first be determined. This is done by calculating the difference between the net premium on the standard table and the corresponding net premium on the multiple of the standard table which is assumed to represent the mortality under the risk in question, and then determining the corresponding advance in age. For example, if a risk were assumed to have 150 per cent. mortality at age 30, and the application were on the ordinary life plan, the American net premium would be taken, amounting to $18.28, and

deducted from the corresponding net premium on 150 per cent. of the American, $23.21, leaving an extra premium of $4.93, which, added to the net premium at age 30 on the American Table, would be equivalent to the net premium at age 38. (The advance in age will not be materially different if the M. A. Table or a modification of the American Table, representing the company's experience, is used.) Before adding this extra in order to determine the equivalent advanced age, a small addition is sometimes added to cover the factor of increased cash values which has just been mentioned.

It is not necessary to consider the question of surrender values or reserves; after the advance in age is determined the case is treated in every respect as a standard life at the advanced age.

When a risk is markedly impaired, it is the part of wisdom to issue only 10-, 15- or 20-year endowment policies. If the continued insurance is not eliminated, then part of the object in obtaining a good selection by limiting to high-priced policies is defeated. A short calculation will show that if the insured carries the policy for a few years and then takes term extension, he is getting the insurance at practically the same rate as would have been charged him for a term policy in the first instance.

Improvement in the Risk.

One of the difficult questions which presents itself is the treatment of policyholders who have improved as insurance risks. It frequently happens, for example, that a man who has a heart murmur, probably functional in its character, is accepted by another insurance company as a standard risk a few years after he had been charged an extra premium or had a lien placed against his policy. If his policy is not changed to a standard one he states that he will surrender his contract. A similar difficulty arises if the applicant receives a policy at the regular rate two or three years after having been granted a policy with an extra premium or an advance in age in the same company. The solution of the problem thus arising depends largely on the method used in determining the relative mortality which forms the basis for the treatment of underaverage lives. If, for example, the basis were the experience of the company on its declined risks, the original policy could not be changed because the grade of declined risks which had improved formed part of the data for the mortality rating. If the basis of the rating were the experience on policies which were formerly issued

as standard risks, the same holds true, because there were no means of determining the number of lives which improved in value as insurance risks. If the basis were the experience of the company with substandard lives, under which the lives which improved in value were eliminated from the data at the time they were given better forms of policy, then the extra premium could be removed or reduced on cases which improved as risks. When the basis upon which the mortality rating was determined did not provide for improvement in the risk, an addition should be made to the mortality rating in order to cover this feature,—otherwise dissatisfaction will be created among both policyholders and agents. The experience of a company which did not make such provision came to my notice. The invariable statement by the home office was that the company had provided for a certain mortality among the insured with each type of impairment, and that as they could not treat those who deteriorated less liberally than stipulated in the contracts, they could not give better policies to those who improved. It was very difficult to satisfy the policyholder in such cases because he could take the surrender value and apply for new insurance. When no better offers are permitted after the policy is issued, "twisting" by rival agents is easy. Where an advance in age has been taken off and the insured has the maximum loan, the policy at the true age would not provide adequate security therefor. Payment of another annual premium which may be made a condition of the change generally meets the difficulty.

In an impairment which lasts only for a very few years some provision for remission of the extra premium is necessary. Take, for example, the case of an applicant who has had two attacks of appendicitis. It is generally believed by medical men that the liability to recurrence will pass in a short time. If, therefore, an extra premium of say $25 per $1,000 were charged for three years on account of the liability to recurrence, what action should a company take if the man one year after being insured undergoes an operation for removal of the appendix? He could get insurance in any good company a reasonable time after a successful operation without extra premium and would naturally object to continue to pay the extra premium for another year. The solution to this problem is to provide a sufficient extra premium to cover those who having had a history of an attack of appendicitis die from that disease, and to provide for the remission of the extra

premium on those who are successfully operated upon. In determining the extra, consideration should be given to the fact that a goodly proportion of policies are terminated at the end of the first year, and these are doubtless on the lives of persons who do not anticipate a return of appendicitis,—i. e., the better class of such risks.

Change from Hazardous to Non-Hazardous Occupations.

When an insured changes from a hazardous occupation to one in which there is little or no hazard, the question of treating him as a standard risk arises. As these policyholders are generally charged an extra premium, or advanced in age, we shall not refer to other methods of providing for the additional mortality. The treatment of such cases depends largely on the basis used in determining the mortality rating. If we knew, for instance, that all the companies which contributed their data to the Medico-Actuarial Mortality Investigation charged saloon-keepers an extra premium and remitted such premium without evidence of health on their leaving the occupation, we could assume that a similar procedure would be safe to adopt for an individual company if the extra premium were based on the M. A. mortality for the occupation in question. We know, however, that there was no such uniformity of treatment by the forty-three companies, and that in furnishing the data most of the companies did not record any change in the occupation after the policy was issued. We are therefore forced to determine from our knowledge of the conditions under which a number of the companies operate and from our own experience how we should treat policyholders who change their occupation. A good practice, in my judgment, is to require a probationary period of a year after the insured has changed his occupation, then to alter the policy from the original date of engaging in the less hazardous occupation, making a refund of the extra premiums paid since that date provided he proves to be a standard risk. In such cases it is customary to require the policyholder to bear the expense of the medical examination. This treatment assumes that the extra premiums paid by all the insured in the particular occupation are not only to cover the risk of death while engaged in the hazardous occupation, but also the liability of the insured's becoming so deteriorated through his occupation that he had to give it up. Companies which reduce or eliminate the extra

premium without such requirements should see that the basis for their occupational mortality ratings provides for the reduction or elimination of the extra premiums of those who change to a less hazardous occupation, irrespective of their then condition of health.

Special Problems.

When a company is issuing policies with liens, a request is occasionally made to give the applicant the option of taking that form or one with an extra premium equivalent to the lien. If a company grants such requests, it is likely to suffer financially because the insured who believe in themselves will generally be contented with the lien, while those who fear that their lives will be cut off early will prefer to pay the extra premium.

The mortality experience of a company which has charged extra premiums on its business during one period of its existence, and liens in another, may be distinctly different on corresponding grades of risk. This is especially true of highly impaired risks. It need hardly be pointed out that the higher the extra premium or the lien, the smaller is the percentage of business placed—that is to say, there is a greater selection exercised against the company the more seriously impaired the risk is. All things being equal, a company issuing policies with lien is likely to deliver a larger percentage of business to highly impaired risks than another company granting policies with extra premium to lives similarly impaired. The experience of at least one company which charges an advance in age has shown that highly impaired risks must be taken with great caution, and that the selection against the company can be offset in a measure by restricting such cases to small amounts of endowment insurance. Of course too much severity would defeat the very object which the company is trying to attain.

Occasionally we hear of a company which insured a man on the 5-year non-renewable term plan, but would not grant him ordinary life, limited payment life or endowment insurance. Is this practice correct? In my judgment, the only reason for such action would arise from the extra risk being an increasing one,—that is, the relative mortality would be practically normal in the first five years, and then distinctly higher thereafter.

A question which is frequently put to the actuary is: Can we not adequately cover certain types of impairments by restricting to the higher priced plans of insurance? There is probably not a

company which during the course of its experience has not limited persons who were overweight, those who have had an attack of syphilis, etc., to 15- and 20-year endowment policies. The Specialized Investigation showed that the companies had a higher mortality than the normal on policies offered by the companies on higher-priced plans than applied for. It seems to me that the question resolves itself into whether a company, rather than lose business, is willing to stand a slightly higher mortality than the normal on part of its 10-, 15- and 20-year endowment policies. If a company on such part of its endowment business experiences 25 per cent. extra mortalty, the monetary loss would be small, and unless an undue proportion of such risks were accepted, the mortality among the endowment insurances as a whole would be distinctly lower than among ordinary life policies. It is therefore a matter of business judgment whether the company is willing to take on the endowment plans a proportion of risks under which the mortality is likely to be slightly above the normal. It has been stated that the psychological effect of acceptance by the applicant on endowment insurance constitutes a selection in favor of the company.

In addition to the papers already mentioned attention is drawn to the contributions of Mr. J. Burn, in the *Transactions of the Fifth International Congress of Actuaries,* of Mr. Ralph Todhunter in Vol. XLIX of the *Journal of the Institute of Actuaries,* and of Messrs. White and Whittall in Vol. XXIV of that Journal.

Several actuaries have been good enough to read this article, and had I attempted to answer the various questions raised by them, the paper would have been too long. I must therefore leave several phases of the subject to be amplified in the discussion, such as the variations in the plans which should be made on account of the different incidence in the extra mortality.

Valuation of the Death Benefits Provided by the Workmen's Compensation Law of New York.

BY

EDWARD OLIFIERS.

Section 16 of Article 2 of the Workmen's Compensation Law of New York may be summarized as follows:

16. *Death Benefits.*—If the injury causes death, compensation shall be known as a death benefit, and shall be payable in the amount and to or for the benefit of the persons following:

1. Reasonable funeral expenses, not exceeding one hundred dollars;

2. If there be a surviving wife (or dependent husband) 30 per-cent. of the average wages of the deceased until the remarriage of the widow, together with two years' compensation in one sum upon such event.

If there be a surviving child or children under the age of 18 years there shall be payable to each such child 15 per cent. of deceased's wages until age 18, except during the lifetime of the surviving wife (or dependent husband) the child's pension is 10 per cent.

The total amount payable shall in no case exceed 66⅔ per centum of such wages.

3. If there be no surviving wife (or dependent husband) 15 per cent. with the same limits as to age (eighteen) and as to amount (66⅔ per cent.) as mentioned in paragraph numbered 2.

4. If there are grandchildren or brothers or sisters, parents or grandparents, dependent upon the deceased, 15 per cent. of deceased's wages (the pensions to the grandchildren, brothers or sisters are payable until age eighteen, while the pensions to parents and grandparents are payable for life).

The aggregate amount payable under this subdivision shall not exceed the difference between 66⅔ per cent. of such wages and the amount payable as provided in subdivisions 2 and 3.

The wife and children of the deceased are a preferred class of claimants.

Maximum salary upon which compensation may be computed is $1,200 a year.

An actuarial expression for the present value of the death benefits payable to a widow and several children may be obtained by allowing the full percentages of the decedent's wages as if there were no maximum limitation of sixty-six and two thirds per cent. fixed by the law and then deducting the value of the percentages in excess of sixty-six and two thirds during the continuance of the following three statuses:

(*a*) During the joint life of the widow and at least four children, as long as the widow does not remarry and the children are less than eighteen years of age.

(*b*) During the life of at least seven children, to begin after the remarriage of the widow, and as long as the children are less than eighteen years of age, the mother being alive and remarried.

(*c*) During the life of at least five children, to begin after the death of the widow, and so long as the children are alive and less than eighteen years of age.

The first status will enter into effect only in the case of a widow and four or more children less than eighteen years of age, as the total percentage of salary payable (if there were no limit) would be 70 per cent. for a widow and four children, and 80 per cent. for a widow and five children, the percentage increasing by 10 per cent. for each additional child.

Assuming a salary of $100 per annum the present value of the excess of those percentages over the limit of 66⅔ per cent. during the life of a widow and exactly four children out of n children is

$$3\tfrac{1}{3}\bar{a}_{x'}\frac{[4]}{y_1:\overline{18-y_1|}\cdots y_n:\overline{18-y_n|}};$$

for a widow and exactly five children out of n children,

$$13\tfrac{1}{3}\bar{a}_{x'}\frac{[5]}{y_1:\overline{18-y_1|}\cdots y_n:\overline{18-y_n|}},$$

where x is the age of the widow; $y_1, y_2, \ldots, y_n$ being the ages of the children, y_1 being the youngest child.

The prime in x' denotes that a mortality table with allowance for remarriage is used.

The second status will have to be considered only when a widow and at least seven children less than eighteen survive the deceased, the widow having since remarried and being alive.

The present value of the excess over the limit 66⅔ per cent. is in this case

$$3\tfrac{1}{3}\left(\bar{a}_x \frac{[7]}{y_1:\overline{18-y_1}|\cdots y_n:\overline{18-y_n}|} - \bar{a}_{x'} \frac{[7]}{y_1:\overline{18-y_1}|\cdots y_n:\overline{18-y_n}|}\right);$$

for a widow and exactly eight children out of n children,

$$13\tfrac{1}{3}\left(\bar{a}_x \frac{[8]}{y_1:\overline{18-y_1}|\cdots y_n:\overline{18-y_n}|} - \bar{a}_{x'} \frac{[8]}{y_1:\overline{18-y_1}|\cdots y_n:\overline{18-y_n}|}\right).$$

Similarly for the third status it will be seen that the expressions to be deducted on account of the limit fixed by the law when exactly five, six, etc., children out of n children are alive after the death of the mother, the children being less than eighteen years of age, are respectively:

$$8\tfrac{1}{3}\left(\bar{a} \frac{[5]}{y_1:\overline{18-y_1}|\cdots y_n:\overline{18-y_n}|} - \bar{a}_x \frac{[5]}{y_1:\overline{18-y_1}|\cdots y_n:\overline{18-y_n}|}\right),$$

$$23\tfrac{1}{3}\left(\bar{a} \frac{[6]}{y_1:\overline{18-y_1}|\cdots y_n:\overline{18-y_n}|} - \bar{a}_x \frac{[6]}{y_1:\overline{18-y_1}|\cdots y_n:\overline{18-y_n}|}\right).$$

The above expressions may be expanded in terms of joint life temporary annuities by the "Z" method (*Textbook Institute of Actuaries,* Part II, p. 133) remembering, however, that Z^r in this particular case is the sum of the joint life temporary annuities for all the combinations of n lives taken r at a time the term being determined by the age of the eldest child involved in the annuity, for example:

$$\bar{a}_{y_1:\overline{18-y_1}|:y_2:\overline{18-y_2}|\ldots y_n:\overline{18-y_n}|}$$

would be considered to be equal to

$$\bar{a}_{y_1:y_2\ldots y_n:\overline{18-y_n}|}.$$ (See T. B., Part II, p. 146.)

If we denote by Z^r, Z_1^r, Z_2^r the value of the sum of joint lives term annuities for all the combinations of r out of n lives when (a) the life of the widow is not involved and (b) the life of the widow is involved, no allowance being made for remarriage; and (c) the life of the widow is involved, allowance being made for remarriage, the expressions for the deduction to be made on account of the limit of 66⅔ per cent. become:

For the first status:

$$3\tfrac{1}{3}\frac{Z_2^4}{(1+Z_2)^4}+10\frac{Z_2^5}{(1+Z_2)^4},$$

since

$$\begin{aligned}3\tfrac{1}{3}\bar{a}_{x'}\frac{[4]}{y_1:\overline{18-y_1|}\cdots y_n:\overline{18-y_n|}}&\\ +13\tfrac{1}{3}\bar{a}_{x'}\frac{[5]}{y_1:\overline{18-y_1|}\cdots y_n:\overline{18-y_n|}}&\\ +23\tfrac{1}{3}\bar{a}_{x'}\frac{[6]}{y_1:\overline{18-y_1|}\cdots y_n:\overline{18-y_n|}}&+\cdots\\ =3\tfrac{1}{3}\frac{Z_2^4}{(1+Z_2)^5}+13\tfrac{1}{3}\frac{Z_2^5}{(1+Z_2)^6}+23\tfrac{1}{3}\frac{Z_2^6}{(1+Z_2)^7}&+\cdots 1.\end{aligned}$$

If we multiply and divide the last expression by $1-\dfrac{Z_2}{1+Z_2}$ and treat the symbol Z_2 as if it represented a quantity we have

$$\left(3\tfrac{1}{3}\frac{Z_2^4}{(1+Z_2)^5}+10\frac{Z_2^5}{(1+Z_2)^6}+10\frac{Z_2^6}{(1+Z_2)^7}+\cdots\right)\div\left(1-\frac{Z_2}{1+Z_2}\right)$$

$$=\frac{3\tfrac{1}{3}\dfrac{Z_2^4}{(1+Z_2)^5}+10\dfrac{Z_2^5}{(1+Z_2)^5}}{\dfrac{1}{1+Z_2}}=3\tfrac{1}{3}\frac{Z_2^4}{(1+Z_2)^4}+10\frac{Z_2^5}{(1+Z_2)^4}.$$

Reasoning in a similar way for the second status we obtain:

$$3\tfrac{1}{3}\frac{Z_1^7}{(1+Z_1)^7}+10\frac{Z_1^8}{(1+Z_1)^7}-3\tfrac{1}{3}\frac{Z_2^7}{(1+Z_2)^7}-10\frac{Z_2^8}{(1+Z_2)^7}.$$

For the third status:

$$8\tfrac{1}{3}\frac{Z^5}{(1+Z)^5}+15\frac{Z^6}{(1+Z)^5}-8\tfrac{1}{3}\frac{Z_1^5}{(1+Z_1)^5}-15\frac{Z_1^6}{(1+Z_1)^5}.$$

To exemplify what precedes, let us assume that the deceased leaves a widow and four children:

$$\begin{aligned} 30\bar{a}_{x'} + 60H_x &+ 15\bar{a}_{y_1\overline{18-y_1}|} - 5\bar{a}_{xy_1\overline{18-y_1}|} \\ &+ 15\bar{a}_{y_2\overline{18-y_2}|} - 5\bar{a}_{xy_2\overline{18-y_2}|} \\ &+ 15\bar{a}_{y_3\overline{18-y_3}|} - 5\bar{a}_{xy_3\overline{18-y_3}|} \\ &+ 15\bar{a}_{y_4\overline{18-y_4}|} - 5\bar{a}_{xy_4\overline{18-y_4}|}. \end{aligned}$$

H_x denotes the present value of 1 to be payable on remarriage of the widow (x).

From the above expression must be deducted

$$3\tfrac{1}{3}\bar{a}_{x':y_1:y_2:y_3:y_4\overline{18-y_4}|},$$

as otherwise 70 per cent. would be paid during the joint life of the four children and the widow as long as she remain a widow (x), the eldest child being less than eighteen.

The actuarial expression for the value of the death benefits payable to a widow, children, and other dependents may be found without much trouble for all practical cases by reasoning as above.

Mr. W. W. Greene, assistant actuary of the New York State Workmen's Compensation Commission, has published in the first number of the proceedings of the "Casualty, Actuarial and Statistical Society of America" a similar general actuarial expression for the value of the death benefits payable to a widow and any number of children.

The great difficulty is to determine the value of the death benefits payable to a widow and several children by a short and practical method, because it must be remembered that:

(*a*) For a widow alone the value of 2 expressions have to be found.

(*b*) For a widow and 1 child the value of four expressions have to be found.

(*c*) For a widow and 2 children 6 expressions.

(*d*) For a widow and 3 children 8 expressions.

(*e*) For a widow and 4 children 11 expressions.

(*f*) For a widow and 5 children 20 expressions.

(*g*) For a widow and 6 children 50 expressions.

(*h*) For a widow and 7 children 138 expressions.

(*i*) For a widow and 8 children 361 expressions.

(*j*) For a widow and 9 children 914 expressions.

The object of this note is to present a short method of finding the reserve for the benefits payable to a widow and any number of children as required by law.

All that need be done by the method is to refer to one table for a widow and one child and to an additional table for each additional child. These tables are entered with the attained age of the widow at date of valuation and the attained ages of the first, second, third, etc., child at valuation date.

The addition of these tabulated values will give the present value of the benefits.

The principle on which the tables are constructed is that the value of a joint term annuity on the lives of a widow and any number of children is approximately equal to a single life annuity for the same term on the life of the widow at a rate of interest dependent upon the number of children involved in the joint term annuity and sometimes, also, upon the term of the annuities. The same principle applies to a joint term annuity on the lives of the children only.

The use of this principle avoids the construction of joint life commutation columns and annuities.

The principle has been applied to the Danish Survivorship Table, the standard used by the New York State Insurance Fund and an American population table, 1901–1910. Values of continuous joint term annuities by the Danish Female Survivorship Table at three and one-half per cent. interest and the approximate values as found by using the above mentioned device are compared in the table on the following page (the ages of the widow have been marked with a dash when allowance for remarriage has been made).

It is to be remembered in examining this table that children by previous marriage of either husband or wife may be included.

The examples chosen are extreme cases and are therefore crucial tests.

The correct values for joint term annuities on children alone, and a widow and children, without allowance for remarriage, have been found by using the "equal ages property" of Makeham's formula, the Danish Female Survivorship Table having been graduated from ages 0 to the limiting age by Makeham's frequency curve.

The approximate values recorded in the fifth column of the above table have been found by the following formulas for a joint term

(1) Age of Widow.	(2) Age of Child or Children.	(3) Term.	(4) Correct Value.	(5) Approximate Value.	(6) (4) — (5)
	1	17	12.373		
	5	13	10.161		
	10	8	6.856		
	17	1	.9805		
20	1	17	11.875		
20	17	1	.9780		
40	1	17	11.645		
40	17	1	.9773		
60	1	17	10.057		
60	17	1	.9717		
	17, 16, 15, 14, 13	1	.9707	.9704	.0003
20	17, 16, 15, 14, 13	1	.9680	.9683	—.0003
40	17, 16, 15, 14, 13	1	.9673	.9676	—.0003
60	17, 16, 15, 14, 13	1	.9618	.9621	—.0003
20′	17, 16, 15, 14, 13	1	.9178	.9180	—.0002
40′	17, 16, 15, 14, 13	1	.9543	.9546	—.0003
60′	17, 16, 15, 14, 13	1	.9614	.9616	—.0002
	5, 4, 3, 2, 1	13	9.007	9.017	—.01
20	5, 4, 3, 2, 1	13	8.735	8.747	—.012
40	5, 4, 3, 2, 1	13	8.629	8.641	—.012
60	5, 4, 3, 2, 1	13	7.837	7.848	—.011
20′	5, 4, 3, 2, 1	13	5.283	5.289	—.006
40′	5, 4, 3, 2, 1	13	7.769	7.779	—.01
60′	5, 4, 3, 2, 1	13	7.810	7.827	—.017
	17, 16, 15, 14, 13, 12, 11	1	.9656	.9655	.0001
20	17, 16, 15, 14, 13, 12, 11	1	.9631	.9635	—.0004
40	17, 16, 15, 14, 13, 12, 11	1	.9624	.9628	—.0004
60	17, 16, 15, 14, 13, 12, 11	1	.9570	.9574	—.0004
20′	17, 16, 15, 14, 13, 12, 11	1	.9134	.9137	—.0003
40′	17, 16, 15, 14, 13, 12, 11	1	.9495	.9499	—.0004
60′	17, 16, 15, 14, 13, 12, 11	1	.9566	.9569	—.0003
	7, 6, 5, 4, 3, 2, 1	11	7.642	7.652	—.010
20	7, 6, 5, 4, 3, 2, 1	11	7.447	7.460	—.013
40	7, 6, 5, 4, 3, 2, 1	11	7.376	7.389	—.013
60	7, 6, 5, 4, 3, 2, 1	11	6.834	6.845	—.011
20′	7, 6, 5, 4, 3, 2, 1	11	4.786	4.792	—.006
40′	7, 6, 5, 4, 3, 2, 1	11	6.707	6.719	—.012
60′	7, 6, 5, 4, 3, 2, 1	11	6.818	6.829	—.011

annuity on m children, the eldest child being y_m

$$\int_{t=0}^{t=18-y_m} |v^t \cdot s^{mt} \cdot dt = \bar{a}^{(m)}_{\overline{18-y_m}|},$$

and for a joint term annuity payable during the joint life of a widow

and m children

$$\int_{t=0}^{t=18-y_m} (vs^m)^t \cdot s^t \cdot g^{c^x(c^t-1)} \cdot dt = \bar{a}_{x\,\overline{18-y_m}|}^{(m)}.$$

In the notations

$$\bar{a}_{\overline{18-y_m}|}^{(m)} \quad \text{and} \quad \bar{a}_{x\,\overline{18-y_m}|}^{(m)}$$

m denotes the number of children involved in the joint life annuity. The value of "s" is the same as that used in the graduation of the Danish Female Survivorship table, *i. e.*, $s = .99486$.

The correct value for a joint term annuity during the life of m children and a widow and for so long as she remains a widow, have been found by the process given in a paper by Mr. M. M. Dawson, "Mortality and Remarriage Tables," (*T. A. S. A.*, XV, 306.)

In passing it may be noted that a frequency curve of the following form may sometimes properly be used for the graduation of a mortality table with allowance for remarriage;

$$\text{colog } p_x' = \alpha + \beta c^x + \beta' r^x,$$

where α and β and c have the same value as the mortality table without allowance for remarriage. The term $\beta' r^x$ takes account of the remarriage factor and the value must be negligible as soon as the remarriage factor vanishes (*i. e.*, about age 65). This frequency curve may sometimes be used instead of the method of graduation described in the above paper and which I have adopted in the graduation of the Danish Female Survivorship Table with allowance for remarriage.

I consider that the approximate values are near enough to the true values for all practical purposes; a closer approximation, however, could be obtained by considering the mortality of the eldest child involved in the annuity so that instead of

$$\bar{a}_{x\,\overline{18-y_m}|}^{(m)} \quad \text{we should have} \quad \bar{a}_{xy_m\,\overline{18-y_m}|}^{(m-1)}.$$

For the sake of clearness I give hereafter the actuarial expressions when values have to be tabulated according to all possible ages of the widows and children. These are the tables which have to be entered with the attained age of the widow and the ages of the first, second, third, etc., youngest child at valuation date.

The expressions are arrived at by grouping together in the correct actuarial expressions all the annuities having the same term and by

using the appropriate notations to signify that the rate of interest is made dependent on the number of children involved in the joint term life annuities.

The first table for a widow and one child must tabulate the value of

$$30\bar{a}_{x'} + 60H_x + 15\bar{a}_{y_1\overline{18-y_1}|} - 5\bar{a}_{xy_1\overline{18-y_1}|}$$

and a second and third table, the value of

$$15\bar{a}_{y_2\overline{18-y_2}|} - 5\bar{a}_{xy_2\overline{18-y_2}|}.$$

I give herewith some specimen values of the first two tables on the basis of the Danish Survivorship Table at $3\frac{1}{2}$ per cent.

TABLE I.

Age of Widow.	Age of Child.		
	17	12	7
20	298.99	342.98	379.56
30	414.83	458.86	495.50
40	486.92	531.04	567.90

TABLE II.

Age of Widow.	Age of Child.		
	17	12	7
20	9.818	53.812	90.386
30	9.819	53.846	90.494
40	9.822	53.940	90.843

For example, from these tables it will be found that the present value of the annuity to be paid to a widow aged 30 and three children whose ages are 7, 12 and 17, respectively, on the basis of $100 salary, is $559.17.

An additional table will be required for each additional child.

Herewith are the complete actuarial expressions of the functions to be tabulated:

(*a*) For a fourth child:

$$15\bar{a}_{y_4\overline{18-y_4}|} - 5\bar{a}_{xy_4\overline{18-y_4}|} - 3\tfrac{1}{3}\bar{a}^{(4)}_{x'\overline{18-y_4}|}.$$

It will be remembered that

$$\bar{a}^{(4)}_{x'\overline{18-y_4}|} = \int_{t=0}^{t=18-y_4} (vs^4)^t \cdot tp_{x'} \cdot dt.$$

(*b*) For a fifth child:

$$15\bar{a}_{y_5\overline{18-y_5}|} - 5\bar{a}_{xy_5\overline{18-y_5}|} - 8\tfrac{1}{3}[\bar{a}^{(5)}_{\overline{18-y_5}|} - \bar{a}^{(5)}_{x\overline{18-y_5}|}]$$
$$- 3\tfrac{1}{3}[{}^4C_3\,\bar{a}^{(4)}_{x'\overline{18-y_5}|} - \bar{a}^{(5)}_{x'\overline{18-y_5}|}].$$

(*c*) For a sixth child:

$$15\bar{a}_{y_6\overline{18-y_6}|} - 5\bar{a}_{xy_6\overline{18-y_6}|} - 8\tfrac{1}{3}\,{}^5C_4[\bar{a}^{(5)}_{\overline{18-y_6}|} - \bar{a}^{(5)}_{x\overline{18-y_6}|}]$$
$$+26\tfrac{2}{3}[\bar{a}^{(6)}_{\overline{18-y_6}|} - \bar{a}^{(6)}_{x\overline{18-y_6}|}] - 3\tfrac{1}{3}[{}^5C_3\,\bar{a}^{(5)}_{x'\overline{18-y_6}|} - {}^5C_4\,\bar{a}^{(5)}_{x'\overline{18-y_6}|}] + 6\tfrac{2}{3}\bar{a}^{(6)}_{x'\overline{18-y_6}|}.$$

(*d*) For a seventh child:

$$15\bar{a}_{y_7\overline{18-y_7}|} - 5\bar{a}_{xy_7\overline{18-y_7}|} - 8\tfrac{1}{3}\,{}^6C_4[\bar{a}^{(5)}_{\overline{18-y_7}|} - \bar{a}^{(5)}_{x\overline{18-y_7}|}]$$
$$+ 26\tfrac{2}{3}\cdot{}^6C_5[\bar{a}^{(6)}_{\overline{18-y_7}|} - \bar{a}^{(6)}_{x\overline{18-y_7}|}] - 50\bar{a}^{(7)}_{\overline{18-y_7}|}$$
$$- 3\tfrac{1}{3}[{}^6C_3\,\bar{a}^{(5)}_{x'\overline{18-y_7}|} - {}^6C_4\,\bar{a}^{(5)}_{x'\overline{18-y_7}|}] + 6\tfrac{2}{3}\cdot{}^6C_5\cdot\bar{a}^{(6)}_{x'\overline{18-y_7}|}$$
$$- 30\bar{a}^{(7)}_{x'\overline{18-y_7}|} + 46\tfrac{2}{3}\bar{a}^{(7)}_{x\overline{18-y_7}|},$$

and so on.

These tables enable us to find the present values for the benefits to be payable to a widow and one, two, three, etc., children.

As far as the accuracy of the tables is concerned it should be noted that for *benefits in connection with a widow and less than four children the values are exact.* The following table shows the error (always negative) in extreme cases, assuming the maximum salary of $1,200 per annum where the number of children is greater than three:

For a widow and 4 children	$ .40
For a widow and 5 children	1.60
For a widow and 6 children	2.80
For a widow and 7 children	4.00

It may be noted that the rates of mortality from ages 0 to 18 increase in the Danish Females Survivorship Table. It may be well to apply the above principle to a mortality table where the rates of mortality decrease to a minimum after the youngest ages, which is the case in practically every mortality table. I have used the graduation of an American life table covering the decade 1901–1910 given in the quarterly publication of the American Statistical Association for September, 1914, by C. H. Forsyth. I used this graduation simply for exemplifying the elasticity of the principle above mentioned and *no other interpretation must be given to the fact of my using this graduation or this life table.*

I give herewith some specimen values of continuous annuities at 3½ per cent.; the female life table has been used for the widow and the males and females combined for the children.

Age of Widow.	Age of Child or Children.	Term.	Exact Value.	Approximate Value.	Difference.
	1	17	12.044		
	5	13	10.270		
	10	8	6.923		
	17	1	.981		
20	1	17	11.517	11.974	−.457
40	1	17	11.150	11.610	−.46
60	1	17	9.209	9.652	−.443
21	2	16	11.306	11.496	−.190
41	2	16	10.953	11.136	−.183
61	2	16	9.028	9.177	−.149
22	3	15	10.880	10.979	−.099
42	3	15	10.554	10.643	−.089
62	3	15	8.687	8.761	−.074
23	4	14	10.410	10.443	−.033
43	4	14	10.096	10.128	−.032
63	4	14	8.305	8.333	−.028
24	5	13	9.882	9.885	−.003
44	5	13	9.589	9.592	−.003
64	5	13	7.892	7.895	−.003
25	6	12	9.315	9.304	.011
45	6	12	9.045	9.034	.011
65	6	12	7.457	7.451	.006
30	11	7	6.020	6.007	.013
50	11	7	5.879	5.866	.013
60	11	7	5.024	5.013	.011
35	16	2	1.911	1.912	−.001
55	16	2	1.892	1.893	−.001
75	16	2	1.778	1.778	
20	17	1	.9787	.9790	−.0003
40	17	1	.9770	.9773	−.0003
60	17	1	.9689	.9692	−.0003

The correct values in the above table have been computed at 3½ per cent. on the joint lives of the widow and one child; the approximations have been found on the single life of a widow at a rate of interest of 3.85 per cent.

From the above table it may be seen that the differencee between the exact values and the approximate values run pretty evenly for the values of annuities of the same term, and that the approximate values are near enough for practical purposes.

A closer approximation may be obtained by varying the rate of interest with the term to run; the interest rate decreasing with the term. The following values were calculated at 4.45 per cent.

Age of Widow.	Age of Child or Children.	Term.	Exact Value.	Value by 2d Approx.	Difference.
20	1	17	11.517	11.496	.021
40	1	17	11.150	11.135	.015
60	1	17	9.209	9.232	−.023

An alternative method which could be used is to find the correct values of the joint life annuities on a widow and one child for the term 17 to 14 for each five ages of the widow and obtain by interpolation the other values for the intermediate ages of the widow. The values of the joint term annuities below 14 are found from the single term life annuities at a modified rate of interest.

The joint term annuities whose values have next to be tabulated are for the joint lives of five children, and a widow and four children with allowance for remarriage; approximate values near enough for practical use may be obtained by either of the above mentioned methods.

Tables of similar form to those above described for a widow and any number of children may be constructed for the most important cases of the widow and any other dependents.

The great practical advantage of the tables is that the present value of the death benefit provided by the Workmen's Compensation Law of New York may be found in a few minutes by anyone without technical knowledge.

The principles of the construction may of course be applied to any mortality table and also to other death benefits provided by the Compensation Act.

Income Tax on Annuities.

(Federal Income Tax Act of 1913.)

BY

JOHN S. THOMPSON.

The treatment of life annuities in connection with laws creating an income tax has nearly always been a matter of some difficulty. The English case of Gresham Life Assurance Society *vs.* Styles (Surveyor of Taxes) opened up a question of interpretation of a clause in the British Income Tax Act of 1853 which provided (Schedule D(4)) "that all interest of money, annuities and other annual profits and gains not charged by virtue of any of the other schedules contained in this Act" should be subject to tax. The point at issue was whether the actual annuity instalments paid by the Society should not be taxed as "profits or gains" and whether the amount of the tax should not, therefore, be withheld by the Society and paid to the Crown. The tax commissioners made this apparently unreasonable claim, and the above-named society carried the case through the Divisional Court and the Court of Appeals into the House of Lords before the judicial decision could be obtained that "annual profits or gains" could be determined only after the annuity payments had been made, and that therefore the company paying the annuities might treat them in the same manner as sums payable on deaths of assured lives. The following reproduction from the *Journal of the Institute of Actuaries,* Vol. XXXI, page 81, of an extract from the Report of the Directors of the Gresham Life Assurance Society for 1893 gives the sequel:

"The Judgment of the House of Lords applied solely to the assessments made and tax paid up to 5th April, 1888. In that year an Act was passed providing that the grantors of 'annuities charged with income tax under Schedule D and not payable or not wholly payable out of profits or gains brought into charge to such tax,' should be accountable to the Crown in respect of the income tax on such annuities. The Directors have lately been in communication with the revenue authorities with reference to the position taken up by the legal advisers of the Society, that annuities granted to persons residing abroad are not within the scope of this enactment. The

Directors have the satisfaction to report that the revenue authorities have lately expressed their acquiescence in this view. The Society is consequently exempted from the liability to account to the Crown for income tax in respect of these annuities, and will continue to pay these annuities without any deduction for income tax."

Because of the construction of the British Act of 1853, under which, by the ruling above discussed, life annuities were not taxable, and because of the wording of the Act of 1888, under which life annuities were specifically chargeable with tax, the question never, of course, arose in this controversy as to the propriety of taxing the *entire* amount of a life annuity instalment, or of exempting that portion of each life annuity instalment which consisted of a part of the "principal" paid in the form of consideration for the contract. Under the British Act, if a loan were to be repaid by a number of instalments-certain, the successive payments of the annuity would be subject to income tax only to the extent of the interest element contained in them. The purpose of the present discussion is to endeavor to regard life annuity contracts logically from the same point of view as that from which it is generally admitted that annuities-certain should be regarded.

The United States Federal Income Tax Act of 1913 has the following provisions bearing on the matter of taxation of annuities:

A. Subdivision 1. "That there shall be levied, assessed, collected and paid annually upon the entire net income arising or accruing from all sources in the preceding calendar year to every citizen of the United States, whether residing at home or abroad, and to every person residing in the United States, though not a citizen thereof, a tax of one per centum per annum upon such income, except as hereinafter provided; and a like tax shall be assessed, levied, collected, and paid annually upon the entire net income from all property owned and every business, trade, or profession carried on in the United States by persons residing elsewhere."

* * * * * * * * * * *

B. "That, subject only to such exemptions and deductions as are hereinafter allowed, the net income of a taxable person shall include gains, profits, and income derived from salaries, wages or compensation for personal service of whatever kind and in whatever form paid, or from professions, vocations, business, trade, commerce, or sales, or dealings in property, whether real or personal, growing out of the ownership or use of or interest in real or personal property, also from interest, rent, dividends, securities, or the transaction of any lawful business carried on for gain or profit, or gains or profits and income derived from any source whatever, including the income

from but not the value of property acquired by gift, bequest, demise or descent:

"Provided, That the proceeds of life insurance policies paid upon the death of the person insured or payments made by or credited to the insured, on life insurance, endowment, or annuity contracts, upon the return thereof to the insured at the maturity of the term mentioned in the contract, or upon surrender of contract, shall not be included as income."

* * * * * * * * * * *

E. "All persons, firms, copartnerships, companies, corporations, joint-stock companies or associations, and insurance companies, in whatever capacity acting . . . having the control, receipt, custody, disposal, or payment of interest, rent, salaries, wages, premiums, annuities, compensation, remuneration, emoluments, or other fixed or determinable annual gains, profits, and income of another person, exceeding $3,000 for any taxable year . . ., are hereby authorized and required to deduct and withhold from such annual gains, profits and income such sums as will be sufficient to pay the normal tax imposed thereon by this section, and shall pay to the officer of the United States Government authorized to receive the same; and they are each hereby made personally liable for such tax."

Under paragraph *E*, the problem confronting an insurance company issuing annuity contracts was simply whether an annuity exceeding $3,000 per annum to a single individual should be subjected to the 1 per cent. tax before payment. Before proposing what is believed to be a technically correct method of treatment, and one which, so far as is known, has not yet been exposed to the test of actual practice, it will perhaps be interesting to review what has already been said on this question.

Mr. Rhodes, writing in October, 1913, immediately after the passage of the act, says ". . . it is clear that . . . payments falling due under annuity contracts, . . . are not taxable under the proviso in paragraph *B* . . . ." A little later, on October 30, 1913, Mr. Alfred Hurrell, attorney to the Association of Life Insurance Presidents, in the *Eastern Underwriter* of that date says:

"This brings us to the second group of payments to policyholders consisting of (4) annuities for life, (5) annuities for a term certain, (6) premium refunds or so-called 'dividends' to policyholders. These payments come within the provision of paragraph *E* as being 'fixed or determinable annual gains, profits and income,' hence, if in excess of $3,000 they would come within the provision requiring the tax thereon to be deducted and withheld by the company paying the same, were it not for the fact that by paragraph *B* they are specifically exempt from the law and are not to be included in the taxable income of the person receiving the same. . . .

"Perhaps it will be best for us to discuss separately the classes of payments just referred to. The specific mention of annuities in paragraph *E* among other payment items on which the tax must be withheld leads us to consider whether there is a conflict between this requirement and the exemption given to payments on annuity contracts in paragraph *B* as quoted above. It seems clear, however, that what was intended was to draw a distinction between payments on annuity contracts by life insurance companies and payments of other kinds of annuities such as arise by reason of gift, trust deed, demise or bequest. We may assume that the tax must be withheld on the class of annuities last referred to, and yet as to payments made by life insurance companies under annuity contracts issued by them no tax need be withheld, because under paragraph *B* none must ultimately be paid by such annuitants.

"The suggestion that life insurance company annuities may be taxable in part and exempt in part leads us to consider the words 'payments made or credited to the insured on annuity contracts upon the return thereof to the insured.' It has been urged that the use of the words 'return thereof' indicates that only such annuity payments or parts of payments as represented moneys paid as a premium by the policyholder to the life insurance company are exempted under this provision. This argument might have some force were it not for the use of the words 'or credited to.' These words seem to indicate that not only payments made by the insured, but payments of interest to the company which have been credited to the insured will be exempt when paid to the policyholder. This view of the matter covers the entire fund out of which annuity payments must be made, since the fund is made up solely of payments of interest to the company which in turn are credited to policyholders. In other words, the funds which life insurance companies hold for the payment of annuities are made up of payments by the insured plus payments credited to the insured. Viewing the matter, therefore, from an insurance company's standpoint, it would seem that the words 'return thereof' can mean nothing more than 'the payment thereof,' so that we reach the conclusion that all payments made by life insurance companies on annuity contracts shall not be included as income of the annuitant."

In the same connection, Mr. F. H. Garrigues, of the Penn Mutual, looking into the spirit rather than the letter of the law, contributes the following to the *Eastern Underwriter* of November 20, 1913:

"While paragraph *E* requires the deduction of the income tax 'at source' when paying 'interest, rent, salaries, wages, premiums, annuities, compensation, . . . exceeding $3,000 for any taxable year' it is contended that this language does not relate to annuities issued by life insurance companies, in view of the provision above quoted, but to annuities payable by executors or administrators in settlement

of estates or other similar annuities. This view is borne out by the fact that Commissioner Osborn, in his rulings under date of October 31, 1913, speaks of the annuities specified in this paragraph as 'taxable annuities,' thereby excluding, by inference at least, annuities payable by life insurance companies.

"A life annuity in the final analysis is a return of principal upon the instalment plan and cannot be considered income in any true sense. A person deposits with an insurance company a principal sum, upon which the company pays the normal income tax, and which theoretically, is exactly adequate to return to him an annual amount during his assumed expectancy. The annuity payment diminishes the principal in the hands of the company from year to year, and, while there is an annual interest improvement of the outstanding fund, the annuitant should not be required to pay an income tax even upon this portion, as such tax is payable yearly by the company issuing the contract, and to exact a tax from the annuitant himself would be double taxation. An individual annuitant, if he survives his expectancy, may receive, with interest accretions, more than the principal sum deposited, but an occasional variation of this kind does not affect the theory, inasmuch as the intent of the company is to come out exactly even, which is the result upon the average if the rates have been correctly calculated.

"Let us assume that an individual had accumulated $10,000 in the conduct of his business during a given year, which fund was subject to the income tax when earned, and that instead of purchasing an annuity from a life insurance company he concluded to invest the sum in bonds of $500 each, maturing in not less than twenty years and at an interest rate of five per cent. per annum. At the end of the first year he has an interest income from this source of $500, but in order to meet certain living or other expenses he finds it necessary to dispose of one bond. If his total income, including this interest item of $500, exceeding the specific exemption he would undoubtedly be required to pay the income tax upon such excess, but neither the Government nor any one else would be likely to contend for a moment that the proceeds of the $500 bond which had been sold was income. If this process were continued for twenty years, with the gradual reduction of interest, the individual would have had, in effect, an annuity during that period, but, as he may possibly survive the twentieth year, instead of personally assuming the risk of being without this needed income thereafter, he deposits the $10,000 with a life insurance company and is thenceforth free of anxiety as to the uncertainty of the duration of his life.

"It is, therefore, submitted that an annuity in all its essential details is a payment on account of principal each year and should be exempt from the operation of the income tax so far as the individual receiving the annuity from year to year is concerned, as the only investment feature of the transaction, i. e., the interest earned annually, is included in the tax return of the company."

In an interpretation of the clause quoted above from Section B of the Income Tax Law, the internal revenue authorities in Treasury Decision Number 1890 of October 31, 1913, expressed the opinion that "this should not be construed to exempt said insurance companies from withholding and paying the normal tax of one per cent. on interest income paid by insurance companies to beneficiaries of policies when said interest exceeds $3,000." To certain inquiries of December 5 and December 15, 1913, concerning the taxation of contracts settled in other than a single sum—"gold bond" policies, continuous instalment contracts, and ordinary policies settled under what were known as "Modes of Settlement" Nos. 2 and 3 of the New York Standard Forms of 1907—the reply on December 29, 1913, was that "the proceeds of insurance policies paid pursuant to contract are held not to be taxable in the hands of the beneficiaries, hence, in the payment of such proceeds, whether the same be in a lump sum or in instalments, at the death of the insured, on a matured policy or as annuities, no withholding of the tax is required." A later query addressed on January 8, 1914, to the Commissioner of Internal Revenue, citing the case of a life annuity of $3,448.40 per annum issued on February 1, 1903, for a cash consideration of $40,-985, and asking whether the one per cent. tax should or should not be withheld from the February 1, 1914, payment, elicited the following ruling:

"So much of annuities paid to annuitant as represents payment made by him on annuity contract and paid back to him, shall not be included in income of annuitant. Any increment on purchase price of annuity is taxable income, and if in excess of three thousand dollars and no exemption is claimed, tax must be withheld."

In reply to an inquiry as follows: "It has been my judgment that annuities sold by . . . insurance companies were not subject to tax under the Income Tax Law, but I realize that the statute is not entirely free from doubt so far as relates to such portions of annuity payments as may be treated as income, though such portions are comparatively small and the determination of their amount is difficult and almost impracticable. I would like to know, however, whether the Department has made any ruling relative to the taxation of annuities, and if any ruling has been made, would be glad to receive a copy of it," the Commissioner replied on February 17, 1914, that "life insurance annuities shall not be included as income."

The next ruling, so far as is known, appeared in T. D. No. 2090 of December 14, 1914, the section relating to insurance and annuity contracts being as follows:

"The amount paid under a life insurance, endowment, or annuity contract is not income when returned to the person making the contract either upon the maturity or the surrender of the contract; but the amount by which the sum received exceeds the sum paid and coming into the hands of the person making the contract and payment is income. When the settlement under such a contract is made in more than one payment, each such payment will be considered as being composed of interest and a proportionate part of the principal. When the entire annuity is composed of an interest return upon the principal paid therefor, the entire annuity is income."

This ruling was amended by T.D. No. 2152, of February 12, 1915, which appears to be the last and, therefore, the governing decision on the question, through the omission of the words: "When the settlement under such a contract is made in more than one payment, each payment will be considered as being composed of interest and a proportionate part of the principal. Where the entire annuity is composed of an interest return upon the principal sum paid therefor, the total annuity is income," so that the ruling, as amended, reads as follows:

"The amount paid under a life insurance, endowment or annuity contract is not income when returned to the person making the contract, either upon the maturity or the surrender of the contract, but the amount by which the sum received exceeds the sum paid and coming into the hands of the person making the contract and payment is income."

Throughout this section of the law and the rulings thereon, insurance and annuity contracts seem to be somewhat confused. It is, however, without the slightest doubt, the intention of the law to protect the beneficiary on whose behalf insurance provision has been made by exempting from taxation the amount paid in a single sum in settlement of an insurance contract. But if the beneficiary, voluntarily or otherwise, leave the proceeds of the policy with the company under any mode of settlement which furnishes an interest-bearing investment, it is also the intent of the law that the interest portion of such a settlement should be taxed and that the tax should be withheld by the paying company if the interest return to a specified individual during a single calendar year is over $3,000. The actual loss incurred by a company under one of its contracts

when it is settled in other than a single sum is no doubt that which is referred to in T. D. No. 2152 as the "sum paid" by the beneficiary to the company; in the reading of the clause, the beneficiary must be understood as "the person making the contract" and any amount received beyond the normal amount above referred to is a taxable interest income. The mathematical analysis of various settlements will be considered from this point of view.

I. Contracts Not Involving Life Contingencies.

(*a*) *Principal Sum Left with the Company at a Guaranteed Rate of Interest.*

Let S be the net sum payable on the maturity of the policy, iS the guaranteed interest income. According to T. D. No. 2090, "when the entire annuity is composed of an interest return upon the principal paid thereon, the entire annuity is income." There is, therefore, no doubt as to the treatment of such cases; whilst the act avoids the taxation, as income, of the proceeds of life insurance policies, the position is taken that if said proceeds are left with the company on an investment basis, the beneficiary should be treated as if the sum had been invested in any other quarter.

(*b*) *Principal Sum Settled in a Specified Number of Equal Annual Annual Instalments-Certain.*

Let S be the net sum payable and i the interest basis of the supplementary contract providing for the payment of the proceeds of the policy in n annual instalments-certain, first payment immediate.

Then, amount of each instalment

$$= \frac{S}{a_{\overline{n}|}}$$

and amount of interest in rth instalment $(r > 1)$

$$= \frac{S}{a_{\overline{n}|}}(1 - v^{n-r+1}) = iS\frac{a_{\overline{n-r+1}|}}{a_{\overline{n}|}}.$$

(*c*) "Gold bond" settlement, under which the issuing company by a provision of the original contract guarantees to pay interest at a specified rate (usually higher than that at which premium and reserves are computed) upon the nominal face of the policy and at the end of that period, said nominal amount in cash.

Let i' be the interest rate provided for on the "gold bond," n the period of the bond and i the rate used as the interest basis of the policy itself.

Then the actual amount of insurance on which the premium paid for the original contract would be based, would be, per unit amount of "bond"

$$\{1 + (i' - i)a_{\overline{n}|}\},$$

and the amount of interest in the rth bond interest payment would be

$$i\{1 + (i' - i)a_{\overline{n-r+1}|}\}.$$

II. Contracts Involving Life Contingencies.

(a) *Life Annuity.*

Before proceeding to a general discussion of this contract, consider, for purposes of illustration, an annuity fund constituted by entrants at age 90 only, the number of entrants at age 90 being l_{90} by the American Experience Table, and the American rates of mortality being supposed to prevail. The progression of the fund may be indicated as in Table A.

If we consider the progression of a single annuity by eliminating from Table A the radix l_{90} and making other necessary modifications, we have Table B, which illustrates how each life annuity instalment may be divided into "interest" and "principal." For a general discussion, consider a life annuity of a unit per annum on a life aged x, upon which the instalment due at age x has just been paid. It is desired to "analyze" the instalment for age $(x+1)$, if paid.

$$a_x = vp_x(1 + a_{x+1}).$$

$$\text{Therefore } 1 = \frac{(1+i)a_x}{p_x} - a_{x+1}$$

$$= (1+i)a_x - a_{x+1} + (1+i)a_x\left(\frac{1}{p_x} - 1\right)$$

$$= \left(a_x + \frac{q_x}{p_x}a_x - a_{x+1}\right) + i\left(a_x + \frac{q_x}{p_x}a_x\right) = \left(1 - \frac{ia_x}{p_x}\right) + \frac{ia_x}{p_x}.$$

The items of this equation correspond in an obvious manner to the various columns of Table B.

TABLE *A*.

NET AMER. 3 PER CENT.—ANNUITY FUND.—ENTRANTS AGED 90.

(1) Age.	(2) No. Living.	(3) Fund at Beginning of Year {(4) + (7)}.	(4) Amt. of Fund Pertaining to Those Who Will Live 1 Yr.	(5) Interest (4) × .03.	(6) Total (4) + (5).	(7) Amt. of Fund Pertaining to Those Who Will Die in Year.	(8) Interest (7) × .03.	(9) Total (7) + (8).	(10) Grand Total (6) + (9).	(11) Payments End of Year.	(12) "Interest" (5) + (8).	(13) "Principal" (11) − (12).
90	847	$745.70	$406.74	$12.20	$418.94	$338.96	$10.17	$349.13	$768.07	$462	$22.37	$439.63
91	462	306.07	143.10	4.29	147.39	162.97	4.89	167.86	315.25	216	9.18	206.82
92	216	99.25	36.30	1.09	37.39	62.95	1.89	64.84	102.23	79	2.98	76.02
93	79	23.23	6.18	.18	6.36	17.05	.51	17.56	23.92	21	.69	20.31
94	21	2.92	.42	.01	.43	2.50	.07	2.57	3.00	3	.08	2.92
95	3											
												Σ = $745.70

TABLE *B*.

INDIVIDUAL ANNUITY AT AGE 90—$1 PER ANNUM.

(1) Age x	(2) n	(3) a_x	(4) $\frac{p_x}{p_x} a_x$	(5) $i \frac{p_x}{p_x} a_x$	(6) $(1+i) \frac{p_x}{p_x} a_x$	(7) $\frac{q_x}{p_x} a_x$	(8) $i \frac{q_x}{p_x} a_x$	(9) $(1+i) \frac{q_x}{p_x} a_x$	(10) Total (6)+(9).	(11) Unit Payment Consists of "Interest" (5) + (8).	(12) "Principal" 1 − (11).	(13) ${}_np_{90}$	(14) (12) × (13)
90	1	.8804	.8804	.0264	.9068	.7337	.0220	.7557	1.6625	.0484	.9516	.5455	.5191
91	2	.6625	.6625	.0199	.6824	.7545	.0226	.7771	1.4595	.0425	.9575	.2550	.2441
92	3	.4595	.4595	.0138	.4733	.7966	.0239	.8205	1.2938	.0377	.9623	.0933	.0898
93	4	.2938	.2938	.0088	.3026	.8117	.0244	.8361	1.1387	.0332	.9668	.0248	.0240
94	5	.1387	.1387	.0042	.1429	.8322	.0249	.8571	1	.0291	.9709	.0035	.0034
													Σ = .8804

Here we have what is believed to be an actuarially correct method of dividing life annuity payments into "principal" and "interest" as compared with the empirical methods of subjecting to income tax all life annuity instalments falling due after n years from date of issue where n is determined either by the equation $S = sa_{\overline{n}|}$ or by $S = ns$ where S is the purchase price and s the annual instalment.

For a concrete illustration, consider a life annuity of $15,000 per annum on a man who is now aged 70. Assume that the reserve basis is McClintock 3 per cent. It is required to analyze the annuity instalment which has just been paid.

$$\begin{aligned} \$15{,}000a_{69} &= \$112{,}425 \\ 15{,}000a_{70} &= \underline{107{,}265} \\ \Delta &= 5{,}160 \\ 15{,}000\frac{q_{69}}{p_{69}}a_{69} &= 6{,}285 \\ \Sigma = \text{"Principal"} & \\ \text{in instalment} &= \$\ 11{,}445 \end{aligned}$$

$$\begin{aligned} .03 \times \$15{,}000a_{69} &= \$3{,}370 \\ .03 \times 15{,}000\frac{q_{69}}{p_{69}}a_{69} &= 185 \\ \Sigma = \text{"Interest"} & \\ \text{in instalment} &= \$3{,}555 \end{aligned}$$

If the principles discussed here are correct, the company paying the annuity should withhold 1 per cent. tax on $3,555 of this instalment.

Suppose also that the annuity was issued at age sixty-nine for the single premium of $120,000. If, in connection with this case, T. D. No. 2152 were interpreted to provide that after the annuity payments to the annuitant had amounted to $120,000 all subsequent annuity payments should be subjected to the 1 per cent. tax, this ruling would mean that the ninth and all later payments would be taxed for their full amount. The following table shows a comparison between the tax that would be paid under this possible construction of T. D. No. 2152 and that which would be paid under such an analysis of each annuity payment as that attempted above. The interest elements are seen to diminish gradually to the limit of the table. In the extreme case of an annuity with an annual payment so large that the interest portion of the annual payment at age $\omega - 1$ was taxable as exceeding $3,000, no further payments would be taxable in the unusual contingency of the annuitant's surviving beyond the limiting age of the mortality table used. Under the suggested construction of the Treasury decision above referred to, the payments after age $\omega - 1$ would, of course, be taxed for their full value. If the experience of the company indicated a pronounced and continual gain from mortality on its annuity busi-

Annuity Payment No.	Annuitant's Age at Successive Anniversaries.	"Principal" in Each Payment.	"Interest" in Each Payment.	1 Per Cent. of "Interest" where Interest Exceeds $3,000.	1 Per Cent. of Each Payment After Consideration is Repaid.
1	70	$11,445	$3,555	$35.55	.....
2	71	11,580	3,420	34.20	.....
3	72	11,730	3,270	32.70	.....
4	73	11,880	3,120	31.20	.....
5	74	12,015	2,985	.....	.....
6	75	12,150	2,850	.....	.....
7	76	12,285	2,715	.....	.....
8	77	12,405	2,595	.....	.....
9	78	12,540	2,460	.....	$150
10	79	12,660	2,340	.....	150
11	80	12,780	2,220	.....	150
16	85	13,290	1,710	.....	150
21	90	13,725	1,275	.....	150
26	95	14,040	960	.....	150

ness, an annuitant might demand a revision of the analysis of his annuity on the ground that because of the use of too low a rate of mortality the interest elements of his various payments had been exaggerated. The deviation of the experience of the company from the valuation table raises a difficult question in connection with the treatment of individual annuity payments which cannot be solved here.

(*b*) *Deferred Life Annuity.*

This benefit could, of necessity, be taxed only at its maturity, after which the payments could be divided as outlined in (*a*). On the other hand, there appears to be the same justice in taxing the interest on the reserve and on the increase in the reserve through the operation of mortality, as in case of the straight life annuity. The act states that "the tax shall be upon the entire net income arising or *accruing* from all sources in the preceding calendar year," and that the "net income of a taxable person" shall include ". . . income derived from any source whatever, including the income but not the value of property acquired by gift, bequest, demise or descent" and "income" has been construed by internal revenue authorities as including such items as "dividends declared and payable, although not collected," "interest good and collectible accruing during year, whether collected or not," "increase in the value of unsold property," "improvement of land in lieu of rent," et cetera. Whilst interest earned by a company upon its annuity reserves is not in any sense "collectible" by individual annuitants at the time of

its being earned, it is held for their benefit and constitutes part of the annuity fund from which annuity instalments will ultimately be paid. In the absence of any payments from which to withhold the tax on this accruing interest, if sufficiently large, account might be taken of the fact, on maturity of a deferred annuity contract, that part of the reserve represents accumulated interest "credits" hitherto untaxed, and the remainder the "principal" to which the reserve at inception of the contract has increased through the effect of mortality. Hence, if an annuity to be deferred n years were issued at age x, the "interest" taxable in the first annuity payment at age $x+n+1$ would be not only the amount

$$i\left(a_{x+n}+\frac{q_{x+n}}{p_{x+n}}a_{x+n}\right)$$

found in the usual way, but also the amount

$$a_{x+n}-\frac{l_x}{l_{x+n}}\cdot {}_{n|}a_x$$

by which the reserve at age $x+n$, one year prior to the payment of the instalment, exceeds the sum

$$\frac{l_x}{l_{x+n}}\cdot {}_{n|}a_x$$

to which the net reserve at the inception of the contract has increased solely by the "benefit of survivorship."

(c) *"Continuous Instalment" Settlement.*

As this benefit, whether arising by direct provision from the original contract or by election under a settlement option, is, in the hands of the beneficiary, simply an annuity certain, followed by a deferred life annuity, it could be treated as a combination of I (*b*) and II (*b*) and the payments analyzed accordingly.

Since, however, the payments under this settlement are continuous, the interest charges, if any, on both the annuity certain reserve and on the deferred annuity reserve, might be deducted from the instalments-certain, notwithstanding these are paid entirely from their own reserve. If this were contemplated, we might consider the progression of the deferred annuity reserve in a manner similar to that utilized above for the life annuity.

Let x be the age of the beneficiary at the date of issue of the supplementary contract and n years the length of the period certain. The reserve on the deferred annuity at the end of the $(r-2)$th year is

$$_{n-r+2|}a_{x+r-2},$$

and

$$_{n-r+2|}a_{x+r-2} \times \frac{1+i}{p_{x+r-2}} = {}_{n-r+1|}a_{x+r-1},$$

so that the principal, at the end of the $(r-1)$th year, has increased to

$$\frac{_{n-r+2|}a_{x+r-2}}{p_{x+r-2}},$$

and the interest accruing to the deferred annuity reserve is

$$i\,\frac{_{n-r+2|}a_{x+r-2}}{p_{x+r-2}}.$$

Hence, including the appropriate item on account of the annuity certain we have as the amount of interest in the rth payment $(r > 1)$ under the continuous instalment settlement

$$i\left\{a_{\overline{n-r+1|}} + \frac{_{n-r+2|}a_{x+r-2}}{p_{x+r-2}}\right\}.$$

While this procedure may be inferior in principle to that discussed in connection with the simple deferred annuity, it is superior in operation, inasmuch as it furnishes a continuous series of "interest" elements.

On the amount of interest derived in each of the above cases tax should be withheld by the paying company if such items were in excess of $3,000. Obviously only very large annuity contracts will give rise to interest earnings sufficiently large, particularly when it is understood that annuity reserves are for the most part accumulated on a 3 per cent. or a $3\frac{1}{2}$ per cent. basis, but by the means above described may be found (for the purpose of individual exhibits) the amount of interest income in life annuity instalments of any size.

A Staff Pension Fund.

BY

DWIGHT A. WALKER.

The subject of staff pension funds seems to have received very little attention in the *Transactions of the Actuarial Society,* possibly because the subject has been so well covered by Messrs. Manly, King, Ackland, M'Lauchlan, and others in the *Journal of the Institute.* In fact the only papers in the *Transactions* bearing on pension funds are "Staff Pension Funds with Special Reference to Retirement Plan for United States Civil Service Employees," by B. D. Flynn, Vol. X, page 275; "Staff Pension Funds, An Investigation of the Mortality of Canadian Civil Service Pensioners," by M. D. Grant, Vol. XI, page 163; and "A Pension Fund Method" by C. C. Ferguson, Vol. XII, page 192.

As pension funds are quite a live topic at the present time it would seem to be not amiss to offer to the Actuarial Society the results of some calculations recently made in connection with a pension fund for the employees of one of the large railroads of America. The question of whether the collective fund method or the individual savings fund plan was preferable in this case does not arise as the former plan had been in actual operation for a short time and the calculations made by the author were consequently based upon this plan.

The benefits to be provided for and the extent of the data furnished are given below:

Benefits.

(*a*) A pension to each member employee for life upon disablement prior to age 70, and in any event upon retirement at age 70.

(*b*) The return to the member of all contributions paid by him, without interest, upon his withdrawal from the service of the company.

(*c*) The payment of a pension for two years certain after the death of each member provided death occur while in the service of the company and prior to retirement.

(*d*) The payment of a pension for two years certain after the death of each member provided death occur after retirement either through disability or on account of having attained the retirement age.

The amount of pension payable for the two years under (*c*) is to be the same as the amount of pension which the employee would have received if he had retired through disablement instead of dying.

The amount of pension payable for the two years under (*d*) is to be the same as the amount of pension which the member was receiving prior to his death.

The pension is to be computed by multiplying the final salary by the length of service and this product by a fixed percentage.

Data.

Data Based upon all Employees in the Operating Department of the Company with the Exception of the Office Forces.

Present Age.	Number of Employees.	Relative Average Yearly Earnings.	Average Years in Service.
Under 20.....	765	1.5691	1.12
20 to 29.....	5,721	2.4551	2.84
30 to 39.....	5,126	3.0067	6.95
40 to 49.....	3,223	3.1733	12.40
50 to 59.....	1,622	3.3002	18.29
60 and over...	677	2.8879	28.77
	17,134		

It has been pointed out repeatedly in the *Journal of the Institute* that, in calculating contributions to provide benefits under a staff pension fund, sufficient data should be obtained, based upon the past experience of the staff in question, to enable the calculation of the mortality and disability rates at each age.

These data, however, were not available in this instance and recourse of necessity was had to rates of mortality and disability based on the published experience of other organizations as nearly similar in character as possible.

Of the material available that which seemed to be most suitable as to rates of death amongst active lives and rates of permanent disability is based upon the experience of the Total Railway Staff of the Union of German Railways and prepared by Dr. Zimmerman. These rates of disability are considerably higher than those shown

by the experience in Germany since the introduction of the Invalidity and Old Age Insurance Law, which latter, however, did not include disability from accident, but are considerably lower than the rates made up by Zimmerman based on the train staff alone of the German railways. Dr. Zimmerman's disability rates are much higher than those experienced by various fraternal orders in the United States and Canada as made up by Messrs. Mead, Pipe, and Landis. Zimmerman's German railway mortality rates amongst active lives are higher than the American Experience rates at ages under 24 and between ages 38 and 57, but are somewhat lower at other ages.

Owing to the fact that it was not intended to make membership in the fund compulsory, although persons employed in either the operating department or in the clerical department were eligible for membership, it was considered wise to be somewhat conservative in making the calculations. It might happen that the pension scheme would appeal more strongly to the operating staff with the consequence that there would be a much larger proportion of members subject to the high train staff disability hazard than was provided for in Zimmerman's Total Railway Staff rates of disability. This is not quite so important a matter as it might at first appear because the rate of mortality amongst the active in such an event would also be abnormally high. The benefit allowed at death is not very large, whereas the pension on retirement due to old age is really the costly benefit. A high rate of death would therefore tend to have the opposite effect upon the contributions to a high rate of disability. With the object of being conservative, therefore, Zimmerman's rates of disability were employed as "the probability of an active life being alive and invalid or disabled at the end of a year."

Taking Zimmerman's rates of disability and of mortality amongst active lives for quinquennial ages from G. W. Richmond's paper, *J. I. A.*, Vol. XLV, page 355, the intermediate values were interpolated by Mr. Henderson's osculatory interpolation formula. Zimmerman's disability rate was followed to age 65 at which point voluntary retirement was assumed to commence, the rate of retirement increasing rapidly to age 69, at which age compulsory retirement was assumed to take place.

Table I exhibits the completed rates as used, the original given values being shown in italics.

For the rate of mortality amongst disabled lives Mr. Arthur Hunter's ultimate rate of mortality (*T. A. S. A.*, XII, 44) based upon the experience of American fraternal societies was employed as the rate of mortality amongst the pensioners. These rates merge into the American Experience rates at age 70, the rates from 65 to 69 being made up arbitrarily. The mortality hazard amongst permanently disabled or retired railway employees does not appear likely to be very different from that amongst other classes of disabled lives. It might be somewhat lighter if the number of men disabled by accident but otherwise in good health comprised an unusually large proportion of those pensioners who were retired upon being disabled. Mr. Hunter's ultimate table would, however, appear to be fairly conservative.

For the rate of withdrawal from the service, it having been assumed that withdrawals from the fund would take place only upon withdrawal from the service of the company, rates of withdrawal made up by Mr. H. W. Manly (*J. I. A.*, Vol. XLV, page 152) and based upon the experience of certain railroads in Great Britain have been employed. It will be noticed on reference to Table I that no withdrawals have been provided for after age 50.

The rate of increase in the salary scale is another important factor in pension fund calculations but owing to the limitation imposed by the lack of detail as to the experience of the company in the matter of individual salary histories it was not thought advisable to employ any elaborate method in the construction of the salary scale. Using the relative average yearly earnings as shown on a previous page the scale shown in Table I was constructed in a somewhat arbitrary manner. It was assumed to increase .16 per annum from age 15 to age 20; then by .12 to age 25; by .08 to age 30; and between 30 and 50 it was increased from 2.80 to 3.20 per annum. The original average earnings for decennial groups show a drop in the scale for the group 60 and over, but for the sake of safety the maximum salary as shown at age 50 was maintained for all higher ages. The scale was constructed down to age 15 as it was required for the younger ages later on in the calculations.

TABLE I.

Present Age.	(1) Probability of an Active Life being Alive and Disabled at end of a Year (Zimmerman's German Railways). p_x^{ai}	(2) Probability of an Active Life Dying within a Year (Zimmerman's German Railways). q_x^a	(3) Probability of Disabled Life Dying within a Year (Hunter's Ultimate). q_x^i	(4) Probability of Withdrawal at end of a Year (Manly's British Railways). q_x^{aw}	(5) Relative Salary Scale based on Yearly Salary. s_x
15					1.00
16					1.16
17					1.32
18					1.48
19					1.64
20	*.00021*	*.00889*	.205	.055	1.80
21	.00030	.00836	.193	.050	1.92
22	.00040	.00792	.182	.047	2.04
23	.00050	.00755	.171	.044	2.16
24	.00061	.00727	.161	.041	2.28
25	*.00072*	*.00705*	.151	.038	2.40
26	.00085	.00689	.141	.035	2.48
27	.00099	.00679	.132	.033	2.56
28	.00115	.00675	.123	.031	2.64
29	.00133	.00677	.114	.029	2.72
30	*.00153*	*.00685*	.106	.027	2.80
31	.00174	.00696	.100	.025	2.84
32	.00199	.00713	.095	.023	2.86
33	.00225	.00735	.092	.021	2.88
34	.00253	.00763	.090	.019	2.90
35	*.00284*	*.00798*	.089	.017	2.90
36	.00315	.00832	.088	.015	2.92
37	.00348	.00873	.088	.014	2.94
38	.00385	.00919	.087	.013	2.96
39	.00427	.00972	.086	.012	2.98
40	*.00474*	*.01031*	.085	.011	3.00
41	.00541	.01046	.085	.010	3.02
42	.00611	.01068	.085	.009	3.04
43	.00682	.01097	.086	.008	3.06
44	.00750	.01135	.086	.007	3.08
45	*.00811*	*.01183*	.087	.006	3.10
46	.00837	.01240	.088	.005	3.12
47	.00869	.01305	.088	.004	3.14
48	.00922	.01376	.089	.003	3.16
49	.01013	.01452	.090	.002	3.18
50	*.01157*	*.01532*	.092	.001	3.20
51	.01399	.01609	.092		3.20
52	.01705	.01687	.094		3.20
53	.02068	.01769	.096		3.20
54	.02480	.01852	.099		3.20
55	*.02935*	*.01940*	.101		3.20
56	.03403	.02033	.103		3.20
57	.03911	.02132	.105		3.20
58	.04464	.02240	.107		3.20
59	.05069	.02360	.109		3.20
60	*.05728*	*.02495*	.111		3.20
61	.06453	.02679	.113		3.20
62	.07241	.02876	.114		3.20

TABLE I.—*Continued.*

Present Age.	(1) Probability of an Active Life being Alive and Disabled at end of a Year (Zimmerman's German Railways). p_x^{ai}	(2) Probability of an Active Life Dying within a Year (Zimmerman's German Railways). q_x^a	(3) Probability of a Disabled Life Dying within a Year (Hunter's Ultimate). q_x^i	(4) Probability of Withdrawal at end of a Year (Manly's British Railways). q_x^{aw}	(5) Relative Salary Scale based on Yearly Salary. s_x
63	.08094	.03080	.114		3.20
64	.09013	.03289	.115		3.20
65	*.10000*	*.03498*	.112		3.20
66	.15000	.03689	.105		3.20
67	.25000	.03877	.095		3.20
68	.50000	.04061	.085		3.20
69	.95757	.04243	.075		3.20
70			.062		
71			.068		
72			.074		
73			.080		
74			.087		
75			.094		
76			.102		
77			.111		
78			.121		
79			.132		
80			.144		
81			.159		
82			.174		
83			.192		
84			.211		
85			.236		
86			.266		
87			.303		
88			.347		
89			.396		
90			.455		
91			.532		
92			.634		
93			.734		
94			.857		
95			1.000		

Starting with a radix of 100,000 at age 20 the service table, Table II, was constructed. Owing to the fact that the scheme contemplated allowing voluntary retirement after attainment of age 65 and provided for compulsory retirement at age 70, the rate of retirement and also the numbers of those retiring increase rapidly between 65 and 70 and all of the members surviving to the end of the year of age 69 are retired on pension at that time. On first thought it might appear best to increase the rate of retirement to a much greater extent at age 65, but this does not appear to me to be

TABLE II.

Service Table.

Age.	(6) Remaining on Active List, l_x^{aa}.	(7) Number Withdrawing, w_x.	(8) Number Dying, d_x.	(9) Number Pensioned, r_x.	(10) Total Decrement.
20	100,000	5,500	889	21	6,410
21	93,590	4,680	782	28	5,490
22	88,100	4,141	698	35	4,874
23	83,226	3,662	628	42	4,332
24	78,894	3,235	573	48	3,856
25	75,038	2,851	529	54	3,434
26	71,604	2,506	493	61	3,060
27	68,544	2,262	465	68	2,795
28	65,749	2,038	444	76	2,558
29	63,191	1,833	428	84	2,345
30	60,846	1,643	417	93	2,153
31	58,693	1,467	409	102	1,978
32	56,715	1,304	404	113	1,821
33	54,894	1,153	403	124	1,680
34	53,214	1,011	406	135	1,552
35	51,662	878	412	147	1,437
36	50,225	753	418	158	1,329
37	48,896	685	427	170	1,282
38	47,614	619	438	183	1,240
39	46,374	556	451	198	1,205
40	45,169	497	466	214	1,177
41	43,992	440	460	238	1,138
42	42,854	386	458	262	1,106
43	41,748	334	458	285	1,077
44	40,671	285	462	305	1,052
45	39,619	238	469	321	1,028
46	38,591	193	479	323	995
47	37,596	150	491	327	968
48	36,628	110	504	338	952
49	35,676	71	518	361	950
50	34,726	35	532	402	969
51	33,757		543	472	1,015
52	32,742		552	558	1,110
53	31,632		560	654	1,214
54	30,418		563	754	1,317
55	29,101		565	854	1,419
56	27,682		563	942	1,505
57	26,177		558	1,024	1,582
58	24,595		551	1,098	1,649
59	22,946		542	1,163	1,705
60	21,241		530	1,217	1,747
61	19,494		522	1,258	1,780
62	17,714		509	1,283	1,792
63	15,922		490	1,289	1,779
64	14,143		465	1,275	1,740
65	12,403		434	1,240	1,674
66	10,729		396	1,609	2,005
67	8,724		338	2,181	2,519
68	6,205		252	3,102	3,354
69	2,851		121	2,730	2,851
70	0		—	—	—

necessary because the amount of yearly pension in the majority of cases would be much less than the yearly salary and it is believed that practically all employees who were physically able to do so would prefer to continue in active service rather than be retired with a consequent reduction in their income. Of course, this depends to quite an extent upon the amount of pension to be allowed, but in this particular fund the employees were to pay the contributions themselves and 2 per cent. of the final salary multiplied by length of service would in all probability be the maximum pension which they could afford.

Having given the bases upon which the calculations are to be made the next step is the deducing of the formulæ required in determining the contributions necessary to provide the benefits (*a*), (*b*), (*c*), and (*d*). Mr. King's notation has been followed where practicable in all of the formulæ.

NOTATION.

$$^{s}D_x^{aa} = s_x \cdot D_x^{aa},$$

$$^{s}\overline{D}_x^{aa} = s_x \cdot \tfrac{1}{2}(D_x^{aa} + D_{x+1}^{aa}),$$

$$^{fs}C_x^{ra} = {}^{s}D_x^{aa} \cdot p_x^{ai} \cdot v \cdot a_{x+1}^{i},$$

$$^{fs}M_x^{ra} = \sum_x^{\infty} {}^{fs}C_x^{ra},$$

$$^{fs}\overline{M}_x^{ra} = {}^{fs}M_x^{ra} - \tfrac{1}{2} \cdot {}^{fs}C_x^{ra},$$

$$^{fs}\overline{R}_x^{ra} = \sum_x^{\infty} {}^{fs}\overline{M}_x^{ra}.$$

$$^{f}C_x^{aw} = l_x^{aa} \cdot q_x^{aw} \cdot v^{x+1},$$

$$^{f}M_x^{aw} = \sum_x^{\infty} {}^{f}C_x^{aw},$$

$$^{fs}M_x^{aw} = s_x \cdot {}^{f}M_x^{aw},$$

$$^{fs}R_x^{aw} = \sum_x^{\infty} {}^{fs}M_x^{aw}.$$

$$^{fs}C_x^{ad\,\overline{2}|} = {}^{s}D_x^{aa} \cdot q_x^{a} \cdot v^{\frac{1}{2}} \cdot \bar{a}_{\overline{2}|},$$

$$^{fs}M_x^{ad\,\overline{2}|} = \sum_x^{\infty} {}^{fs}C_x^{ad\,\overline{2}|},$$

$$^{fs}\overline{M}_x^{ad\,\overline{2}|} = {}^{fs}M_x^{ad\,\overline{2}|} - \tfrac{1}{2}\,{}^{fs}C_x^{ad\,\overline{2}|},$$

$$^{fs}\overline{R}_x^{ad\,\overline{2}|} = \sum_x^{\infty} {}^{fs}\overline{M}_x^{ad\,\overline{2}|}.$$

$$^{fs}C_x^{r\,|\,d\,\overline{2}|} = {}^{s}D_x^{aa} \cdot p_x^{ai} \cdot \overline{A}_{x+1}^{\,i} \cdot v \cdot \bar{a}_{\overline{2}|},$$

$$^{fs}M_x^{r\,|\,d\,\overline{2}|} = \sum_x^{\infty} {}^{fs}C_x^{r\,|\,d\,\overline{2}|},$$

$$^{fs}\overline{M}_x^{r\,|\,d\,\overline{2}|} = {}^{fs}M_x^{r\,|\,d\,\overline{2}|} - \tfrac{1}{2}\,{}^{fs}C_x^{r\,|\,d\,\overline{2}|},$$

$$^{fs}\overline{R}_x^{r\,|\,d\,\overline{2}|} = \sum_x^{\infty} {}^{fs}\overline{M}_x^{r\,|\,d\,\overline{2}|}.$$

Present Value of Future Contributions.

$^{f}F_x^{c}$ = present value of future contributions of full salary, based on present salary of 1.

$$= \frac{1}{s_x}\left\{\sum_{n=0}^{n=69-x} s_{x+n} \cdot v^n \cdot {}_np_x^{aa} \cdot \left(\text{value at beginning of a year of } t \text{ payments of } \frac{1}{t} \text{ each made at equal intervals throughout the year}\right)\right\}$$

$$= \frac{1}{{}^{s}D_x^{aa}}\left\{\sum_{n=0}^{n=69-x} s_{x+n} \cdot D_{x+n}^{aa} \cdot \left(\sum_{r=0}^{r=t-1} \frac{1}{t} \cdot v^{\frac{r}{t}} \cdot {}_{\frac{r}{t}}p_{x+n}^{aa}\right)\right\}$$

$$= \frac{1}{{}^{s}D_x^{aa}}\left\{\sum_{n=0}^{n=69-x} \tfrac{1}{2}(D_{x+n}^{aa} + D_{x+n+1}^{aa}) \cdot s_{x+n}\right\}$$

$$= \frac{1}{{}^{s}D_x^{aa}} \sum_x^{69} {}^{s}\overline{D}_x^{aa}$$

$$= \frac{{}^{s}\overline{N}_x^{aa}}{{}^{s}D_x^{aa}}.$$

This formula assumes that contributions will be made at frequent intervals throughout the year.

Present Value of Pension on Disablement or Retirement—Due to Future Service.

${}^{fs}F_x^{ra}$ = present value of a pension based on full final salary multiplied by years of future service, on the basis of present salary of 1.

$$= \frac{1}{s_x}\left\{\begin{array}{l} \frac{s_x}{2}\cdot p_x^{ai}\cdot v\cdot(\bar{a}_{x+1}^{i}+\tfrac{1}{2}) \\ \quad +\frac{3s_{x+1}}{2}\cdot p_x^{aa}\cdot p_{x+1}^{ai}\cdot v^2\cdot(\bar{a}_{x+2}^{i}+\tfrac{1}{2}) \\ \quad + \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot \\ \quad +\frac{2n-1}{2}\cdot s_{x+n-1}\cdot {}_{n-1}p_x^{aa}\cdot p_{x+n-1}^{ai}\cdot v^n\cdot(\bar{a}_{x+n}^{i}+\tfrac{1}{2}) \\ \quad + \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot \end{array}\right\}$$

$$= \frac{1}{{}^{s}D_x^{aa}}\left\{\begin{array}{l} s_x\cdot v^x\cdot l_x^{aa}\cdot p_x^{ai}\cdot v\cdot \mathrm{a}_{x+1}^{i} - \frac{s_x}{2}\cdot v^x\cdot l_x^{aa}\cdot p_x^{ai}\cdot v\cdot \mathrm{a}_{x+1}^{i} \\ \quad +2s_{x+1}\cdot v^{x+1}\cdot l_{x+1}^{aa}\cdot p_{x+1}^{ai}\cdot v\cdot \mathrm{a}_{x+2}^{i} \\ \qquad\qquad -\frac{s_{x+1}}{2}\cdot v^{x+1}\cdot l_{x+1}^{aa}\cdot p_{x+1}^{ai}\cdot v\cdot \mathrm{a}_{x+2}^{i} \\ \quad + \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot \\ \quad +n\cdot s_{x+n-1}\cdot v^{x+n-1}\cdot l_{x+n-1}^{aa}\cdot p_{x+n-1}^{ai}\cdot v\cdot \mathrm{a}_{x+n}^{i} \\ \qquad\qquad -\frac{s_{x+n-1}}{2}\cdot v^{x+n-1}\cdot l_{x+n-1}^{aa}\cdot p_{x+n-1}^{ai}\cdot v\cdot \mathrm{a}_{x+n}^{i} \\ \quad + \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot \end{array}\right\}$$

$$= \frac{1}{{}^{s}D_x^{aa}}\left\{\begin{array}{l} {}^{fs}C_x^{ra}+{}^{fs}C_{x+1}^{ra}+\cdots+{}^{fs}C_{69}^{ra}-\tfrac{1}{2}{}^{fs}C_x^{ra} \\ \qquad +{}^{fs}C_{x+1}^{ra}+\cdots+{}^{fs}C_{69}^{ra}-\tfrac{1}{2}{}^{fs}C_{x+1}^{ra} \\ \qquad\qquad + \cdot\cdot\cdot\cdot\cdot\cdot\cdot \\ \qquad\qquad\qquad +{}^{fs}C_{69}^{ra}-\tfrac{1}{2}{}^{fs}C_{69}^{ra} \end{array}\right\}$$

$$= \frac{1}{{}^{s}D_x^{aa}}\left\{({}^{fs}M_x^{ra}-\tfrac{1}{2}{}^{fs}C_x^{ra})+\cdots+({}^{fs}M_{69}^{ra}-\tfrac{1}{2}{}^{fs}C_{69}^{ra})\right\}$$

$$= \frac{1}{{}^{s}D_x^{aa}}\sum^{69}{}^{fs}\overline{M}_x^{ra}$$

$$= \frac{{}^{fs}\overline{R}_x^{ra}}{{}^{s}D_x^{aa}}.$$

It has been assumed, in deducing this formula, that retirements will occur on the average in the middle of the year of service, and that the pension will take effect as of that time, but that the pension instalments due during the last half of such year will not be actually paid until the end of the year. Theoretically $(\frac{1}{2}v^{\frac{3}{2}} + v\bar{a}^{i}_{x+1})$ should be used instead of $v(\bar{a}^{i}_{x+1} + \frac{1}{2})$ but the data upon which the figures are based do not warrant too much refinement.

Present Value of Benefit on Withdrawal of Contributions —Due to Future Service.

${}^{fs}F^{aw}_{x}$ = present value of benefit on withdrawal, based on contribution of full salary and present salary of 1.

$$= \frac{1}{{}^{s}D^{aa}_{x}} \left\{ \begin{array}{l} s_x \cdot \sum\limits_{n=0}^{n=69-x} l^{aa}_{x+n} \cdot q^{aw}_{x+n} \cdot v^{x+n+1} + \cdots \\ \qquad + s_{x+r} \cdot \sum\limits_{n=0}^{n=69-x-r} l^{aa}_{x+r+n} \cdot q^{aw}_{x+r+n} \cdot v^{x+r+n+1} + \cdots \end{array} \right\}$$

$$= \frac{1}{{}^{s}D^{aa}_{x}} \left(s_x \sum_{x}^{69} {}^{f}C^{aw}_{x} + \cdots + s_{x+r} \sum_{x+r}^{69} {}^{f}C^{aw}_{x+r} + \cdots \right)$$

$$= \frac{1}{{}^{s}D^{aa}_{x}} ({}^{fs}M^{aw}_{x} + \cdots + {}^{fs}M^{aw}_{x+r} + \cdots)$$

$$= \frac{{}^{fs}R^{aw}_{x}}{{}^{s}D^{aa}_{x}} .$$

Present Value of Benefit at Death Before Retirement—Due to Future Service.

${}^{fs}F^{ad\overline{2}|}_{x}$ = present value of an annuity certain for two years commencing at death of an active life, the amount of the annuity being equal to the amount of pension which would have been allowed on account of future service if disablement had occurred instead of death.

$$= \frac{1}{s_x} \left(\frac{s_x}{2} \cdot q^{a}_{x} \cdot v^{\frac{1}{2}} \cdot \bar{a}_{\overline{2}|} + \frac{3s_{x+1}}{2} \cdot p^{aa}_{x} \cdot q^{a}_{x+1} \cdot v^{\frac{3}{2}} \cdot \bar{a}_{\overline{2}|} + \cdots \right.$$

$$\left. + \frac{2n-1}{2} \cdot s_{x+n-1} \cdot {}_{n-1}p^{aa}_{x} \cdot q^{a}_{x+n-1} \cdot v^{\frac{2n-1}{2}} \cdot \bar{a}_{\overline{2}|} + \cdots \right)$$

$$= \frac{1}{{}^{s}D_x^{aa}} \left\{ \begin{array}{l} s_x \cdot v^x \cdot l_x^{aa} \cdot q_x^a \cdot v^{\frac{1}{2}} \cdot \bar{a}_{\overline{2}|} - \frac{1}{2} s_x \cdot v^x \cdot l_x^{aa} \cdot q_x^a \cdot v^{\frac{1}{2}} \cdot \bar{a}_{\overline{2}|} \\ \quad + 2 s_{x+1} \cdot v^{x+1} \cdot l_{x+1}^{aa} \cdot q_{x+1}^a \cdot v^{\frac{1}{2}} \cdot \bar{a}_{\overline{2}|} \\ \qquad\qquad - \frac{1}{2} s_{x+1} \cdot v^{x+1} \cdot l_{x+1}^{aa} \cdot q_{x+1}^a \cdot v^{\frac{1}{2}} \cdot \bar{a}_{\overline{2}|} \\ \quad + \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \quad + n \cdot s_{x+n-1} \cdot v^{x+n-1} \cdot l_{x+n-1}^{aa} \cdot q_{x+n-1}^a \cdot v^{\frac{1}{2}} \cdot \bar{a}_{\overline{2}|} \\ \qquad\qquad - \frac{1}{2} s_{x+n-1} \cdot v^{x+n-1} \cdot l_{x+n-1}^{aa} \cdot q_{x+n-1}^a \cdot v^{\frac{1}{2}} \cdot \bar{a}_{\overline{2}|} \\ \quad + \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{array} \right\}$$

$$= \frac{1}{{}^{s}D_x^{aa}} \left\{ \begin{array}{l} {}^{fs}C_x^{ad\overline{2}|} + {}^{fs}C_{x+1}^{ad\overline{2}|} + \cdots + {}^{fs}C_{69}^{ad\overline{2}|} - \frac{1}{2}\, {}^{fs}C_x^{ad\overline{2}|} \\ \qquad + {}^{fs}C_{x+1}^{ad\overline{2}|} + \cdots + {}^{fs}C_{69}^{ad\overline{2}|} - \frac{1}{2}\, {}^{fs}C_{x+1}^{ad\overline{2}|} \\ \qquad\qquad + \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \qquad\qquad\qquad + {}^{fs}C_{69}^{ad\overline{2}|} - \frac{1}{2}\, {}^{fs}C_{69}^{ad\overline{2}|} \end{array} \right\}$$

$$= \frac{1}{{}^{s}D_x^{aa}} \left\{ \left({}^{fs}M_x^{ad\overline{2}|} - \frac{1}{2}\, {}^{fs}C_x^{ad\overline{2}|} \right) + \left({}^{fs}M_{x+1}^{ad\overline{2}|} - \frac{1}{2}\, {}^{fs}C_{x+1}^{ad\overline{2}|} \right) + \cdots \right.$$
$$\left. + \left({}^{fs}M_{69}^{ad\overline{2}|} - \frac{1}{2}\, {}^{fs}C_{69}^{ad\overline{2}|} \right) \right\}$$

$$= \frac{1}{{}^{s}D_x^{aa}} \sum_{x}^{69} {}^{fs}\bar{M}_x^{ad\overline{2}|}$$

$$= \frac{{}^{fs}\bar{R}_x^{ad\overline{2}|}}{{}^{s}D_x^{aa}} .$$

Present Value of Benefit at Death After Retirement—Due to Future Service.

${}^{fs}F_x^{r|d\overline{2}|}$ = present value of an annuity certain for two years commencing at death of a pensioner, the amount of the annuity being equal to the amount of pension which was being received by the pensioner on account of future service.

$$= \frac{1}{s_x} \left(\frac{s_x}{2} \cdot p_x^{ai} \cdot \bar{A}_{x+1}^{i} \cdot v \cdot \bar{a}_{\overline{2}|} + \frac{3 s_{x+1}}{2} \cdot p_x^{aa} \cdot p_{x+1}^{ai} \cdot \bar{A}_{x+2}^{i} \cdot v^2 \cdot \bar{a}_{\overline{2}|} \right.$$
$$\left. + \cdots + \frac{2n-1}{2} \cdot s_{x+n-1} \cdot {}_{n-1}p_x^{aa} \cdot p_{x+n-1}^{ai} \cdot \bar{A}_{x+n}^{i} \cdot v^n \cdot \bar{a}_{\overline{2}|} + \cdots \right)$$

$$= \frac{1}{{}^{s}D_x^{aa}} \left\{ \begin{array}{l} s_x \cdot v^x \cdot l_x^{aa} \cdot p_x^{ai} \cdot \bar{A}_{x+1}^{i} \cdot v \cdot \bar{a}_{\overline{2}|} \\ \qquad\qquad - \frac{s_x}{2} \cdot v^x \cdot l_x^{aa} \cdot p_x^{ai} \cdot \bar{A}_{x+1}^{i} \cdot v \cdot \bar{a}_{\overline{2}|} \\ + \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\ + n \cdot s_{x+n-1} \cdot v^{x+n-1} \cdot l_{x+n-1}^{aa} \cdot p_{x+n-1}^{ai} \cdot \bar{A}_{x+n}^{i} \cdot v \cdot \bar{a}_{\overline{2}|} \\ \qquad - \frac{s_{x+n-1}}{2} \cdot v^{x+n-1} \cdot l_{x+n-1}^{aa} \cdot p_{x+n-1}^{ai} \cdot \bar{A}_{x+n}^{i} \cdot v \cdot \bar{a}_{\overline{2}|} \\ + \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \end{array} \right\}$$

$$= \frac{1}{{}^{s}D_x^{aa}} \sum_{x}^{69} \left({}^{fs}M_x^{r\,|\,d\,\overline{2}|} - \tfrac{1}{2}\, {}^{fs}C_x^{r\,|\,d\,\overline{2}|} \right)$$

$$= \frac{1}{{}^{s}D_x^{aa}} \sum_{x}^{69} {}^{fs}\bar{M}_x^{r\,|\,d\,\overline{2}|}$$

$$= \frac{{}^{fs}\bar{R}_x^{r\,|\,d\,\overline{2}|}}{{}^{s}D_x^{aa}}.$$

PRESENT VALUE OF PENSION ON DISABLEMENT OR RETIREMENT—DUE TO PAST SERVICE.

$(x-y)\,{}^{ps}F_x^{ra}$ = present value of a pension based on full final salary multiplied by years of past service, on the basis of present salary of 1.

$$= \text{length of past service } \frac{1}{s_x} \left(s_x \cdot p_x^{ai} \cdot v \cdot \mathrm{a}_{x+1}^{i} + s_{x+1} \cdot p_x^{aa} \cdot p_{x+1}^{ai} \cdot v^2 \cdot \mathrm{a}_{x+2}^{i} + \cdots \right)$$

$$= \frac{1}{{}^{s}D_x^{aa}} (x-y)\left({}^{fs}C_x^{ra} + {}^{fs}C_{x+1}^{ra} + \cdots + {}^{fs}C_{69}^{ra} \right)$$

$$= \frac{1}{{}^{s}D_x^{aa}} (x-y) \cdot {}^{fs}M_x^{ra}.$$

It should be remembered that y, the age at entry into the service of the company, will not be the same in practice for all employees now aged x, so that $(x-y)$ should be computed for each person now aged x. Nor will the salary at time of entry into the fund be the same for all persons aged x, so that to determine the present value of this benefit in actual practice we must multiply the present salary of each person aged x by the number of years of past service. The sum of all these products multiplied by ${}^{fs}M_x^{ra} \div {}^{s}D_x^{aa}$ will then be the present value of the benefit on the basis of a pension of full salary multiplied by years of past service.

The present value of the benefit on withdrawal due to past service does not enter into our calculations, as contributions are assumed not to have commenced as yet. The valuing of this benefit would, however, be required whenever a subsequent valuation of the fund was required. The proper formula may be easily deduced.

PRESENT VALUE OF BENEFIT AT DEATH BEFORE RETIREMENT—DUE TO PAST SERVICE.

$(x-y)\,{}^{ps}F_x^{ad\overline{2}|}$ = present value of an annuity certain for two years commencing at death of an active life, the amount of the annuity being equal to the amount of pension which would have been allowed on account of past service if disablement had occurred instead of death.

$$= \text{length of past service}\,\frac{1}{s_x}\,(s_x \cdot q_x^a \cdot v^{\frac{1}{2}} \cdot \bar{a}_{\overline{2}|} + s_{x+1} \cdot p_x^{aa} \cdot q_{x+1}^a \cdot v^{\frac{3}{2}} \cdot \bar{a}_{\overline{2}|} + \cdots)$$

$$= \frac{1}{{}^{s}D_x^{aa}}\,(x-y) \cdot ({}^{fs}C_x^{ad\overline{2}|} + {}^{fs}C_{x+1}^{ad\overline{2}|} + \cdots + {}^{fs}C_{69}^{ad\overline{2}|})$$

$$= \frac{1}{{}^{s}D_x^{aa}}\,(x-y) \cdot {}^{fs}M_x^{ad\overline{2}|}.$$

PRESENT VALUE OF BENEFIT AT DEATH AFTER RETIREMENT—DUE TO PAST SERVICE.

$(x-y)\,{}^{ps}F_x^{r|d\overline{2}|}$ = present value of an annuity certain for two years commencing at death of a pensioner, the amount of the annuity being equal to the amount of pension which was being received by the pensioner on account of past service.

$$= \text{length of past service}\,\frac{1}{s_x}\,(s_x \cdot p_x^{ai} \cdot \bar{A}_{x+1}^{i} \cdot v \cdot \bar{a}_{\overline{2}|} + s_{x+1} \cdot p_x^{aa} \cdot p_{x+1}^{ai} \cdot \bar{A}_{x+2}^{i} \cdot v^2 \cdot \bar{a}_{\overline{2}|} + \cdots)$$

$$= \frac{1}{{}^{s}D_x^{aa}}\,(x-y)\,({}^{fs}C_x^{r|d\overline{2}|} + {}^{fs}C_{x+1}^{r|d\overline{2}|} + \cdots + {}^{fs}C_{69}^{r|d\overline{2}|})$$

$$= \frac{1}{{}^{s}D_x^{aa}}\,(x-y)\,{}^{fs}M_x^{r|d\overline{2}|}.$$

TABLE III.

4 PER CENT.

Age.	(11) D_x^{aa}	(12) l_x^i	(13) D_x^i	(14) N_x^i	(15) a_x^i	(16) $\bar{A}_x^i = 1 - \delta\left(a_x^i - \frac{1}{2}\right)$
20	45,639	1,000,000	456,387	2,374,026	5.2018	.81559
21	41,070	795,000	348,873	1,917,639	5.4967	.80403
22	37,174	641,565	270,712	1,568,766	5.7950	.79233
23	33,767	524,800	212,925	1,298,054	6.0963	.78051
24	30,778	435,059	169,726	1,085,129	6.3934	.76886
25	28,148	365,015	136,923	915,403	6.6855	.75740
26	25,827	309,898	111,777	778,480	6.9646	.74645
27	23,772	266,202	92,323.4	666,703.1	7.2214	.73638
28	21,926	231,063	77,054.2	574,379.7	7.4542	.72725
29	20,262	202,642	64,977.4	497,325.5	7.6538	.71942
30	18,760	179,541	55,355.9	432,348.1	7.8103	.71328
31	17,400	160,510	47,584.8	376,992.2	7.9225	.70888
32	16,167	144,459	41,179.2	329,407.4	7.9994	.70587
33	15,046	130,735	35,833.7	288,228.2	8.0435	.70414
34	14,025	118,707	31,285.5	252,394.5	8.0675	.70320
35	13,092	108,023	27,374.6	221,109.0	8.0772	.70282
36	12,238	98,409	23,979.2	193,734.4	8.0793	.70273
37	11,456	89,749	21,027.9	169,755.2	8.0729	.70299
38	10,727	81,851	18,439.8	148,727.3	8.0656	.70327
39	10,046	74,730	16,188.1	130,287.5	8.0484	.70395
40	9,408.2	68,303	14,226.8	114,099.4	8.0200	.70506
41	8,810.6	62,497	12,516.8	99,872.6	7.9791	.70666
42	8,252.6	57,185	11,012.4	87,355.8	7.9325	.70849
43	7,730.4	52,324	9,688.73	76,343.40	7.8796	.71057
44	7,241.3	47,824	8,514.87	66,654.67	7.8280	.71259
45	6,782.7	43,711	7,483.24	58,139.80	7.7693	.71489
46	6,352.6	39,908	6,569.42	50,656.56	7.7110	.71718
47	5,950.8	36,396	5,760.87	44,087.14	7.6529	.71946
48	5,574.6	33,193	5,051.81	38,326.27	7.5866	.72206
49	5,220.9	30,239	4,425.21	33,274.46	7.5193	.72470
50	4,886.4	27,517	3,872.00	28,849.25	7.4507	.72739
51	4,567.4	24,985	3,380.50	24,977.25	7.3886	.72982
52	4,259.6	22,686	2,951.38	21,596.75	7.3175	.73261
53	3,956.9	20,554	2,571.16	18,645.37	7.2517	.73519
54	3,658.7	18,581	2,234.96	16,074.21	7.1922	.73753
55	3,365.7	16,741	1,936.20	13,839.25	7.1476	.73928
56	3,078.4	15,050	1,673.67	11,903.05	7.1119	.74068
57	2,799.1	13,500	1,443.56	10,229.38	7.0862	.74168
58	2,528.8	12,083	1,242.34	8,785.82	7.0720	.74224
59	2,268.5	10,790	1,066.73	7,543.48	7.0716	.74226
60	2,019.2	9,614	913.907	6,476.751	7.0869	.74166
61	1,781.8	8,547	781.230	5,562.844	7.1206	.74034
62	1,556.9	7,581	666.287	4,781.614	7.1765	.73814
63	1,345.5	6,717	567.640	4,115.327	7.2499	.73526
64	1,149.2	5,951	483.566	3,547.687	7.3365	.73187
65	969.08	5,267	411.527	3,064.121	7.4457	.72758
66	806.05	4,677	351.374	2,652.594	7.5492	.72353
67	630.20	4,186	302.388	2,301.220	7.6102	.72113
68	431.00	3,788	263.114	1,998.832	7.5968	.72166
69	190.41	3,466	231.487	1,735.718	7.4981	.72553
70	0	3,206	205.886	1,504.231	7.3061	.73306
71		3,007	185.679	1,298.345		

TABLE III.—*Continued.*

Age.	(11) D_x^{aa}	(12) l_x^i	(13) D_x^i	(14) N_x^i	(15) a_x^i	(16) $\bar{A}_x^i = 1 - \delta\left(a_x^i - \frac{1}{2}\right)$
72		2,803	166.425	1,112.666		
73		2,596	148.208	946.241		
74		2,388	131.089	798.033		
75		2,180	115.069	666.944		
76		1,975	100.2392	551.8750		
77		1,774	86.5730	451.6358		
78		1,577	73.9991	365.0628		
79		1,386	62.5363	291.0637		
80		1,203	52.1910	228.5274		
81		1,030	42.9675	176.3364		
82		866	34.7361	133.3689		
83		715	27.5768	98.6328		
84		578	21.4351	71.0560		
85		456	16.2605	49.6209		
86		348	11.9319	33.3604		
87		255	8.40710	21.42849		
88		178	5.64278	13.02139		
89		116	3.53580	7.37861		
90		70	2.05163	3.84281		
91		38	1.07092	1.79118		
92		18	.48776	.72026		
93		7	.18239	.23250		
94		2	.05011	.05011		
95		0	.0	.0		

Having deduced the necessary formulæ we now proceed to apply them to the Service Table. Tables III to XIV inclusive show the calculations in detail.

TABLE IV.

FUTURE CONTRIBUTIONS.

Age.	(17) ${}^{s}\bar{D}_{x}^{aa}$	(18) ${}^{s}\bar{N}_{x}^{aa}$	(19) ${}^{f}F_{x}^{c} = (18) \div (20)$
20	78,038.00	1,396,532.67	16.9999
21	75,114.33	1,318,494.67	16.7204
22	72,359.67	1,243,380.34	16.3958
23	69,708.66	1,171,020.67	16.0554
24	67,175.67	1,101,312.01	15,6939
25	64,770.00	1,034,136.34	15.3080
26	61,502.67	969,366.34	15.1345
27	58,493.33	907,863.67	14.9181
28	55,688.00	849,370.34	14.6737
29	53,070.00	793,682.34	14.4009
30	50,624.00	740,612.34	14.0994
31	47,665.00	689,988.34	13.9628
32	44,634.67	642,323.34	13.8917
33	41,862.33	597,688.67	13.7930
34	39,319.67	455,826.34	13.6663
35	36,728.67	516,506.67	13.6042
36	34,593.33	479,778.00	13.4258
37	32,609.00	445,194.67	13.2175
38	30,744.03	412,575.67	12.9941
39	28,986.77	381,831.64	12.7551
40	27,328.20	352,844.87	12.5012
41	25,765.43	325,516.67	12.2338
42	24,294.16	299,751.24	11.9480
43	22,906.70	275,457.08	11.6448
44	21,596.97	252,550.38	11.3234
45	20,359.70	230,953.41	10.9840
46	19,193.30	210,593.71	10.6251
47	18,094.87	191,400.41	10.2432
48	17,056.90	173,305.54	9.8382
49	16,070.60	156,248.64	9.4113
50	15,126.07	140,178.04	8.9649
51	14,123.20	125,051.97	8.5560
52	13,146.40	110,928.77	8.1382
53	12,184.96	97,782.37	7.7223
54	11,239.03	85,597.41	7.3110
55	10,310.57	74,358.38	6.9040
56	9,404.00	64,047.81	6.5017
57	8,524.64	54,643.81	6.1007
58	7,675.67	46,119.17	5.6993
59	6,860.33	38,443.50	5.2958
60	6,081.60	31,583.17	4.8880
61	5,341.93	25,501.57	4.4725
62	4,643.83	20,159.64	4.0465
63	3,991.54	15,515.81	3.6035
64	3,389.23	11,524.27	3.1337
65	2,840.21	8,135.04	2.6233
66	2,298.00	5,294.83	2.0528
67	1,697.92	2,996.83	1.4860
68	994.26	1,298.91	.94179
69	304.65	304.65	.50000

TABLE V.

Benefit (a)—Future Service.

Age.	(20) ${}^{s}D_x^{aa}$	(21) $v \cdot p_x^{ai} \times a_{x+1}^{i}$	(22) ${}^{fs}C_x^{ra}$ = (20) × (21)	(23) ${}^{fs}M_x^{ra}$	(24) ${}^{fs}\overline{M}_x^{ra}$ = (23) − ½(22)	(25) ${}^{fs}\overline{R}_x^{ra}$	(26) ${}^{fs}P_x^{ra}$ = (25) ÷ (20)
20	82,149.67	.00111	91.19	73,165.29	73,119.69	2,530,651.22	30.8054
21	78,855.33	.00167	131.69	73,074.10	73,008.26	2,457,531.53	31.1651
22	75,835.33	.00235	178.21	72,942.41	72,853.30	2,384,523.27	31.4434
23	72,936.33	.00308	224.64	72,764.20	72,651.88	2,311,669.97	31.6944
24	70,174.33	.00392	275.08	72,539.56	72,402.02	2,239,018.09	31.9065
25	67,555.33	.00482	325.62	72,264.48	72,101.67	2,166,616.07	32.0717
26	64,050.33	.00590	377.90	71,938.86	71,749.91	2,094,514.40	32.7011
27	60,856.67	.00710	432.08	71,560.96	71,344.92	2,022,764.49	33.2382
28	57,884.00	.00846	489.70	71,128.88	70,884.03	1,951,419.57	33.7126
29	55,113.33	.00999	550.58	70,639.18	70,363.89	1,880,535.54	34.1212
30	52,528.00	.01165	611.95	70,088.60	69,782.63	1,810,171.65	34,4611
31	49,416.33	.01338	661.19	69,476.65	69,146.05	1,740,389.02	35.2189
32	46,238.00	.01539	711.60	68,815.46	68,459.66	1,671,242.97	36,1444
33	43,332.67	.01745	756.16	68,103.86	67,725.78	1,602,783.31	36.9879
34	40,671.33	.01965	799.19	67,347.70	66,948.10	1,535,057.53	37.7430
35	37,966.67	.02207	837.92	66,548.51	66,129.55	1,468,109.43	38.6684
36	35,735.67	.02445	873.74	65,710.59	65,273.72	1,401,979.88	39.2319
37	33,681.33	.02699	909.06	64,836.85	64,382.32	1,336,706.16	39.6869
38	31,751.00	.02980	946.18	63,927.79	63,454.70	1,272,323.84	40.0719
39	29,935.67	.03293	985.78	62,981.61	62,488.72	1,208,869.14	40.3822
40	28,224.67	.03637	1,026.53	61,995.83	61,482.57	1,146,380.42	40.6163
41	26,608.00	.04126	1,097.85	60,969.30	60,420.37	1,084,897.85	40.7734
42	25,088.00	.04629	1,161.32	59,871.45	59,290.79	1,024,477.48	40.8354
43	23,655.00	.05134	1,214.45	58,710.13	58,102.90	965,186.69	40.8027
44	22,303.33	.05603	1,249.66	57,495.68	56,870.85	907,083.79	40.6703
45	21,026.33	.06013	1,264.31	56,246.02	55,613.86	850,212.94	40.4356
46	19,820.33	.06159	1,220.73	54,981.71	54,371.35	794,599.08	40.0901
47	18,685.67	.06339	1,184.48	53,760.98	53,168.74	740,227.73	39.6147
48	17,615.67	.06666	1,174.26	52,576.50	51,989.37	687,058.99	39.0027
49	16,602.33	.07258	1,205.00	51,402.24	50,799.74	635,069.62	38.2518
50	15,636.33	.08220	1,285.31	50,197.24	49,554.58	584,269.88	37.3662
51	14,615.67	.09843	1,438.62	48,911.93	48,192.62	534,715.30	36.5851
52	13,630.67	.11888	1,620.41	47,473.31	46,663.11	486,522.68	35.6932
53	12,662.33	.14301	1,810.84	45,852.90	44,947.48	439,859.57	34.7376
54	11,708.00	.17044	1,995.51	44,042.06	43,044.30	394,912.09	33.7301
55	10,770.33	.20070	2,161.61	42,046.55	40,965.75	351,867.79	32.6701
56	9,851.00	.23187	2,284.15	39,884.94	38,742.86	310,902.04	31.5605
57	8,957.00	.26595	2,382.11	37,600.79	36,409.74	272,159.18	30.3851
58	8,092.10	.30354	2,456.28	35,218.68	33,990.54	235,749.44	29.1333
59	7,259.23	.34541	2,507.41	32,762.40	31,508.69	201,758.90	27.7934
60	6,461.33	.39218	2,534.00	30,254.99	28,987.99	170,250.21	26.3491
61	5,701.87	.44529	2,538.99	27,720.99	26,451.49	141,262.22	24.7747
62	4,981.93	.50478	2,514.78	25,182.00	23,924.61	114,810.73	23.0454
63	4,305.73	.57098	2,458.49	22,667.22	21,437.98	90,886.12	21.1082
64	3,677.53	.64527	2,373.00	20,208.73	19,022.23	69,448.14	18.8845
65	3,101.07	.72588	2,251.00	17,835.73	16,710.23	50,425.91	16.2608
66	2,579.37	1.09763	2,831.19	15,584.73	14,169.13	33,715.68	13.0713
67	2,016.67	1.82615	3,682.74	12,753.54	10,912.17	19,546.55	9.6925
68	1,379.20	3.60486	4,971.82	9,070.80	6,584.89	8,634.38	6.2604
69	609.33	6.72702	4,098.98	4,098.98	2,049.49	2,049.49	3.3635

TABLE VI.

Benefit (b)—Future Service.

Age.	(27) ${}^{f}C_x^{aw}$	(28) ${}^{f}M_x^{aw}$	(29) ${}^{fs}M_x^{aw}$	(30) ${}^{fs}R_x^{aw}$	(31) ${}^{fs}F_x^{aw}$ = (30) ÷ (20)
20	2,413.6	15,685.50	28,233.90	233,472.33	2.8420
21	1,974.6	13,271.90	25,482.05	205,238.43	2.6027
22	1,680.0	11,297.30	23,046.49	179,756.38	2.3704
23	1,428.6	9,617.30	20,773.37	156,709.89	2.1486
24	1,213.4	8,188.70	18,670.24	135,936.52	1.9371
25	1,028.5	6,975.30	16,740.72	117,266.28	1.7359
26	869.20	5,946.80	14,748.06	100,525.56	1.5695
27	754.33	5,077.60	12,998.66	85,777.50	1.4095
28	653.55	4,323.27	11,413.43	72,778.84	1.2573
29	564.99	3,669.72	9,981.64	61,365.41	1.1134
30	487.01	3,104.73	8,693.24	51,383.77	.9782
31	418.25	2,617.72	7,434.32	42,690.53	.8639
32	357.53	2,199.47	6,290.48	35,256.21	.7625
33	303.84	1,841.94	5,304.79	28,965.73	.6684
34	256.23	1,538.10	4,460.49	23,660.94	.5818
35	213.98	1,281.87	3,717.42	19,200.45	.5057
36	176.49	1,067.89	3,118.24	15,483.03	.4333
37	154.22	891.395	2,620.70	12,364.79	.3671
38	134.08	737.175	2,182.04	9,744.09	.3069
39	115.89	603.095	1,797.22	7,562.05	.2526
40	99.507	487.205	1,461.62	5,764.83	.2042
41	84.729	387.698	1,170.85	4,303.21	.1617
42	71.438	302.969	921.027	3,132.356	.1249
43	59.449	231.531	708.483	2,211.329	.0935
44	48.724	172.082	530.013	1,502.846	.0674
45	39.144	123.358	382.410	972.833	.0463
46	30.525	84.214	262.748	590.423	.0298
47	22.896	53.689	168.583	327.675	.0175
48	16.080	30.793	97.303	159.092	.0090
49	10.025	14.713	46.787	61.789	.0037
50	4.6880	4.6880	15.002	15.002	.0010

TABLE VII.

Benefit (c)—Future Service.

Age.	(32) $q_x^a \cdot v^{\frac{1}{2}} \cdot \bar{a}_{\overline{2}\rceil}$	(33) ${}^{fs}C_x^{ad\,\overline{2}\rceil}$ $= (20) \times (32)$	(34) ${}^{fs}M_x^{ad\,\overline{2}\rceil}$	(35) ${}^{fs}\overline{M}_x^{ad\,\overline{2}\rceil}$ $=(34)-\frac{1}{2}(33)$	(36) ${}^{fs}\overline{R}_x^{ad\,\overline{2}\rceil}$	(37) ${}^{fs}F_x^{ad\,\overline{2}\rceil}$ $= (36) \div (20)$
20	.0167725	1,377.85	26,462.94	25,774.02	476,919.23	5.8055
21	.0157723	1,243.73	25,085.09	24,463.23	451,145.21	5.7212
22	.0149452	1,133.37	23,841.36	23,274.67	426,681.98	5.6264
23	.0142335	1,038.14	22,707.99	22,188.92	403,407.31	5.5309
24	.0137142	962.39	21,669.85	21,188.66	381,218.39	5.4325
25	.0132910	897.87	20,707.46	20,258.52	360,029.73	5.3294
26	.0130025	832.82	19,809.59	19,393.18	339,771.21	5.3048
27	.0128102	779.58	18,976.77	18,586.98	320,378.03	5.2645
28	.0127333	737.06	18,197.19	17,828.66	301,791.05	5.2137
29	.0127717	703.89	17,460.13	17,108.19	283,962.39	5.1523
30	.0129256	678.95	16,756.24	16,416.77	266,854.20	5.0802
31	.0131179	648.24	16,077.29	15,753.17	250,437.43	5.0679
32	.0134449	621.66	15,429.05	15,118.22	234,684.26	5.0756
33	.0138681	600.95	14,807.39	14,506.91	219.566.04	5.0670
34	.0143874	585.15	14,206.44	13,913.87	205,059.13	5.0419
35	.0150606	571.80	13,621.29	13,335.39	191,145.26	5.0345
36	.0156953	560.88	13,049.49	12,769.05	177,809.87	4.9757
37	.0164647	554.56	12,488.61	12,211.33	165,040.82	4.9001
38	.0173303	550.25	11,934.05	11,658.93	152,829.49	4.8134
39	.0183304	548.74	11,383.80	11,109.43	141,170.56	4.7158
40	.0194461	548.85	10,835.06	10,560.63	130,061.13	4.6081
41	.0197346	525.10	10,286.21	10,023.66	119.500.50	4.4911
42	.0201385	505.23	9,761.11	9,508.50	109,476.84	4.3637
43	.0206963	489.57	9,255.88	9,011.09	99,968.34	4.2261
44	.0214080	477.47	8,766.31	8,527.58	90,957.25	4.0782
45	.0223120	469.14	8,288.84	8,054.27	82,429.67	3.9203
46	.0233891	463.58	7,819.70	7,587.91	74,375.40	3.7525
47	.0246201	460.05	7,356.12	7,126.09	66,787.49	3.5743
48	.0259473	457.08	6,896.07	6,667.53	59,661.40	3.3868
49	.0273899	454.73	6,438.99	6,211.62	52,993.87	3.1919
50	.0288902	451.73	5,984.26	5,758.40	46,782.25	2.9919
51	.0303520	443.62	5,532.53	5,310.72	41,023.85	2.8068
52	.0318138	433.64	5,088.91	4,872.09	35,713.13	2.6201
53	.0333718	422.56	4,655.27	4,443.99	30,841.04	2.4357
54	.0349298	408.96	4,232.71	4,028.23	26,397.05	2.2546
55	.0365840	394.02	3,823.75	3,626.74	22,368.82	2.0769
56	.0383535	377.83	3,429.73	3,240.81	18,742.08	1.9026
57	.0402193	360.24	3,051.90	2,871.78	15,501.27	1.7306
58	.0422581	341.96	2,691.66	2,520.68	12,629.49	1,5607
59	.0445086	323.10	2,349.70	2.188.15	10,108.81	1.3926
60	.0470668	304.11	2,026.60	1,874.54	7,920.66	1.2259
61	.0505289	288.11	1,722.49	1,578.44	6,046.12	1.0604
62	.0542412	270.23	1,434.38	1,299.26	4,467.68	.8968
63	.0580881	250.11	1,164.15	1,039.10	3,168.42	.7359
64	.0620312	228.12	914.04	799.98	2,129.32	.5790
65	.0659742	204.59	685.92	583.63	1,329.34	.4287
66	.0695711	179.45	481.33	391.60	745.71	.2891
67	.0731295	147.48	301.88	228.14	354.11	.1756
68	.0765917	105.63	154.40	101.59	125.97	.0913
69	.0800346	48.77	48.77	24.38	24.38	.0400

$\bar{a}_{\overline{2}\rceil}$ has been taken as equal to $v^{\frac{1}{2}}(1 + v)$.

TABLE VIII.

Benefit (D)—Future Service.

Age.	(38) $v \cdot \bar{d}_{\overline{2}\rceil} \times p_x^{ai} \cdot \bar{A}^{i}_{x+1}$	(39) ${}^{fs}C_x^{r\|d\overline{2}\rceil} = (20) \times (38)$	(40) ${}^{fs}M_x^{r\|d\overline{2}\rceil}$	(41) ${}^{fs}\overline{M}_x^{r\|d\overline{2}\rceil} = (40) - \frac{1}{2}(39)$	(42) ${}^{fs}\overline{R}_x^{r\|d\overline{2}\rceil}$	(43) ${}^{fs}F_x^{r\|d\overline{2}\rceil} = (42) \div (20)$
20	.00031	25.47	13,795.05	13,782.32	480,565.09	5.8499
21	.00044	34.70	13,769.58	13,752.23	466,782.77	5.9195
22	.00057	43.23	13,734.88	13,713.26	453,030.54	5.9739
23	.00070	51.06	13,691.65	13,666.12	439,317.28	6.0233
24	.00085	59.65	13,640.59	13,610.77	425,651.16	6.0656
25	.00100	67.56	13,580.94	13,547.16	412,040.39	6.0993
26	.00117	74.94	13,513.38	13,475.91	398,493.23	6.2216
27	.00133	80.94	13,438.44	13,397.97	385,017.32	6.3266
28	.00154	89.14	13,357.50	13,312.93	371,619.35	6.4201
29	.00176	97.00	13,268.36	13,219.86	358,306.42	6.5013
30	.00200	105.06	13,171.36	13,118.83	345,086.56	6.5696
31	.00227	112.18	13,066.30	13,010.21	331,967.73	6.7178
32	.00259	119.76	12,954.12	12,894.24	318,957.52	6.8982
33	.00292	126.53	12,834.36	12,771.09	306,063.28	7.0631
34	.00329	133.81	12,707.83	12,640.93	293,292.19	7.2113
35	.00370	140.48	12,574.02	12,503.78	280,651.26	7.3920
36	.00409	146.16	12,433.54	12,360.46	268,147.48	7.5036
37	.00453	152.58	12,287.38	12,211.09	255,787.02	7.5943
38	.00501	159.07	12,134.80	12,055.27	243,575.93	7.6714
39	.00557	166.74	11,975.73	11,892.36	231,520.66	7.7339
40	.00620	174.99	11,808.99	11,721.49	219,628.30	7.7814
41	.00708	188.38	11,634.00	11,539.81	207,906.81	7.8137
42	.00803	201.46	11,445.62	11,344.89	196,367.00	7.8271
43	.00899	212.66	11,244.16	11,137.83	185,022.11	7.8217
44	.00991	221.03	11,031.50	10,920.98	173,884.28	7.7963
45	.01076	226.24	10,810.47	10,697.35	162,963.30	7.7504
46	.01113	220.60	10,584.23	10,473.93	152,265.95	7.6823
47	.01160	216.75	10,363.63	10,255.26	141,792.02	7.5883
48	.01235	217.55	10,146.88	10,038.10	131,536.76	7.4670
49	.01363	226.29	9,929.33	9,816.19	121,498.66	7.3182
50	.01561	244.08	9,703.04	9,581.00	111,682.47	7.1425
51	.01896	277.11	9,458.96	9,320.40	102,101.47	6.9858
52	.02317	315.82	9,181.85	9,023.94	92,781.07	6.8068
53	.02820	357.08	8,866.03	8,687.49	83,757.13	6.6147
54	.03390	396.90	8,508.95	8,310.50	75,069.64	6.4118
55	.04021	433.07	8,112.05	7,895.51	66,759.14	6.1984
56	.04668	459.84	7,678.98	7,449.06	58,863.63	5.9754
57	.05369	480.90	7,219.14	6,978.69	51,414.57	5.7402
58	.06127	495.80	6,738.24	6,490.34	44,435.88	5.4913
59	.06952	504.66	6,242.44	5,990.11	37,945.54	5.2272
60	.07844	506.83	5,737.78	5,484.36	31,955.43	4.9456
61	.08809	502.28	5,230.95	4,979.81	26,471.07	4.6425
62	.09847	490.57	4,728.67	4,483.38	21,491.26	4.3138
63	.10956	471.74	4,238.10	4,002.23	17,007.88	3.9501
64	.12129	446.05	3,766.36	3,543.34	13,005.65	3.5365
65	.13381	414.95	3,320.31	3,112.84	9,462.31	3.0513
66	.20006	516.03	2,905.36	2,647.34	6,349.47	2,4616
67	.33368	672.92	2,389.33	2,052.87	3,702.13	1.8358
68	.67093	925.35	1,716.41	1,253.73	1,649.26	1.1958
69	1.29825	791.06	791.06	395.53	395.53	.6491

TABLE IX.

Benefits (a), (c), (d)—Past Service.

Age.	(44) ${}^{ps}F_x^{ra} = (23) \div (20)$	(45) ${}^{ps}F_x^{ad\overline{2}\rceil} = (34) \div (20)$	(46) ${}^{ps}F_x^{r\rceil d\overline{2}\rceil} = (40) \div (20)$
20	.8906	.3221	.1679
21	.9267	.3181	.1746
22	.9619	.3144	.1811
23	.9976	.3113	.1877
24	1.0337	.3088	.1944
25	1.0697	.3065	.2010
26	1.1232	.3093	.2110
27	1.1759	.3118	.2208
28	1.2288	.3144	.2308
29	1.2817	.3168	.2408
30	1.3343	.3190	.2508
31	1.4060	.3253	.2644
32	1.4883	.3337	.2802
33	1.5717	.3417	.2962
34	1.6559	.3493	.3124
35	1.7528	.3587	.3312
36	1.8388	.3652	.3479
37	1.9250	.3708	.3648
38	2.0134	.3759	.3822
39	2.1039	.3803	.4001
40	2.1965	.3839	.4184
41	2.2914	.3866	.4372
42	2.3865	.3891	.4562
43	2.4819	.3913	.4753
44	2.5779	.3930	.4946
45	2.6750	.3942	.5141
46	2.7740	.3945	.5340
47	2.8771	.3937	.5546
48	2.9846	.3915	.5760
49	3.0961	.3878	.5981
50	3.2103	.3827	.6205
51	3.3465	.3785	.6472
52	3.4828	.3733	.6736
53	3.6212	.3676	.7002
54	3.7617	.3615	.7268
55	3.9039	.3550	.7532
56	4.0488	.3482	.7795
57	4.1979	.3407	.8060
58	4.3522	.3326	.8327
59	4.5132	.3237	.8599
60	4.6825	.3136	.8880
61	4.8617	.3021	.9174
62	5.0547	.2879	.9492
63	5.2644	.2704	.9843
64	5.4952	.2485	1.0242
65	5.7515	.2212	1.0707
66	6.0421	.1866	1.1264
67	6.3241	.1497	1.1848
68	6.5769	.1119	1.2445
69	6.7270	.0800	1.2983

TABLE X.

Age.	(47) = (26) + (37) + (43)	(48) = (19) − (31)	Age.	(47) Cont'd.	(48) Cont'd.
20	42.4608	14.1579	45	52.1063	10.9377
21	42.8058	14.1177	46	51.5249	10.5953
22	43.0437	14.0254	47	50.7773	10.2257
23	43.2486	13.9068	48	49.8565	9.8292
24	43.4046	13.7568	49	48.7619	9.4076
25	43.5005	13.5721	50	47.5006	8.9639
26	44.2275	13.5650	51	46.3777	8.5560
27	44.8293	13.5086	52	45.1201	8.1382
28	45.3464	13.4164	53	43.7880	7.7223
29	45.7748	13.2875	54	42.3965	7.3110
30	46.1109	13.1212	55	40.9454	6.9040
31	47.0046	13.0989	56	39.4385	6.5017
32	48.1182	13.1292	57	37.8559	6.1007
33	49.1180	13.1246	58	36.1853	5.6993
34	49.9962	13.0845	59	34.4132	5.2958
35	51.0949	13.0985	60	32.5206	4.8880
36	51.7112	12.9925	61	30.4776	4.4725
37	52.1813	12.8504	62	28.2560	4.0465
38	52.5567	12.6872	63	25.7942	3.6035
39	52.8319	12.5025	64	23.0000	3.1337
40	53.0058	12.2971	65	19.7408	2.6233
41	53.0782	12.0721	66	15.8220	2.0528
42	53.0262	11.8231	67	11.7039	1.4860
43	52.8505	11.5513	68	7.5475	.94179
44	52.5448	11.2560	69	4.0526	.50000

Column (47) represents the sum of ${}^{fs}F_x^{ra}$, ${}^{fs}F_x^{ad\overline{2}|}$ and ${}^{fs}F_x^{r|d\overline{2}|}$, while column (48) represents ${}^{f}F_x^{c}$ less ${}^{fs}F_x^{aw}$, or, we might say, the present value of the net contributions. (47) ÷ (48) gives us the contributions required as a percentage of salary to provide benefits (*a*), (*b*), (*c*), and (*d*), based on a pension of 1 per cent. of final salary multiplied by years of future service. (26), (37), and (43), divided in turn by (19), give us the percentage contributions required to provide benefits (*a*), (*c*), and (*d*), respectively, while the percentage contribution to provide benefit (*b*) is obtained by deducting the sum of the percentage contributions for (*a*), (*c*), and (*d*) from the total percentage contribution. Table XI shows these different percentages for ages from 15 down to 69 inclusive. The values for ages 15 to 19 have been obtained by extrapolation.

TABLE XI.

Age.	(49) Contribution required as a Percentage of Salary to provide Benefit (*a*) (pension 1% final salary × years of future service).	(50) Contribution required as a Percentage of Salary to provide Benefit (*b*).	(51) Contribution required as a Percentage of Salary to provide Benefit (*c*) (pension 1% final salary × years of future service).	(52) Contribution required as a Percentage of Salary to provide Benefit (*d*) (pension 1% final salary × years of future service).	(53) Contribution required as a Percentage of Salary to provide Benefits (*a*), (*b*), (*c*) and (*d*) (pension 1% final salary × years of future service).
15	1.59%	.67%	.33%	.29%	2.88%
16	1.63	.64	.33	.30	2.90
17	1.67	.61	.33	.31	2.92
18	1.71	.58	.33	.32	2.94
19	1.76	.55	.33	.33	2.97
20	1.81	.51	.34	.34	3.00
21	1.86	.48	.34	.35	3.03
22	1.92	.45	.34	.36	3.07
23	1.97	.42	.34	.38	3.11
24	2.03	.39	.35	.39	3.16
25	2.10	.36	.35	.40	3.21
26	2.16	.34	.35	.41	3.26
27	2.23	.32	.35	.42	3.32
28	2.30	.28	.36	.44	3.38
29	2.37	.26	.36	.45	3.44
30	2.44	.24	.36	.47	3.51
31	2.52	.23	.36	.48	3.59
32	2.60	.19	.37	.50	3.66
33	2.68	.18	.37	.51	3.74
34	2.76	.16	.37	.53	3.82
35	2.84	.15	.37	.54	3.90
36	2.92	.13	.37	.56	3.98
37	3.00	.12	.37	.57	4.06
38	3.08	.10	.37	.59	4.14
39	3.17	.08	.37	.61	4.23
40	3.25	.07	.37	.62	4.31
41	3.33	.06	.37	.64	4.40
42	3.41	.04	.37	.66	4.48
43	3.50	.04	.36	.68	4.58
44	3.59	.02	.36	.70	4.67
45	3.68	.01	.36	.71	4.76
46	3.77	.01	.35	.73	4.86
47	3.87	.01	.35	.74	4.97
48	3.96	.01	.34	.76	5.07
49	4.06		.34	.78	5.18
50	4.17		.33	.80	5.30
51	4.28		.32	.82	5.42
52	4.38		.32	.84	5.54
53	4.50		.31	.86	5.67
54	4.61		.31	.88	5.80
55	4.73		.30	.90	5.93
56	4.86		.29	.92	6.07
57	4.99		.28	.94	6.21
58	5.12		.27	.96	6.35
59	5.25		.26	.99	6.50
60	5.39		.25	1.01	6.65
61	5.53		.24	1.04	6.81

TABLE XI.—*Continued.*

Age.	(49) Contribution Required as a Percentage of Salary to provide Benefit (*a*) (pension 1% final salary × years of future service).	(50) Contribution required as a Percentage of Salary to provide Benefit (*b*).	(51) Contribution required as a Percentage of Salary to provide Benefit (*c*) (pension 1% final salary × years of future service).	(52) Contribution required as a Percentage of Salary to provide Benefit (*d*) (pension 1% final salary × years of future service).	(53) Contribution required as a Percentage of Salary to provide Benefits (*a*), (*b*), (*c*) and (*d*) (pension 1% final salary × years of future service).
62	5.69		.22	1.07	6.98
63	5.86		.20	1.10	7.16
64	6.03		.18	1.13	7.34
65	6.21		.16	1.16	7.53
66	6.37		.14	1.20	7.71
67	6.52		.12	1.24	7.88
68	6.64		.10	1.27	8.01
69	6.73		.08	1.30	8.11

In actual practice it would probably be sufficient to employ only one percentage for each quinquennial age group by using, say, the percentage at the middle age of the group raised to the next higher 1/10 per cent.

Liability at the Commencement of the Fund.

The percentages of contribution given above in Table XI are applicable to new employees of the company who immediately join the fund, but, if applied to old employees of the company who now or subsequently join the fund, they will provide only for benefits based on service dating from their membership in the Pension Fund Association, in other words on service dating from their commencement of contributions.

It is clear that a new entrant into the Association at, say, age 40 who has already had 20 years service with the company, if paying only the same percentage contribution as another new entrant who has just entered the service of the company, cannot expect to receive benefits based on his total length of service including the past 20 years while the other receives benefits based only on future service.

If it is desired to allow benefits based on service with the company prior to commencement of contributions then a special fund must be in some way provided. In order to show the nature and magnitude of this initial burden tables have been prepared which enable us to determine for each individual entrant the average

initial liability for benefits based on past service. On the assumption that a number of employees equal to the whole operating force joins the Association we shall be able to determine the total initial liability and the percentage of the same to the payroll. It must be borne in mind that we are not assuming that the whole operating force joins the Association but that a number of employees from amongst the various departments of the company in proportion to the numbers in such departments and equal (simply for convenience sake) to the number in the operating department joins the fund.

The first step is to determine the number of employees at each exact age and the length of past service at each such age. This has been done by determining the total number of employees and the total years of past service respectively under ages 20, 30, 40, 50, 60, and 70 in turn. The following table shows this calculation.

Present Age.	(m) Number of Employees.	(n) Average Years of Service.	(o) Product = Total Years of Service.	Present Age.	(p) Accumulation of (*m*).	(q) Accumulation of (*o*).
Under 20..	765	1.12	857	Under 20..	765	857
20 to 29..	5,721	2.84	16,247	" 30..	6,486	17,104
30 to 39..	5,126	6.95	35,626	" 40..	11,612	52,730
40 to 49..	3,223	12.40	39,965	" 50..	14,835	92,695
50 to 59..	1,622	18.29	29,667	" 60..	16,457	122,362
60 and over	677	28.77	19,477	" 70..	17,134	141,839
	17,134					

Intermediate values for (*p*) and (*q*) were then obtained by interpolation using Mr. Henderson's formula. In the process of interpolation the values in each case were taken as zero at age 15 but the resulting values between ages 15 and 20 were discarded and new values obtained by graphical means. The resulting values are given in Table XII together with their first differences, the latter being the values which we set out to obtain. The original decennial values are shown in italics.

TABLE XII.

Present Age.	(54) Accumulated Number of Employees under Age stated.	(55) Number of Employees between Ages x and $x+1$.	(56) Accumulated Length of Service under, the Age stated.	(57) Length of Past Service between Ages x and $x+1$.
15	*0*	20	*0*	20
16	20	71	20	52
17	91	143	72	108
18	234	223	180	200
19	457	308	380	477
20	*765*	398	*857*	718
21	1,163	459	1,575	909
22	1,622	512	2,484	1,104
23	2,134	557	3,588	1,303
24	2,691	592	4,891	1,506
25	3,283	625	6,397	1,695
26	3,908	643	8,092	1,910
27	4,551	650	10,002	2,133
28	5,201	648	12,135	2,365
29	5,849	637	14,500	2,604
30	*6,486*	607	*17,104*	2,886
31	7,093	586	19,990	3,106
32	7,679	564	23,096	3,298
33	8,243	542	26,394	3,464
34	8,785	521	29,858	3,602
35	9,306	502	33,460	3,687
36	9,808	481	37,147	3,784
37	10,289	460	40,931	3,867
38	10,749	441	44,798	3,938
39	11,190	422	48,736	3,994
40	*11,612*	405	*52,730*	4,038
41	12,017	387	56,768	4,068
42	12,404	368	60,836	4,084
43	12,772	350	64,920	4,088
44	13,122	331	69,008	4,077
45	13,453	312	73,085	4,082
46	13,765	294	77,167	4,030
47	14,059	276	81,197	3,950
48	14,335	259	85,147	3,842
49	14,594	241	88,989	3,706
50	*14,835*	225	*92,695*	3,497
51	15,060	208	96,192	3,346
52	15,268	193	99,538	3,209
53	15,461	179	102,747	3,086
54	15,640	165	105,833	2,976
55	15,805	153	108,809	2,908
56	15,958	141	111,717	2,810
57	16,099	130	114,527	2,712
58	16,229	119	117,239	2,612
59	16,348	109	119,851	2,511
60	*16,457*	99	*122,362*	2,405
61	16,556	91	124,767	2,303
62	16,647	83	127,070	2,201
63	16,730	75	129,271	2,099
64	16,805	68	131,370	1,998
65	16,873	62	133,368	1,897
66	16,935	56	135,265	1,795
67	16,991	52	137,060	1,694
68	17,043	47	138,754	1,593
69	17,090	44	140,347	1,492
70	*17,134*		*141,839*	

As the data furnished by the company for any group of ages, say 30 to 40, were considered as covering those persons who had passed their thirtieth birthday, but had not yet reached their fortieth birthday, columns (55) and (57) therefore show for any particular age the number of employees or years of service for those who have passed the birthday for that particular age but have not yet attained the next birthday. In other words these columns on the average give the values of the functions at one half age older than indicated. In order to obtain the corresponding values of the functions for exact ages columns (55) and (57) were plotted on a curve and the intermediate values for exact ages were read off. These latter values are shown as columns (59) and (61) below.

TABLE XIII.

Age Nearest Birthday at Entry into Fund.	(58) Initial Liability to Cover Past Service per Person Admitted at the Age Stated, Yearly Salary at Entry 1 and Past Service 1 Year, Pension Based on Full Final Salary × Years of Service = (44) + (45) + (46).	(59) Number of Employees at each Age Stated.	(60) Relative Yearly Salary Scale, see (5).	(61) Total Years of Past Service for all Employees Assumed Admitted to Fund at Age Stated = (59) × Average Past Service.	(62) Initial Liability due to all Persons Admitted at each Age Stated Based on Assumed Salaries at that Time = (58) × (60) × (61) ÷ 100 (Pension Based on 1 % Final Salary × Years of Service).
15	1.1906	0	1.00	0	.0
16	1.2286	44	1.16	38	.54
17	1.2666	100	1.32	80	1.34
18	1.3046	182	1.48	148	2.86
19	1.3426	264	1.64	324	7.13
20	1.3806	350	1.80	570	14.16
21	1.4194	430	1.92	800	21.80
22	1.4574	480	2.04	1,016	30.21
23	1.4966	532	2.16	1,202	38.86
24	1.5369	573	2.28	1,392	48.78
25	1.5772	608	2.40	1,586	60.03
26	1.6435	635	2.48	1,790	72.96
27	1.7085	647	2.56	2,010	87.91
28	1.7740	654	2.64	2,240	104.91
29	1.8393	643	2.72	2,470	123.57
30	1.9041	625	2.80	2,720	145.02
31	1.9957	595	2.84	2,980	168.90
32	2.1022	574	2.86	3,190	191.79
33	2.2096	552	2.88	3,370	214.45
34	2.3176	530	2.90	3,525	236.92
35	2.4427	513	2.90	3,640	257.85
36	2.5519	490	2.92	3,740	278.69
37	2.6606	472	2.94	3,827	299.35
38	2.7715	452	2.96	3,904	320.27
39	2.8843	432	2.98	3,964	340.71
40	2.9988	415	3.00	4,012	360.94
41	3.1152	395	3.02	4,048	380.83
42	3.2318	377	3.04	4,080	400.85
43	3.3485	358	3.06	4,090	419.08
44	3.4655	340	3.08	4,088	436.34
45	3.5833	320	3.10	4,080	453.22
46	3.7025	302	3.12	4,064	469.47
47	3.8254	284	3.14	4,002	480.71
48	3.9521	267	3.16	3,900	487.06
49	4.0820	250	3.18	3,784	491.19
50	4.2135	232	3.20	3,617	487.69
51	4.3722	215	3.20	3,420	478.49
52	4.5297	200	3.20	3,265	473.26
53	4.6890	185	3.20	3,148	472.35
54	4.8500	170	3.20	3,030	470.26
55	5.0121	156	3.20	2,942	471.86
56	5.1765	145	3.20	2,855	472.93
57	5.3446	135	3.20	2,758	471.69

TABLE XIII.—*Continued.*

Age Nearest Birthday at Entry into Fund.	(58) Initial Liability to Cover Past Service per Person Admitted at the Age Stated, Yearly Salary at Entry 1 and Past Service 1 Year, Pension Based on Full Final Salary × Years of Service = (44) + (45) + (46).	(59) Number of Employees at each Age Stated.	(60) Relative Yearly Salary Scale, see (5).	(61) Total Years of Past Service for all Employees Assumed Admitted to Fund at Age Stated = (59) × Average Past Service.	(62) Initial Liability due to all Persons Admitted at each Age Stated Based on Assumed Salaries at that Time = (58) × (60) × (61) ÷ 100 (Pension Based on 1% Final Salary × Years of Service).
58	5.5175	124	3.20	2,660	469.65
59	5.6968	114	3.20	2,564	467.41
60	5.8841	104	3.20	2,460	463.20
61	6.0812	94	3.20	2,358	458.86
62	6.2918	87	3.20	2,252	453.41
63	6.5191	79	3.20	2,150	448.51
64	6.7679	72	3.20	2.044	442.67
65	7.0434	64	3.20	1,943	437.93
66	7.3551	59	3.20	1,845	434.25
67	7.6586	53	3.20	1,742	426.92
68	7.9333	48	3.20	1,646	417.86
69	8.1053	45	3.20	1,541	399.69
		17,066			16,567.59

There are two reasons for the difference between 17,066 and 17,134, this latter figure representing the total of column (55). In reading off column (59) from the graph a certain number of employees came under age 70, namely those whose nearest age at entry was 70. As we are principally interested in getting the percentage of liability to the payroll the few cases at age 70 have not been included. Of course, in actual practice the liability arising from such cases should be taken account of, but this might be done by considering them as pensioners. In addition part of the difference mentioned above arose from the difficulties of drawing the graph and of reading off the values correctly to the last figure.

TABLE XIV.

Age Nearest Birthday at Entry into Fund.	(63) Total Assumed Yearly Pay-roll at Age stated = (59) × (60)	(64) Initial Liability at each Age as a Percentage of the Salary at the Time of Admission to Fund = (62) ÷ (63) × 100 (pension based on 1 % Final salary × years of service).	(65) Future Extra Percentage Contribution per Year of Past Service in Addition to Column (53) to be paid by Present Employees entering Fund in order to receive a Pension based also on Past Service (pension based on 1 % Final salary × years of Service = (58) ÷ (48).	(66) Total Contributions at each Age to cover Future Service Benefits = (53) × (63).	(67) Amount of Money Necessary to be Furnished the First Year by each Age Group to cover Benefits based on Past Service alone using Percentages in (65) = (65) × Average Past Service = (65) × (60) × (61).
15	0	.0%	.08%	.0	.0
16	51.04	1.06	.09	1.48	.04
17	132.00	1.02	.09	3.85	.10
18	269.36	1.06	.09	7.92	.20
19	432.96	1.65	.09	12.86	.48
20	630.00	2.25	.10	18.90	1.03
21	825.60	2.64	.10	25.02	1.54
22	979.20	3.09	.10	30.06	2.07
23	1,149.12	3.38	.11	35.74	2.86
24	1,306.44	3.73	.11	41.28	3.49
25	1,459.20	4.11	.12	46.84	4.57
26	1,574.80	4.63	.12	51.34	5.33
27	1,656.32	5.31	.13	54.99	6.69
28	1,726.56	6.08	.13	58.36	7.69
29	1,748.96	7.07	.14	60.16	9.41
30	1,750.00	8.29	.15	61.43	11.42
31	1,689.80	10.00	.15	60.66	12.69
32	1,641.64	11.68	.16	60.08	14.60
33	1,589.76	13.49	.17	59.46	16.50
34	1,537.00	15.41	.18	58.71	18.40
35	1,487.70	17.33	.19	58.02	20.06
36	1,430.80	19.48	.20	56.95	21.84
37	1,387.68	21.57	.21	56.34	23.63
38	1,337.92	23.94	.22	55.39	25.42
39	1,287.36	26.47	.23	54.46	27.17
40	1,245.00	28.99	.24	53.66	28.89
41	1,192.90	31.92	.26	52.49	31.78
42	1,146.08	34.98	.27	51.34	33.49
43	1,095.48	38.26	.29	50.17	36.29
44	1,047.20	41.67	.31	48.90	39.03
45	992.00	45.69	.33	47.22	41.74
46	942.24	49.82	.35	45.79	44.38
47	891.76	53.91	.37	44.32	46.50
48	843.72	57.73	.40	42.78	49.30
49	795.00	61.78	.43	41.18	51.74
50	742.40	65.69	.47	39.35	54.40
51	688.00	69.55	.51	37.29	55.81
52	640.00	73.95	.56	35.46	58.51
53	592.00	79.79	.61	33.57	61.45
54	544.00	86.44	.66	31.55	63.99

TABLE XIV.—*Continued.*

Age nearest birthday at Entry into Fund.	(63) Total Assumed Yearly Pay-roll at Age stated = (59) × (60).	(64) Initial Liability at each Age as a Percentage of the Salary at the Time of Admission to Fund = (62) ÷ (63) × 100 (pension based on 1% final salary × years of service).	(65) Future Extra Percentage Contribution per Year of Past Service in Addition to Column (53) to be paid by Present Employees entering Fund in order to receive a Pension based also on Past Service (pension based on 1% final salary × years of service) = (58) ÷ (48).	(66) Total Contributions at each Age to cover Future Service Benefits = (53) × (63).	(67) Amount of Money Necessary to be Furnished the First Year by each Age Group to cover Benefits based on Past Service alone using Percentages in (65) = (65) × Average Past Service = (65) × (60) × (61).
55	499.20	94.52	.73	29.60	68.73
56	464.00	101.92	.80	28.16	73.09
57	432.00	109.19	.88	26.83	77.67
58	396.80	118.36	.97	25.20	82.57
59	364.80	128.13	1.08	23.71	88.61
60	332.80	139.18	1.20	22.13	94.46
61	300.80	152.55	1.36	20.48	102.62
62	278.40	162.86	1.55	19.43	111.70
63	252.80	177.42	1.81	18.10	124.53
64	230.40	192.13	2.16	16.91	141.28
65	204.80	213.83	2.68	15.42	166.63
66	188.80	230.01	3.58	14.56	211.36
67	169.60	251.72	5.15	13.36	287.08
68	153.60	272.04	8.42	12.30	443.50
69	144.00	277.56	16.21	11.68	799.35
	46,891.80	35.33% of total payroll		1,983.24 or 4.23% of total payroll	3,807.71 or 8.12% of total payroll

It is seen from column (62) that, assuming 17,134 persons of whom 17,066 are under 70 at nearest birthday immediately join the fund, 16,567.59 is the initial liability on the basis of 1 per cent. pension, this being 35.33 per cent. of the total payroll. If the pension guaranteed were based on 2 per cent. of final salary multiplied by years of service (including past service) then the liability at the outset to cover past service would be twice the above amount.

Column (65) shows how this initial burden could be carried by the members themselves. As an example, suppose that immediately upon starting the fund two employees become members, *A* being a new employee at age nearest birthday 30, yearly salary $800, past service none; *B* an old employee at age 30, yearly salary $800, and 10 years past service. Suppose that *A* and *B* are both to receive a pension based on 2 per cent. of final salary multiplied by length

of future service but that *B* is in addition to receive a pension based on 1 per cent. of final salary multiplied by length of service prior to joining the fund. *A*'s contribution during active service with the company will be only 3.51 per cent. $\times$ 2 = 7.02 per cent. of his salary each year, viz., $56.16 while his salary remains at $800. If *A* becomes permanently disabled and is pensioned at age 60, final salary $1,000, he will receive a pension for life of 30 $\times$ 2 per cent. $\times$ 1,000 = $600. On the other hand *B* will pay the newcomer's contribution of $56.16 a year plus .15 per cent. $\times$ 10 $\times$ 800 = $12.00 extra contribution making a total of $68.16 so long as his salary remains at $800. If he is pensioned on account of disablement at age 60, final salary $1,000, he will receive a pension for life of 30 $\times$ 2 per cent. $\times$ 1,000 = $600 plus an extra pension of 10 $\times$ 1 per cent. $\times$ 1,000 = $100, making in all $700 a year.

It should be noticed, however, that the extra contribution jumps up very rapidly as the age at entry becomes high so that the old employees who had been with the company a long time before the fund started would be unable to pay this extra based on 1 per cent. of final salary. If the company itself did not care to assume any of this charge, it would be practically necessary to reduce the percentage benefit at the older ages and durations so that the employee could himself pay the contributions. If the company were to pay a portion of these extra contributions, it would be possible to reduce the same somewhat as the employee would not in such event receive back also the company's contributions upon his withdrawal.

From column (66) we see that if all of the employees joined the fund the total yearly contribution would be 4.23 per cent. of the payroll, to provide a pension based on 1 per cent. of final salary $\times$ future length of service or 8.46 per cent. to provide a pension based on 2 per cent. etc.

Data to Be Kept and Periodical Valuation Made.—In establishing a fund of this nature certain assumptions, as stated before, must be made as to interest, mortality, disability and withdrawal rates. As these rates may not be actually experienced in practice it is most advisable that about once every five years an Actuary be consulted and a valuation be made of the liabilities to determine whether the fund is solvent or not and whether the contributions should be adjusted. In order to be able to make a reasonably accurate valuation records should be kept showing the following information for each entrant into the fund:

Date of birth;
Date of entry into fund;
Date of first employment with the company and total period of such employment to date;
Initial salary;
Changes in salary with dates of same;
Date and cause of death and payments to beneficiary;
Date and cause of retirement and amount of pension paid;
Date of withdrawal and amount of contributions returned on withdrawal.

Expenses.—No provision has been made in the above contributions for the expenses which will be entailed yearly in carrying on the work of the fund. The chief expense will be due to clerical work, stationery, etc., and expense incident to investing the funds. These might be covered by making a small uniform charge each year on each member, the total being sufficient to cover the expenses of the current year. The expenses may be expected to increase somewhat as years go by on account of the number of pensioners increasing. Another method would be to increase the contributions by an amount which would remain constant from year to year. In this case the total charge for expenses should be somewhat higher than needed during the early years of the fund so that this excess might be accumulated for the higher expenses of future years.

Before concluding there are several things to which attention should be drawn. They are as follows:

On the basis of the calculations and tables given in this paper the age of each new member who joins the fund should be determined for the purposes of fixing his contribution as the age at nearest birthday at entry into the fund.

Owing to membership in the fund being voluntary, care should be taken to guard against the admission of persons practically disabled at time of entry.

It would be well in actual practice to make a rule to the effect that no pension greater than a certain maximum percentage, say 80 per cent. of the final salary should be allowed, and a rule limiting the maximum amount of salary upon which contributions shall be paid or pensions based.

One of the objects of interpolating on columns (p) and (q) instead of on (m) and (n) was that the interpolated figures might be

consistent with each other, or, in other words, that the average years of past service as calculated from the interpolated values might correspond closely with the average length of service as given by the company. As a matter of fact the method adopted of necessity exactly reproduces the average years of service for the various age groups as given in (n).

This same method might have been applied to produce the salary scale by multiplying the number of employees in the various age groups by the average salaries as furnished by the company for the corresponding age groups and by then interpolating on these products. The first difference of the interpolated series would be the total salaries for each respective age. A test was made to determine how closely the salary scale as used conformed with the scale for certain age groups as furnished by the company. The following table exhibits the results of this test.

At Exact Ages.	(r) Group Totals taken from Column (59).	(s) Sum of Products of (5) × (59).	At Exact Ages.	(t) See Column (59).	(u) Product of (5) × (59).	(v) $(r) + \frac{1}{2}(t)_x - \frac{1}{2}(t)_{x-10}$.	(w) $(s) + \frac{1}{2}(u)_x - \frac{1}{2}(u)_{x-10}$.
15 to 19 inc.	590	885.36	20	350	630.00	765	1,200.36
20 " 29 "	5,552	13,056.20	30	625	1,750.00	5,689	13,616.20
30 " 39 "	5,235	15,139.70	40	415	1,245.00	5,131	14,887.20
40 " 49 "	3,308	10,191.34	50	232	742.40	3,216	9,940.04
50 " 59 "	1,676	5,363.20	60	104	332.80	1,612	5,158.40
60 " 69 "	705	2,256.00	70	0	0	653	2,089.60

	(x) Average Salary for age groups = (w) ÷ (v).	(y) Relative average yearly earnings (see data).
Under age 20.....	1.5691	1.5691
20 to 29.....	2.3934	2.4551
30 to 39.....	2.9014	3.0076
40 to 49.....	3.0908	3.1733
50 to 59.....	3.2000	3.3002
60 and over.....	3.2000	2.8879

It may happen that the figure in the last place, as shown in some of the tables given on the previous pages, may appear to be incorrect. A difference of one unit or in some cases possibly two units may arise from the fact that the original tables were made up on the basis of the actual salary scale whereas the figures given throughout this paper have been made up on the basis of a relative salary scale. In changing the tables to conform to the relative scale

the figures were simply divided by a common divisor and were not entirely recalculated. It is believed that the perecentage contributions and other percentages have not been changed at all on this account.

In the practical adoption of a pension scheme with death and withdrawal benefits similar to those just described the rules of the scheme should be explicit as to the application of the withdrawal benefit, and the benefit at death while in active service, otherwise the number of withdrawals might be larger than anticipated and the deaths while in active service smaller than expected. The object, of course in granting only small death benefits was that the pension might be as large as possible without making the contributions unreasonably high. A rule to the effect that two weeks or a month must elapse after actual withdrawal before any withdrawal benefit shall be paid, the death benefit applying if death should occur during that time, would probably cover the point in case the company were not willing to continue the employee's salary for such a length of time after illness might occur as to make the salary paid during illness together with the death benefit at least equal to the withdrawal benefit. If the company should be willing to continue the salary for such a time as suggested there should be a stipulation to the effect that if the employee withdrew or were dismissed after receiving his salary for a certain length of time after illness occurred then such salary paid should be considered as a part of the withdrawal benefit.

Legal Notes.

BY

WENDELL M. STRONG.

Mutual Benefit Insurance, Increase of Assessments:—(Green *vs.* Royal Arcanum, Supreme Court of U. S., not yet reported.) The Royal Arcanum, which is a Massachusetts order, increased its assessments a number of times; the increase in 1905 Green objected to, but paid the increased assessment under protest down to February, 1910. Then he began suit in New York, in which state was situated the council of which he was a member, assailing the validity of the increase "on the ground that it was void as exceeding the powers of the corporation and because conflicting with his contract rights as a member of the corporation and a certificate holder."

In Green's application was contained the following:—

"I agree to make punctual payment of all dues and assessments for which I may become liable, and to conform in all respects to the Laws, Rules and Usages of the Order now in force, or which may hereafter be adopted by the same."

When he was admitted, it was upon the condition

"that the said member complies, in the future, with the laws, rules and regulations now governing the said Council and Fund, or that may hereafter be enacted by the Supreme Council to govern said Council and Fund."

A similar question had been taken into court in Massachusetts, the home state of the Royal Arcanum, and there, in the higher court, it was decided that the increase was valid, impaired no contract right to the certificate holder, and could be enforced. The present case went to the United States Supreme Court on the contention that

"a violation of the full faith and credit clause of the Constitution of the United States resulted from refusing to hold that the rights of the parties were to be determined by the Massachusetts law, and to apply that law, and in further refusing to give due effect to the decree rendered in Massachusetts concerning the subject of the controversy."

The United States Supreme Court held, contrary to the decision of the Court of Appeals of New York, that the law of Massachusetts controlled, and that the increased assessment was valid. The following extract shows the basis of the decision:—

"It is not disputable that the corporation was exclusively of a fraternal and beneficiary character and that all the rights of the complainant concerning the assessment to be paid to provide for the Widows' and Orphans' Benefit Fund had their source in the constitution and by-laws and therefore their validity could be alone ascertained by a consideration of the constitution and by-laws. This being true, it necessarily follows that resort to the constitution and by-laws was essential unless it can be said that the rights in controversy were to be fixed by disregarding the source from which they arose and by putting out of view the only considerations by which their scope could be ascertained. Moreover, as the charter was a Massachusetts charter and the constitution and by-laws were a part thereof, adopted in Massachusetts, having no other sanction than the laws of that state, it follows by the same token that those laws were integrally and necessarily the criterion to be resorted to for the purpose of ascertaining the significance of the constitution and by-laws. Indeed, the accuracy of this conclusion is irresistibly manifested by considering the intrinsic relation between each and all the members concerning their duty to pay assessments and the resulting indivisible unity between them in the fund from which their rights were to be enjoyed. The contradiction in terms is apparent which would rise from holding on the one hand that there was a collective and unified standard of duty and obligation on the part of the members themselves and the corporation, and saying on the other hand that the duty of members was to be tested isolatedly and individually by resorting not to one source of authority applicable to all but by applying many divergent, variable and conflicting criteria. . . . it is certain that when reduced to their last analysis the contentions relied upon in effect destroy the rights which they are advanced to support, since an assessment which was one thing in one State and another in another, and a fund which was distributed by one rule in one State and by a different rule somewhere else, would in practical effect amount to no assessment and no substantial sum to be distributed."

The importance of this decision as affecting fraternal orders will readily be recognized as will also the chaos (hinted at in the above opinion) which might well result had the decision been the opposite.

Presumption of Death Because of Disappearance:—(Linke *vs.* Canadian Order of Foresters, Supreme Court of Ontario, Appellate Division, 33 Ontario Law Reports 159.) The claimant, the

wife of the insured, based her claim upon the fact that the insured had been absent and not heard from for seven years. She had during that time kept up the assessments upon the policy. In the town where he lived he was well known as was also his wife, and and any information received by anyone in the town about the insured would undoubtedly have been communicated to her. The trial court held that the seven years' absence was proof of the death of the insured under the circumstances, but on appeal the higher court held that further inquiry should have been made and that a new trial should be granted. The importance of this decision lies in the fact that it was held not to be sufficient to show that insured disappeared, that he was not heard from for seven years by his immediate family, and that no news of him was obtained by certain investigations they undertook, when certain avenues of probable or possible information were overlooked.

Bankruptcy, Effect of Right to Change Beneficiary:—(In *re* Hammel & Co., United States Circuit Court of Appeals, Second Circuit, 34 American Bankruptcy Reports 46.) In this case the policy provided for the right to change the beneficiary and provided for a loan value, but did not provide for a cash value except as such could be obtained indirectly by means of taking a loan. The question before the court was whether the insured could be compelled to exercise the right to change the beneficiary to his estate and obtain the loan value of the policy, such loan value to go to the trustee in bankruptcy. The court held that, under the circumstances of the case, the insured could not be compelled to take these steps, and since a loan could not be obtained without changing the beneficiary except by the beneficiary joining in the request, the policy was held to be free from the claim of the trustee.

(In *re* Jamison Bros. & Co., United States District Court, Eastern District of Pennsylvania, 34 American Bankruptcy Reports 231.) This was a petition of the trustee in bankruptcy for a ruling on an order of the referee. The court laid down two rulings: (1) "Where there has been merely a designation of a beneficiary to receive the moneys payable on the death of the insured, and this designation is open to recall or change by the insured to whom also belongs the right to cancel or surrender the policy if the insured be bankrupt, the surrender value of the policy passes to his trustee. (2) Where, however, the wife, children or a dependent relative of

the insured has been made the owner of the policy within the meaning of the Pennsylvania statutes by it having been taken out for or bona fide assigned to them, then nothing passes to the trustee."

These cases do not add greatly to our knowledge of the rights of the beneficiary in case of bankruptcy under a policy where such a policy has a surrender value. The first case indicates that the court was not willing to stretch the meaning of surrender value to include a cash value obtainable for the policy indirectly by means of loan. The second case states specifically that in certain cases, at least, where there is a right to change the beneficiary, the trustee in bankruptcy does obtain a right to a surrender value. This last, as has been stated in preceding Legal Notes, is not believed to be the correct interpretation of the statute.

Suicide under Policy Issued in Exchange for Term Policy:—(Gans *vs.* Aetna Life Insurance Co., N. Y. Court of Appeals, 214 N. Y. 326.) A term policy, after being in force several years, was exchanged, without medical examination, in accordance with a provision contained therein, for a new policy dating as of the date of exchange. This policy had a provision against suicide within one year from its date, and the insured committed suicide within that year. The company did not concede any liability, contending that according to the provisions of the policy it was void because of the suicide. The question before the court was whether the real date of the policy was the date of the exchange or the date of the original term policy, for in the former case suicide would avoid the policy and in the latter it would not. The court held that the language of the contract determined the rights of the parties and that under this language the new policy dated as of the date of the exchange and, consequently, was avoided by suicide. This is shown in the following extract from the opinion:

"The intent of the insured and the company existing April 5, 1912, as expressed, binds and obligates both of the parties. Presumptively, their intent is expressed by the natural and ordinary meaning of their language referable to it and such meaning cannot be perverted or destroyed by the courts through construction. Where the parties by their words have left no fair reason for doubt, there is no just or defensible excuse for construction. . . .

"The policy in suit is dated April 5, 1912, and in it the insured

and the defendant said: 'If the insured shall commit suicide within one year *from the date hereof,* while sane or insane, this policy shall be null and void.' This is a rigid and certain agreement from which no doubt or hesitation as to its meaning can spring. They also said: 'This policy and the application herefor constitute the entire contract between the parties hereto. . . .' It is and must be conceded that there is no language or stipulation within the policy or the application for it which conflicts with or affects the stipulations above quoted. A reference to the application in consideration of which the policy was issued is a strong confirmation of the integrity of those stipulations. It applies for a 'new contract or policy,' specifying the kind desired. The policy of April 5, 1912, is in form and subject an independent, complete and isolated contract. . . .

"The appellant asserts as a reason for such action that either of the options of the policy of April 5, 1907, which the insured did not exercise, would, if exercised, have made April 5, 1907, the date of the present policy, and, therefore, the option which the insured did exercise and which expressly made the date of the present policy that of its issuance should be, through construction, rewritten by the court, so as to provide that the date of the new should be that of the old policy. . . ."

(Silliman *vs.* International Life Ins. Co., Supreme Court of Tenn., 174 S. W. Rep. 1131.) A term policy was issued in 1910 which contained the contractual provision:

". . . on any anniversary of this policy and while in force, the company will, upon its surrender, grant in exchange therefor, without medical re-examination, a new policy on any plan then written for an amount not greater than the sum insured by this policy, and at the rate of premium required for the kind of policy chosen at the age then attained. . . ."

In 1914 this policy was exchanged, in accordance with the above provisions, for an ordinary life policy to which was attached a rider stating:

"This policy was issued in exchange for and in lieu of Tennessee Life policy No. 757 issued by The Tennessee Life Insurance Company."

The new policy provided against suicide within one year "from the date on which this insurance begins." The insured committed suicide within one year and the company contested the claim. The court held that the insurance began with the issuance of the term policy and not with the date of the exchange.

This case is not necessarily in conflict with Gans *vs.* The Aetna and the court took pains to distinguish it from that case. Emphasis was laid upon the fact that here the prohibition of suicide is "within one year from the date on which this insurance begins." This clause, viewed in the light of the exchange of the term policy for the new policy, the fact that no new policy application was made and that the policy specifically referred back to the term policy, was held to refer back the date of the beginning of the insurance to the issue of the term policy.

In Gans *vs.* The Aetna, the corresponding clause with regard to suicide in the new policy read:

"If the insured shall commit suicide within one year from the date hereof. . . ."

which is unquestionably more specific than the expression

"the date on which this insurance begins."

One of the distinctions drawn by the court seems not to have been justified. The printed report in quoting from the policy involved in the Gans case stated that it could be exchanged "upon medical re-examination." This is incorrect since it could be exchanged without medical re-examination. In the opinion the court does not mention this, but does speak of the application for the new policy certifying in regard to the physical condition of the applicant, and, at the same time, it is noted that the new policy requested was one with a disability provision. It would seem from the report that the certification of physical condition was for the sake of obtaining the disability provision, since the original contract gave the right, without such certification, to exchange for a new policy. It does not seem then, since the new application in the Gans case was simply an application for exchange and there must also have been a request for exchange made in the present case, that any distinction drawn between the two cases in this respect was of great importance.

Materiality of Representation:—(Van Woert et al. *vs.* Modern Woodmen of America, Supreme Court of North Dakota, 151 N. W. Rep. 224.) The misrepresentations entering this case were: first, that the insured had not consulted a physician within seven years, and, second, that he had not had "tumors." He afterwards died of a tumor of the brain and the medical testimony was

undisputed that he must have had such tumor at the time of applying for such insurance, but it was also undisputed that he did not know of this. He had, however, consulted physicians in regard to failing eyesight.

The court held that his statements which were not true avoided the policy, and spoke particularly of his statement that he had not consulted a physician, emphasizing in the opinion the fact that the insurance company had the right to know whether he had consulted physicians in order to give it an opportunity to investigate the nature of the complaint and to form its own opinion whether the complaint was of importance or not.

(Royal Neighbors of America *vs.* Spore, Court of Appeals of Kentucky, 169 S. W. Rep. 984.) The testimony in this case showed without contradiction that the insured had been treated by a physician in the years immediately preceding her application for insurance, whereas in the application she stated that she had not consulted any physician for seven preceding years. There was also conflicting testimony in regard to her health. The court held that the false statements in regard to consultation of a physician were material, and that, inasmuch as there was no contradiction of the evidence as to such consultation or treatment, there was nothing to submit to the jury, and that the trial court should have directed a verdict for the defendant.

Such cases are of importance because of the tendency in certain states to hold no misrepresentation material unless it can be shown that the matter misrepresented contributed to the death of the insured (which requirement has, in some states, been enacted into a statute), thus making it almost impossible for an insurance company to defend itself even in a case of actual fraud.

Illness Subsequent to Application and Prior to Delivery of Policy:—(Harris *vs.* Security Mutual Life Ins. Co., Supreme Court of Tenn., 170 S. W. Rep. 474.) One of the questions asked in the application was whether the applicant had been afflicted with renal colic, to which he answered "No." Between the date of the examination and the delivery of the policy, the applicant had an attack of renal colic which lasted several days and he was informed by the physician that his sickness was due to "renal colic." The question was whether it was the duty of the applicant to inform

the company of this illness, and whether his failure to do so avoided the policy. It is to be noted that the defence of the company here rested upon the duty of the applicant to disclose any events having a material bearing upon the question of his insurability taking place between the time he applied and before the contract was finally consummated by the delivery of the policy, rather than upon any provision of the policy that it should not take effect unless the insured was in good health at the time of its delivery. In other words, the company's case rested upon the general law of contracts, on the ground that the principles applicable thereto should be applicable to an insurance contract.

The court upheld the company in its contention, stating:

"We hold that the duty to make disclosure to the company was incumbent on him, and that failure to make same constituted a fraud which voids the policy at the option of the company."

Insanity of Insured:—(New York Life Ins. Co. *vs.* Hagler, Court of Civil Appeals of Texas, 169 S. W. Rep. 1064.) Certain policies were surrendered to the company by the insured while laboring under an insane delusion that his children were planning to murder him for his insurance. The company accepted surrender without any knowledge of the fact that the insured was not entirely sane. Two principal questions were involved. One of these was whether the fact that the insured had an insane delusion which was an inducing cause to the surrender of the policy, although he knew at the time the surrender was made the nature and effect of his act, was sufficient to invalidate the surrender; the court held that it was, thus going very far in the doctrine that insanity avoids a contract. It is easily seen how far-reaching such a doctrine may be when it is considered that a very great number of people have fixed ideas on some subject which border on delusions and which may seem delusions to others, but are yet perfectly sane in all the ordinary affairs of life. The other important question involved in the decision was the claim of the company that, even though the insured was insane, the contract of surrender, being a Missouri contract, was binding, since, under the Missouri Statute, a contract fairly made with an insane person not under guardianship is valid and will not be set aside unless the parties can be restored to their original status, and, in this case, the contract was fairly made and the parties could not be restored to their original status. The

court held that the company could be and was restored to its original status by putting the extended insurance in force which would have been in force had the contract not been surrendered. In this it seemed as if the court erred, because after the risk had gone against the company by the death of the insured, it held that the contract of surrender could be rescinded; had the result worked out the other way and had the insured not died during the term of extended insurance, there would have been no attempt to rescind the contract and it would have been allowed to stand; in other words, the company would be supposed to carry the risk under the extended insurance only in the cases in which death occurred, and for these cases would have had in its assets only the premiums payable for average risks.

It would seem that this was a case in which the company was fully justified in every sense in making the contest that it did, and should not, in any way, have been penalized for so doing, yet there was assessed against it a 12 per cent. penalty, attorneys' fees of $5,000 (about 25 per cent. of the insurance), and interest from the time of death.

Limitation of Beneficiary to "Family":—(Peterson *vs.* National Council of Knights and Ladies of Security, St. Louis Court of Appeals, 175 S. W. Rep. 284.) The insured, after his wife's death and when about fifty years of age, being alone and without relatives, went to live with friends, the Petersons, at their invitation. He lived with these friends until his death, about two years later, worked only occasionally, and paid no board. He changed the beneficiary to Mrs. Peterson, in whose house he was living, and, in so doing, designated her as a "dependent," to bring her within the required class. It was acknowledged that this designation was not correct, and the question whether the certificate could be paid to her turned on the further question whether she could be included in the other class who could be beneficiaries under such certificate, namely "family" of the insured. The court held that she could and, therefore, that the certificate was payable to her.

It is to be noted that, for a case where no blood relationship existed, the relations were unusually close between Mr. and Mrs. Peterson and the insured. They appear to have been lifelong friends, and when the insured's wife died and he was in poverty, the Petersons took him to their home and practically adopted him

as a member of the family, he not being in any sense a guest or a boarder. Consequently, the decision does not seem to extend the use of the meaning of the word "family" in a benefit certificate much, if any, beyond the usual significance of that word.

Period of Contestability under Reinstated Policy. Effect of Assignment of Policy Reinstated on False Representations:—(State Mutual Life Assurance Company *vs.* Rosenberry, Court of Civil Appeals of Texas, 175 S. W. Rep. 757.) Premiums were paid on the policy four years, a length of time extending beyond the period of contestability. At the end of the fourth year, a default was made in payment, and, some nine months later, the policy was reinstated. Immediately on reinstatement, the policy was assigned and the company received notice of the assignment. The insured died within a year after reinstatement, and it was found that representations on the basis of which the policy was reinstated were false. Two questions arose: one was whether, after reinstatement, the policy, being then past the contestable period set out in the contract, could be contested; the second was as to the rights of the assignee, who received assignment of the policy after reinstatement, without notice of the lapse and reinstatement and the possibility of defences arising from such reinstatement. In the first of these, the court decided that the reinstatement of the policy revived the period of contestability so that it would be contestable for fraud in the reinstatement within the same period from the date of reinstatement as the policy was originally contestable from its date of issue. On the other hand, the case was decided against the company on the ground that as the company had knowledge of the assignment and there was no mention made in the renewed policy of the fact that it was a renewal or of any circumstances making it subject to forfeiture the company could not contest payment on the ground of misrepresentation since the company's assent to the assignment created a new contract between it and the assignee.

Power of Insurance Commissioner:—(Phoenix Mutual Life Ins. Co. *vs.* McMaster, Supreme Court of U. S., 237 U. S. 63.) The company applied for a license for the year beginning April 1, 1912, and offered a surety bond for $20,000 to comply with the South Carolina statute requiring either an approved surety bond

or a deposit of securities "at the discretion of the Commissioner" for $20,000. The Commissioner refused to issue a license unless the company would deposit with him acceptable South Carolina securities of a certain description, or unless the company would furnish him an affidavit that at least one-fourth of its reserve on South Carolina policies was invested in South Carolina securities. The company fought the case through the state courts and lost (*T. A. S. A.*, XIV, 337.) It then carried the case to the Supreme Court of the United States on the basis of the Fourteenth Amendment upon the ground that there was discrimination between individuals of the same class and a deprivation of property without due process of law. The fact that the Mutual Benefit had been licensed was particularly mentioned in respect to the charge of discrimination. This was taken up and discussed by the court which found that different conditions existed, in that the Mutual Benefit, while not then having one-fourth of its South Carolina reserve invested in South Carolina policies, had approved real estate mortgage loans bringing these investments up to more than one-fourth the reserve. The court's decision in favor of the Commissioner thus rested on the fact that there were no acts constituting a violation of the equal protection clause of the Fourteenth Amendment. The following sentence from the opinion raises the doubt whether, in any case, whatever the discrimination, the Supreme Court would have held that the Fourteenth Amendment applied to the case of licensing a life insurance company doing business in a state not its home state:

"Assuming, without deciding, that the Phoenix Company occupied such attitude in the State of South Carolina as to entitle it to claim the benefit of the equal protection clause of the Fourteenth Amendment, we are of opinion that upon this record no such facts are shown as would lead to the conclusion that the action of the Insurance Commissioner in this case amounted to a deprivation of the equal protection of the law."

On the other hand, in several places the court speaks as if arbitrary discrimination against an insurance company as to the right to do business in a state might violate the Fourteenth Amendment, thus:

"We are only concerned with the question whether this conduct of the state authority was so arbitrary and discriminatory in its character as to amount to a deprivation of the real protection of

the laws, within the meaning of the Federal Constitution. We think the action here challenged was based upon real and substantial differences, and was not that merely arbitrary classification which this court has condemned because of the Fourteenth Amendment."

REBATE, WHAT CONSTITUTES:—(People *vs.* American Life Ins. Co., Supreme Court of Illinois, 108 N. E. Rep. 679.) This was an action brought by the state to recover statutory penalty for a rebate, the statute of Illinois being similar to the usual statutes in regard to "discrimination." The agent, in lieu of a premium of $145.75, had accepted a note for $48.50, the remainder of the premium of nearly $100 to be paid as stated in the insured's testimony as follows:

"He agreed to let me have the policy for one-third of that sum in money, and I was to work and help him around through the neighborhood in working up his business to pay off the difference. The character of work I was to do was booming up his work and driving him through the country, trying to work up prospects. I cannot say exactly how much work I did. I did a good deal of it as he was in the neighborhood several times."

The court held that there was a rebate, saying:

". . . we are convinced the arrangement made by and between Dixon and Erwin was a mere shift or device to evade the provisions of the law."

CHANGE OF BENEFICIARY:—(Rumsey *vs.* New York Life Ins. Co., Supreme Court of Colorado, 147 Pac. Rep. 336.) The policy was originally made out to Benson, Smith & Co., Limited, but reserved the right to the insured to change the beneficiary under a clause reading as follows:

"The insured, having reserved the right, may change the beneficiary, or beneficiaries, at any time during the continuance of this policy, by written notice to the company, at the home office, provided the policy is not then assigned. . . . No designation or change of beneficiary . . . shall take effect until endorsed on this policy, by the company at the home office."

There also appears on the policy the following:

"Register of Change of Beneficiary. Note.—No notice of change of beneficiary or declaration of the absolute beneficiary shall take effect until indorsed on this policy by the company at the home office."

The policy was in the possession of Benson, Smith & Co., who refused to surrender, so that the insured, while having performed the other necessary acts to change the beneficiary, was unable to furnish the policy for endorsement. While correspondence was going on in regard to the matter, the insured died and the new beneficiary whom he had attempted to appoint brought suit. The original beneficiary was not made a party and could not be because not within the jurisdiction of the court. The court held that no change of beneficiary had been made, since the endorsement required was an essential part of such change and had never been made, nor had the company been put in a position where it was able to make such endorsement, since the policy had not been furnished to it. In the course of the opinion the court discussed the possibility that the company would be compelled to pay the same policy twice if any other conclusion were reached, since in some other jurisdiction the original beneficiary might claim and recover on the ground that no change had been effected.

This case is not to be regarded as contradicting such cases as Mutual Life Insurance Company *vs.* Lowther (*T. A. S. A.*, XIV, 81), for in that case the insured had performed all the acts required of him in order to effect a change of beneficiary but died before endorsement had actually been made on the policy, whereas in this case the insured had not and was not able to perform all the acts required by the policy as conditions to the change of beneficiary.

Payment of Premiums "in Fraud of Creditors":—(Lanning *vs.* Parker, Court of Chancery, of N. J., 94 At. Rep. 64.) The New Jersey statute after providing that the beneficiary should take the proceeds of a life policy free from claims of the creditors of the insured, further provided:

"that subject to statute of limitation the amount of any premiums for said insurance paid in fraud of creditors with interest thereon shall inure to their benefit from the proceeds of the policy."

It was argued that the right of action did not accrue to the creditor until the policy became a claim and that consequently the "statute of limitation" began to run at that time.

The court, however, rejected this argument and held that the words referred to the length of time before the filing of the bill within which the premiums had been paid, so that with the "statute

of limitation" which exists in New Jersey, the recourse of the creditor was only to premiums paid within six years. A similar statute in Massachusetts has been given the opposite interpretation.

False Statement in Reinstatement of Policy:—(Thorner vs. John Hancock, Supreme Court of N. Y., Appellate Division, 149 N. Y. Supp. 345.) The policy gave the right, at any time within five years after lapse, to reinstatement "upon the production of evidence of insurability satisfactory to the Company and approved at its home office and upon payment. . . ." The insured, in requesting reinstatement, signed the following certificate:

"I certify that I am now in good health; that within one year prior to the date of this certificate I have had no injury, ailment, illness or disease, nor symptoms of such. *Neither have I consulted a physician.*"

The insured died some six months after reinstatement and it appeared that he had consulted physicians a number of times within the few months before the reinstatement. The certificate was the only evidence of insurability furnished for reinstatement; thus, the decision of the case resolved itself into one of law, as the facts were not disputed.

The court held that the company was not liable and directed judgment in its favor, thus reversing the lower court. Extracts from the opinion are:

"Two of the facts which the insured was asked to certify to were: First, whether within the past year he had any injury, ailment, illness, or disease, or symptoms of such; and, second, whether he had consulted any physician. The course of conduct of defendant with regard to further medical examination would undoubtedly be governed by the answers given thereto. Each of his statements respecting these vital facts was knowingly false. . . . He did know that he had consulted a physician. Suppose it were the fact, although the indications are entirely to the contrary, that he had then been advised by Dr. Fuhs that he was suffering from no disease, illness or ailment whatever, defendant was entitled to know all of the facts respecting his conduct, that it might govern its conduct accordingly. The materiality of a representation is largely measured by the known consequences thereof. It is impossible to attribute to the insured but a single purpose in falsely stating to defendant that he had no 'symptoms of ailment' and had not 'consulted a physician.' It was to avoid, if possible, further examinations by defendant's physicians, the consequences of which he might well dread. It is unfortunate that the insured

suffered his first annual premium to lapse, but defendant owes a duty to its other policyholders to see to it that its treasury is not depleted by the payment of fraudulent claims. In defending this action, it is but discharging that duty."

The interest of this case lies in the fact that the false statements were in an application to restore the policy rather than in the original application, and that the court apparently applied the same rules as if they had been in the original application.

(Rome Industrial Insurance Company *vs.* Eidson, Supreme Court of Georgia, 82 S. E. Rep. 641.) The policy gave the right to reinstatement "upon payment of all arrears and the presentation of evidence satisfactory to the company of the sound health of the insured." The insured offered evidence of good health to the company and paid the premium to the agent of the company.

The company received the application on August 8 and held it without action until after the date of death of the insured, September 2. The decision was against the company and was affirmed in the Supreme Court without opinion, but the syllabus would seem to indicate (1) that the burden of proof is on the plaintiff in such a case to show that the conditions of restoration were satisfactorily complied with; (2) that the responsibility of the company could not be avoided by a declination of the application after the death of the insured.

Due Date of Premium:—(Cilek *vs.* N. Y. Life Ins. Co., Supreme Court of Nebraska, 149 N. W. Rep. 49.) The application for the policy was made June 13, 1899; the application was approved June 23 and a few days later, probably on June 26, the policy was delivered. The insured died on July 23, 1906, without having paid the premium due June 13. It was contended that the policy was in force, on the ground that the original date of the policy was really June 23, 1899, or a few days later, since, according to its terms, the contract of insurance was not a binding contract upon both parties prior to that date, at the earliest.

The court upheld this contention which, inasmuch as there were thirty days grace, made the policy in force on the date of the death of the insured.

This decision is contrary to the weight of authority which is that

the date specified in the policy as the due date of premiums should stand and not be changed by the court. This is in accord with the general law of contracts that the written instrument governs. Any other doctrine in the case of insurance policies would always leave uncertainty as to what date the premium was due, and thus make great confusion. Nevertheless, in a few cases it has been held that the date the insurance actually went into force (usually through the delivery of the policy) was the date of the policy and thus determined the due date of future premiums. In these cases, without exception, the question occurred when the second premium came due, so that the present case goes far beyond any previous decision. The reasons for regarding the date specified in the policy as being the real premium due date are even stronger after the policy has been several years in force than at the the end of the first policy-year. The insurance contract is made between the insured and the company, and the date can be made whatever is agreed upon. It is conceivable that the insured may have paid no attention to the question of date when taking out the policy; when, however, the policy is kept in force for a number of years by the payment of premium, the premium notices are reminders of the due dates, so that in such a case the insured must have had repeated reminders of the date specified by the policy and by failing to object have ratified such date in which respect the situation differs from the case where the question arises at the end of the first policy-year. This fact makes it the more surprising that a court, after the death of one of the parties to the contract who himself, notwithstanding repeated reminders of the point at issue, showed that the contract was the one he intended to make by raising no question or objection, should undertake to rewrite the contract for the parties on a theory of its own.

The McMaster case cited by the court (183 U. S. 25), which was decided in the Supreme Court of the United States, is not at all parallel for the reason that in that case there was evidence that the insured understood and believed the contract was to date as of the date that the insurance went into force, the insurance to continue for a year, plus thirty days of grace, from such date, without further payment of premium. Again, the question arose by reason of the death of the insured shortly after the end of the first policy-year, so that his attention was not specifically called, by repeated premium notices, to the date entered in the policy and the

requirement of paying the premium within the time set by such notices. There was, therefore, no estoppel against McMaster.

WAIVER OF LAPSE:—(Clifton *vs.* Mutual Life Life Ins. Co., Supreme Court of North Carolina, 84 S. E. Rep. 817.) The insured became sick during the days of grace and failed to pay the premium. After the end of the days of grace, while the insured was seriously ill, the premium was sent to the company, together with the premium notice but without any letter. The company advised the insured immediately that it could not accept the money as a payment of premium until the insured furnished a certificate of good health, blank for which was forwarded. No such certificate was ever furnished. A second premium also was paid, but was also not accepted and was held in suspense. After the death of the insured, his administrator brought action to recover on the policy on the theory that the holding of the amount offered as premium after lapse amounted to a waiver of the lapse.

The court held that there was no evidence in the circumstances of a waiver of the conditions of the policy:

"The defendant had a right to receive the premium and hold it awaiting the return of the health certificate. . . . Receiving the premium under such circumstances is no evidence of a waiver."

APPROVAL OF APPLICATION AFTER DEATH OF APPLICANT:—(Erickson *vs.* Brotherhood of Locomotive Firemen and Enginemen, Supreme Court of Minnesota, 152 N. W. Rep. 537.) After the application, medical examination and initiation, but before the approval of the examination, the applicant was killed. The by-laws of the order provide that an applicant should not become a "beneficiary" member, i. e., be insured, until the medical examination was approved by the chief medical officer, and that applicants should make no payment to the "beneficiary" fund, i. e., no payment for insurance, until the beginning of the second month whether accepted or not, so that the question of payment not having been made could not enter. There was also no provision that the insurance should take effect as of the date of application.

The court held that the contract was never consummated and that consequently the order was not liable.

MIS-STATEMENT OF AGE:—(Criscuolo *vs.* Societa Monarchica di Mutuo Soccorso Vittorio Emanuele III, Supreme Court of Errors

of Connecticut, 93 At. Rep. 532.) The by-laws of the order provided that no person over fifty years of age could become a member and that any member mis-stating his age should, on discovery of the fraud, be expelled and forfeit all dues paid to the order theretofore. The insured represented himself to be forty-eight years of age at the time of becoming a member whereas he was actually fifty-two.

The court held that the insured was ineligible and could be expelled by the order. It further held that the order was not bound to return the dues paid, calling attention to the fact that the insured received sick benefits and enjoyed the advantage of membership "and continued to pay his dues with the knowledge, actual or imputed, of the by-law of the society to the effect that any member found guilty of misrepresenting his age should forfeit all dues theretofore paid."

Abstract of the Discussion of the Papers Read at the Previous Meeting.

HEALTH INSURANCE FROM A THEORETICAL AND PRACTICAL ASPECT —JAMES D. CRAIG.

VOL. XV, PAGE 271.

WRITTEN DISCUSSION.

MR. WILLIAM YOUNG.

We are indebted to Mr. Craig for bringing before the Society a very important subject,—health insurance; and giving us what is practically the pioneer paper on that subject. Health insurance is a subject that this Society has all but ignored, but it is one that will without doubt be frequently before us in the future. It is a large subject of great importance and has many difficulties, and it is one that this Society cannot continue to ignore. We ought therefore to be very grateful to Mr. Craig for his courage in breaking the ground, and for his industry in bringing before us such an array of facts and ideas for discussion. His paper shows that he has delved deeply into the literature of this great subject and given it much thought.

As I read his paper it seems to me that in writing it he had in mind a corporation granting sickness benefits to the population at large, and doing so on a strictly business basis. This thought gives the bias of the whole paper. In discussing his paper we have therefore to keep in mind that this is a project that has never yet been attempted. Putting aside the various policies of accident companies in this country and Great Britain granting sickness benefits to professional and business men, we find that the industrial classes have been provided with health insurance by the friendly societies in Great Britain and elsewhere, and more recently by the governments of Germany, Britain and other countries; but no corporation has offered health insurance to the industrial population on a purely business basis. The mass of the industrial population in the United States has no means of obtaining sickness insurance, and no corporation has offered sickness benefits to the industrial population in the way that life insurance has been offered. So Mr. Craig in his paper is in a double sense a pioneer: he is not only introducing a subject we have failed hitherto to discuss, but he is attacking a new problem in that subject.

We find, then, that at the very start he meets a grave difficulty in the fact that the Manchester Unity has never been able from the various investigations it has made to forecast with any precision its future experience from the experience of the past. The general trend of sickness rates has been to increase, whereas the general trend of mortality rates has been in the opposite direction, so that a corporation, to grant sickness benefits, would have to make some provision for the possibility of a higher sickness rate in the future than in the past. Mr. Craig suggests tables in select form, so that the sickness rate assumed at age 40 for lives now aged 30 would be more than the sickness rate assumed at age 40 for lives now 40; but the important question of how much more is not answered and is unanswerable. It seems to me that for a contingency such as this an addition to the premium is as good as the assumptions involved in forming tables in select form, and is a much more simple solution of the difficulty.

After a very interesting and instructive discussion of sickness experiences, Mr. Craig says:

"It would seem that the Manchester Unity tables can be used as a basis in this country, provided rules and regulations are adopted analogous to those in the societies from which the experience was deduced, but the premiums should be amply loaded to cover any increase in the morbidity rate which might occur."

It seems to me that the basis for the conclusion that the Manchester Unity Table can be used in this country is very slender. In the Manchester Unity each small lodge administers its own sickness claims, and the members know that any of their number malingering is doing so at their cost. Each lodge is a small group, and the natural supervision that results from the members knowing one another and looking out for their own interests is a very important safeguard. This would be absent in the case of a corporation granting sickness benefits; and, consequently, the experience of a corporation would probably be very different from the experience of a large society doing the same class of business with the lodge system. If health insurance were compulsory in the United States, the average cost would, I think, be much less than in England because of the larger percentage of the rural population in the United States, but it is questionable if a corporation would obtain its proportionate share of the rural business.

Considering the variation of the Manchester Unity experience from time to time as its investigations were made, considering the wide range of its experience at the same time in different localities and occupations, considering, further, the differences between Manchester Unity experience and that of other friendly societies in England, and the difference between the English experience and the German experience, it seems, as a guess, more likely that United States experience will differ widely from Manchester Unity experience than that it will be similar. Whether the sickness experience

of a corporation transacting business in the United States would be more or less than Manchester Unity experience is, in my opinion, a mere guess, on the basis of our present knowledge. Possibly all that Mr. Craig means when he says that Manchester Unity tables can be used as a basis in this country is that they are the best tables available. I wonder if he expects American sickness rates to be more or to be less than the Manchester Unity experience.

In the calculations which he has prepared he does not use the Manchester Unity experience for the whole society, but that part of it called the A. H. J. group. The A. H. J. group is the main group, and excludes hazardous occupations, especially those which show a high sickness rate. With the A. H. J. sickness experience he has combined the mortality experience of the whole society. Does he mean us to infer that in his opinion the A. H. J. sickness experience can be used as representing a possible American sickness experience, and that the "whole society" mortality experience can be used as representing American industrial mortality?

Perhaps the most important part of Mr. Craig's paper is the discussion of the calculation of annual premiums and reserves for a sickness benefit as it might presumably be granted by a corporation in this country. He calculates an annual premium payable during good health for a sickness benefit of $1 a week, terminating at age 65. I shall not discuss the formulas used for the premium and the reserve for this benefit. The ideas underlying the calculation are that a person applying for a sickness benefit at age 30 would not be granted the benefit if he were sick at the time of application, and that the payment of premium would cease while he was in receipt of the sickness benefit. The tables for sickness benefits as usually published are theoretically not applicable to the case. Such tables are based upon schedules of the average number of weeks' sickness per person between ages 30 and 31, 31 and 32, and so on, without regard to the age at which the sickness commences, the number of weeks' sickness in any year of age being divided up according as it is first three months' sickness, second three months' sickness, second six months' sickness, second twelve months' sickness, and after two years' sickness. This form of tabulation corresponds to the general practice of the friendly societies in England, and is the most useful form for the purposes for which the tables are designed, the principal purpose being the valuation of the liabilities of friendly societies. To calculate a theoretically accurate premium for age 30, taking account only of sickness happening after that age, would mean a complete retabulation of the original data, or the adoption of some theory such as that set forth by Mr. Craig. It is obvious that the distinction made by Mr. Craig would have some effect on the premium at the older ages, but would a corporation grant sickness benefits at ages 55 or 60? The effect of the distinction at insurable ages is negligible, and it would seem to me to be sufficiently accurate for all practical purposes to use the tables in the

form in which they are customarily published. It is not quite clear to me that the method of calculating the premium which has been adopted in the paper attains all that is claimed for it; especially I do not see how it "safeguards against any adverse selection that may be exercised against the company." I do not think any method of calculation can do that.

Mr. Craig makes the interesting remark, "as the insurance cannot be compulsory" in this country. Why can't it?

MR. B. D. FLYNN:

Mr. Craig has made a careful examination of published Health Insurance experiences and, having fixed upon that of the Manchester Unity (1893–97) as best suited to the purpose, used it as a basis for working out premiums and reserves for a contract providing benefits similar to those of the British friendly societies but suited to American business conditions. This policy contemplates a weekly benefit to the policyholder during the period of sickness even if such disability extend to age 65 or 70 years. Further, it is what might be called a "non-cancelable" contract in that it will remain in force so long as the insured continues to pay premiums. Until recently all contracts issued in this country differed in several important particulars from that just referred to; (1) indemnity was paid in case of sickness for a limited period only, generally 52 weeks; (2) the renewal of the contract was subject to the approval of the company; (3) the company could cancel the policy at any time upon the payment of the unearned portion of the premium and in this way be relieved from further insurance upon an undesirable risk. At the present time several companies are providing indemnity for life in case of sickness, and at least two companies are issuing contracts which are non-cancelable, i. e., with no right upon the part of the company to decline a renewal or to cancel the insurance. The issuance of a contract of this nature has of late been receiving the attention of company managers, so that Mr. Craig's comprehensive paper is timely and a most valuable contribution to our knowledge upon this subject.

In the preparation of premiums Mr. Craig has made the assumption that policyholders in this country will not be willing to pay premiums during incapacity—unlike the practice in friendly societies of paying dues regularly even during periods of disability. Accordingly in the calculation the number of persons paying premiums is taken as the healthy living persons at the particular age. In the writer's opinion, based upon the experience of the companies which have been writing health insurance in this country, this is not a safe assumption to make. It has been found here that the policyholder will pay the renewal premium when due even if incapacitated by sickness, provided such incapacity is not clearly of a permanent nature, rather than allow the policy to lapse. The per-

son who has illness of a nature which is probably temporary at the time of renewal is glad to pay the next premium, as he realizes then the value of his protection and will not allow his policy to lapse because of the possibility of difficulty in obtaining new insurance. These remarks apply particularly to the large volume of business written on what are known as the preferred classes—bankers, professional men, etc. The policies written upon the basis of a monthly premium covering industrial workers also require the payment of the renewal premium in order to maintain the insurance in force regardless of whether the policyholder is incapacitated or not.

In view of these considerations it would seem better to assume that the renewal premium will be paid during incapacity from illness except when the disability is practically of a total and permanent nature. My suggestion would be to use in the calculation a "force of sickness" based upon the rate of sickness of more than one year's duration. In other words, in the preparation of premiums it should be considered that all persons living who are not totally and permanently disabled by sickness (a period of disability of more than one year's duration) will pay renewal premiums when due.

The writer agrees with Mr. Craig that the increase in the rate of sickness as indicated by certain foreign experiences is probably traceable to causes such as laxity of management, poorly drawn contracts or rules, unduly liberal benefits, seasonal occupations of the insured and unemployment. The gradual education of the claimant in the practice of malingering is undoubtedly a large factor in this situation—just as it figures in the gradual increase in the cost of workmen's compensation insurance. It would seem a reasonable assumption to make that the true rate of sickness is really decreasing as a result of general education in medical and sanitary science.

The experience of the Manchester Unity (1893–97) appears to be the best available basis of rates for a "non-cancelable" contract of insurance on industrial operatives in this country. It is probable, however, that some loading of these basic rates will be necessary to provide adequate rates for a contract of this description in this country, principally for the reason that an insurance company here will not have the advantages of the "visiting committee" and other practical aids resulting from the plan of operation of a friendly society. As a basis of rates for industrial workers upon a one-year renewable term plan with right of the company to cancel, the Manchester Unity experience should be satisfactory—at least for the first year or two of business. In the early years of insurance of this character, there is what might be called a period of selection during which the knowledge of the claimant in malingering is increasing. This effect upon experience is most noticeable in workmen's compensation and constitutes a large factor in the calculation of rates for that class of business.

The scale of benefits of the Manchester Unity is particularly well adapted to the insurance of workmen in this country. The first seven days of disability are eliminated—which is a most desirable limitation in the insurance of industrial workers—and protection is provided for accidents which occur away from employment, thereby supplementing the protection given under workmen's compensation acts. The only weakness would seem to be that it provides only for total disability and makes no provision for partial indemnity during the period of convalescence when the policyholder is not confined to the house. Practically all other insurance contracts issued in this country have contained the partial indemnity provision and it is probable that such protection will be required in an American contract. It is possible that a thorough knowledge of the method by which claims in the Manchester Unity experience were paid would provide a basis for the additional charge necessary to allow this partial indemnity protection. Then again, it is possible that American health experience can be utilized to determine the extra charge necessary for this feature.

In this connection it should be stated that the experience of companies which have written health insurance in this country is of but little value in determining the cost of a "non-cancelable" policy similar to that discussed by Mr. Craig. The effect of the right of the company to decline a renewal or to cancel the insurance is undoubtedly of considerable importance and will make the American experience an unsafe guide for the cost of a "non-cancelable" contract unless a proper modification of the rates of sickness can be made to offset the effects of this underwriting privilege. The experience of The Travelers Insurance Company is now being examined and the writer hopes to present it at the next meeting of the Society. An effort will be made to trace claims which ran to the limit of the period of indemnity under health contracts and, in addition, an attempt will be made to weigh the effect of the company's privilege to discontinue insurance on undesirable risks. If this enlargement of the experience can be made with reasonable accuracy, we should have a helpful addition to our knowledge of health experience in this country as a basis for a "non-cancelable" policy.

ORAL DISCUSSION:

Mr. J. K. Gore: What I have to say on this subject is of no scientific value, but it may be of interest historically. From what has been said here this morning, I think the Prudential may have been the first corporation to issue sickness insurance policies. The company began to do business in November 1875, and issued three forms of industrial policies; one, a regular life policy, one a sickness policy, and the third a combination of the two. The sickness policy was founded as to its premiums on the old Neison table, the only table I believe that was available at that time; and perhaps

three or four thousand policies were issued during the next two or two and a half years. The company then discontinued them. There was a great deal of malingering, and the company lost money on this part of its business. At the end of last year, there were still fifty-four policies in force. The policy provided for weekly premiums averaging about twenty-five cents, payable until age 65, at which age the contract terminated.

The fact that only fifty-four policies remain in force does not mean that the rest have lapsed, or terminated by death, but that very many have attained age 65.

At the end of last year, the reserve was about $3,500, and the company was still losing on the policies at the rate of about four hundred dollars a year. Of course, it would be of no scientific value to tabulate the experience. Most of the policyholders lived in Newark, or vicinity, because the company did no business outside of New Jersey during the first three years of its history.

In response to an inquiry I would say that the title of the company at the outset was "The Prudential Friendly Society."

Mr. J. M. Craig: In regard to the statement just made, and also to get the fact into the record, I might say there was an organization transacting sick insurance called the Hildise Fund, that operated from 1869, which was a little earlier than 1875. There was life insurance connected with the premiums that the insured paid. These premiums were paid weekly and the Hildise Fund retained that part which was to be applied toward the payment of sick benefits, turning the balance over to the Metropolitan Life Insurance Company for the payment of the death claims. That was the largest organization for sick insurance transacting business at that time or for some years thereafter, but their experience was somewhat similar to that described by Mr. Gore in the Prudential, and they were compelled to reorganize, and formed a new organization called The Prudential League, which is still receiving premiums and paying sick benefits, but to a very, very limited extent.

Mr. Arthur Coburn: Mr. Flynn referred to the issuing of a noncancelable contract. I was connected with a company that issued both a permanent and a yearly contract, and our experience was quite a general one. We had a very much higher sickness rate on the contract that could not be cancelled than on the contract that you could cancel at any renewal date. Our sickness rate was so high on the permanent contract that the company in self-defense had to cease issuing it. I saw also the actual sickness experience of the company which I believe has written more permanent sickness contracts than any other British company. They had succeeded in getting comparatively low sickness rates, but in taking risks for that policy they were exceedingly careful. They obtained a full medical examination, and if there was a flaw in the case, they

turned it down. That is to say, many lives could apply to the company for life insurance policies, and they would get their policies, but if they would apply for sickness policies, they would be declined. I believe that a company that writes a permanent sickness contract can hardly be too careful.

Mr. Flynn referred to one thing in a way such that I want to make an adverse criticism of what he said. He said that the Manchester Unity Table could safely be used in this country for writing permanent sickness contracts, because that table would cover the rate of sickness in this country during what he termed the select period. Under a new form of insurance, before the public are aware of the advantages of it the rate of sickness or of accidents is very much lower than it is, say, after five or ten years. Now, if a corporation is going to write a permanent contract, it is not sufficient to fix your premium rates to cover the select period. You must try to fix your premium rates so that they will cover the ultimate sickness rate.

MR. J. D. CRAIG.

(AUTHOR'S REVIEW OF DISCUSSION.)

My main object in presenting this paper was to get the views of the Society, as well as to give a few warnings.

You will notice that the word "safely" is not included in referring to the fact that the Manchester Unity could be used. I tried to word that particular clause very carefully. If you will allow me to repeat it, it reads: "It would seem that the Manchester Unity Table can be used as a basis in this country, provided rules and regulations are adopted analogous to those in the Societies from which the experience was deduced, but the premiums should be amply loaded to cover any increase in the morbidity rate which might occur."

As Mr. Flynn has pointed out, there are two policies now being issued that are non-lapsable. While we do not know what the experience will be under those policies, it is just as well that this Society should put forth its warning. The policies that Mr. Flynn referred to, where the premiums are a dollar a month, give about the same benefit for each age? And that is another point about which I thought we ought to give a warning.

Mr. Young has said that if the suggestion followed in the rates of the A. H. J. group were used, it would require a new tabulation of the data. That is true. The A. H. J. group was used only as an illustration, and tables should be prepared for each group.

In regard to the lapsable feature, there is a point that occurs to me: that even though the policies are cancelable, nevertheless, if disability has occurred within the year, the benefits must be continued. That is, the company can only exercise the option of can-

cellation provided the insured is not in benefit at the end of the year. It seems to me that the condition is analogous to the issuance of a one-year term life insurance policy, whereby if death occurs, the amount insured is equal to the present value of an annuity to the beneficiary. If death does not occur within the year, the amount is not required. But, if it does occur, the annuity payment to the beneficiary must be continued through life, or through the definite period stated.

So, in the health policies of the companies that have the cancellation clause, if disability occur, the amount at risk is the present value of the future payments according to the terms of the contract, as long as disability, either total or partial, continues. And it was partly to bring out that phase of the question that I started on these calculations.

There is just one question I would like to ask Mr. Flynn in his experience in the Travelers, where he says the experience will probably show a decrease. Will that decrease exist at all ages, or only at certain ones? That is, will the experience show a marked decrease at certain ages, which more than offsets an increase at other ages, or will the decrease run practically at all ages?

Mr. Flynn: As I remember, there is a slight decrease all through the ages. That is, there is no particular difference in the range of ages.

MORTALITY EXPERIENCE OF THE PROVIDENT LIFE AND TRUST COMPANY OF PHILADELPHIA UNDER PAID-UP INSURANCE. —M. ALBERT LINTON.

VOL. XV, PAGE 291.

WRITTEN DISCUSSION.

MR. E. E. CAMMACK.

In the investigation presented by Mr. Linton, 99 per cent. of the policies for which reduced paid-ups were issued contain no cash surrender value for at least ten years, and no loan provision whatever. The experience, therefore, may be taken as a fair measure of the mortality prevailing amongst discontinuing policyholders in the Provident Life. The rates of mortality experienced under these paid-up policies showed no marked difference from the rates of mortality experienced under premium-paying contracts.

There has not been a selection experienced against the company by deteriorated lives ever failing to lapse their policies. In other words, some of the policyholders at the time of the lapse could not have been in good health, and this is clearly shown by the high percentages of actual to expected deaths in the first year after lapse.

An examination of the experience of the Aetna Life in its non-participating department under reduced paid-up policies confirms the conclusion drawn by Mr. Linton from the experience of the Provident Life that the vitality of policyholders who lapse is not much above the average vitality of policyholders who do not lapse.

The investigation I have made covers the experience of 10,424 paid-up policies granted for $5,338,938.00 of insurance in exchange for policies originally issued in the years 1885 to 1899, inclusive. The exposures are traced to the anniversary dates in 1914. The experience includes non-participating paid-up policies issued to discontinuing policyholders in the participating department on the surrender of their policies.

On all policies issued in 1900 and since, policies contain guaranteed cash and loan values, and on some plans prior to that date guaranteed cash values were granted, but not before the expiration of ten years. The automatic privilege in the Aetna Life policies from the beginning of 1900 has been extended insurance. Prior to that date, with some exceptions, policies, on failure of the policyholder to pay the premium, became automatically paid-up for a reduced amount.

In the table below is shown a comparison of the actual losses in

this experience with the expected by the M. A. Table by attained ages. The percentages of actual to expected are 91 per cent. by amounts and 84.6 per cent. by policies. The percentage of actual losses to expected by the M. A. Table for the company's general experience, excluding the first two years, was 93.5 per cent. by amounts.

MORTALITY EXPERIENCE ÆTNA LIFE INSURANCE COMPANY.—REDUCED PAID-UP POLICIES.

By Amounts of Insurance.

Attained Age.	Exposed.	Actual Loss.	Expected Loss M. A. Table.	Percentage.	Percentage of Actual to Expected of Company's General Experience (Excluding first two Years).
18/29	$ 1,772,070	$ 5,050	$ 8,570	58.9	87.9
30/39	10,606,860	49,850	54,780	91.0	88.8
40/49	16,203,220	121,280	120,270	100.8	98.0
50/59	12,749,330	164,420	192,730	85.3	96.5
60/69	5,405,820	159,070	188,040	84.6	92.8
70/79	1,093,380	87,560	84,150	104.1	78.5
80/89	31,650	7,500	5,070	147.9	27.5
Total . .	$47,862,330	$594,730	$653,610	91.0	93.5

By Numbers of Policies.

Attained Age.	Exposed.	Actual Loss.	Expected Loss M. A. Table.	Percentage.
18/29	6,734	19	32.32	58.8
30/39	32,609	143	168.34	84.9
40/49	40,743	264	299.45	88.2
50/59	25,413	309	377.29	81.9
60/69	9,406	265	326.31	81.2
70/79	1,694	120	128.50	93.4
80/89	48	16	9.94	161.0
Total	116,647	1,136	1,342.15	84.6

The experience of the Aetna Life upon reduced paid-up policies issued in exchange for premium-paying contracts originally issued in the years 1900 to 1913, i. e., with cash and loan values, is small. On an exposure of 9,996 years for $3,508,170.00 with actual deaths of 81 for $30,370.00, the percentages of actual to expected losses were 66.9 by amounts of insurance and 85.7 by numbers of policies. The mortality experience by amounts of insurance is much lighter than under paid-ups granted in exchange for policies without cash or loan values.

Under modern conditions, it may be contended that policyholders in poor health, unable to pay their premiums in cash, will borrow on their policies to keep them in force. There will also be a certain number of policyholders fallen into dissolute habits who, under modern contracts now issued, will surrender their policies

for cash, but who, under the old form of contract, would have allowed their policies to become automatically paid-up for reduced amounts.

The experience under paid-up policies under contracts of the present day cannot be taken as a reliable guide of the mortality amongst the general body of discontinuing policyholders.

ORAL DISCUSSION.

Mr. J. D. Craig: The principal point that struck me as I read this paper was the fact of the heavy mortality in the first year after lapse, both on paid-up insurance and on continued insurance. And in connection therewith, and possibly not strictly germane to the question, but of general interest, it occurred to me that the question of the waiving of premiums in the event of total and permanent disability would enter. Mr. Linton, I think, referred to fifteen or seventeen cases where the insurers were sick at the time of surrender, of diseases which caused their death. Whether or not they were sick in the sense that they would be considered totally and permanently disabled, I do not know. But if those cases were eliminated and included in the class where the premiums were waived, it would have a very marked effect upon the paid-up experience. Possibly it would under the continued insurance experience also. While as yet we have not sufficient data to make an interesting paper for the Society, we have some facts that I thought might interest the Society on the question of disability insurance. At the beginning of 1914, the Metropolitan made the waiver of premium option retroactive on all its regular ordinary policies, so that at the end of 1914 we had over 425,000 policies that were entitled to the benefit. During 1914, we waived the premiums on 268 cases, and so far this year on 177 cases. There was one admitted in 1913. So that we have today on our books 446 policies where disability was admitted. On these we have thus far paid 55 claims, divided as follows, 12 in the first year of disability, 18 in the second, 10 in the third, and a few in each year thereafter. If any considerable proportion of those fifty-five claims had allowed their policies to lapse and to go on continued or paid-up insurance, it would have had a marked effect on any future experience. This is such an important feature, I can surely agree with Mr. Cammack that the experience in the future, with the option included, or as an element of consideration, might naturally be very different.

MR. M. A. LINTON:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

The experience of the Aetna Life presented by Mr. Cammack furnishes an interesting analogy to the results attained under policies reduced to a paid-up basis in the Provident Life and Trust.

It also supports the conclusion that under conditions existing prior say to 1905, the lapsing policyholders possessed about average vitality and therefore did not exercise a selection against the company. I agree with Mr. Cammack's statement that the experience under paid-up policies granted under present day contracts, cannot be taken as a reliable guide of the mortality amongst the general body of discontinuing policyholders. At the same time, to repeat one of the conclusions in the original paper, the Provident experience of an abnormally high rate of mortality in the first year on the paid-up basis, indicates a tendency on the part of policyholders that should not be ignored in the discussion relating to the vitality of those who lapse their policies.

MORTALITY AND REMARRIAGE TABLES FOR VALUATION OF COMPENSATION TO WIDOWS AND OTHER DEPENDENTS—MILES M. DAWSON.

VOLUME XV, PAGE 306.

WRITTEN DISCUSSION.

MR. A. H. MOWBRAY:

Mr. Dawson's paper specifies the tables adopted for the solution of a practical compensation problem in New York, and a very interesting way in which these tables were adapted by graduation to simplify the work involved.

The mortality table has already been presented to the Society and discussed at our meetings. It should be pointed out, however, that many of the beneficiaries under this provision of the New York Act are children under the age of eighteen years whose compensation runs only to that age, therefore, for the purposes in hand the extension of the Danish Survivorship Mortality Table, as presented in *Transactions,* Vol. XI, page 451, is the more important. The graduation of this table by the Makeham formula has removed one feature which is characteristic of infantile mortality tables, the decrease in the mortality rate during the years of very early childhood. As the table is to be used for annuity computations only, this departure is on the side of safety, and the convenience in using a Makehamized table in computation work probably justifies its use.

Mr. Dawson's paper might have been more interesting to many of us if, instead of the brief statement that the Dutch Remarriage Table had been tested by comparison with certain British tables, more particulars of these tests had been given. The assistant actuary of the New York State Compensation Commission has kindly furnished me with a copy of a memorandum in which this matter among others was discussed, and the comparisons are very interesting.

The gradation of the $l_x^{(2)}$ column would probably be more easily followed by students if fuller particulars had been given of the method of obtaining the constants, although the same methods appear to have been used as are described in Mr. Jensen's paper in *Transactions,* Vol. X, page 253.

It is to be regretted that fuller particulars of the Dutch Remarriage Table, including the number of cases brought under observation, are not given. Due to the absence of these particulars the usual test of the good graduation by comparison of actual and

expectant remarriages cannot be applied. The rates themselves, giving equal weight to the value at all ages, do not meet these tests, as the differences between the graduated and ungraduated rates do not display many changes of sign, and the accumulated difference is a uniform excess of the ungraduated rate, the excess at some ages running as high as 10 per cent. of the ungraduated rate. Although the amounts are small, the annuities payable until death or remarriage, according to the graduated table, are almost uniformly in excess of the annuities according to the ungraduated table. The following table shows this comparison:

MR. DAWSON'S GRADUATION OF DUTCH REMARRIAGE RATE.

Age.	Excess Ungraduated Rate over Graduated.	Accumulated Excess.	Excess Annuity by Graduated Table over Annuity by Ungraduated.	Age.	Excess Ungraduated Rate over Graduated.	Accumulated Excess.	Excess Annuity by Graduated Table over Annuity by Ungraduated.
20	.02022	.02022	0.392	45	.00131	.05904	.001
21	.01493	.03515	.250	46	−.00027	.05877	−.021
22	.01090	.04605	.131	47	−.00156	.05721	−.016
23	.00610	.05215	.022	48	−.00153	.05568	.008
24	.00258	.05473	−.046	49	−.00117	.05451	.033
25	−.00021	.05452	−.085	50	−.00098	.05353	.054
26	−.00229	.05223	−.095	51	−.00042	.05311	.072
27	−.00367	.04856	−.079	52	−.00050	.05261	.082
28	−.00438	.04418	−.040	53	.00031	.05292	.094
29	−.00390	.04028	.015	54	.00051	.05343	.095
30	−.00290	.03738	.072	55	.00122	.05465	.093
31	−.00178	.03560	.122	56	.00164	.05629	.082
32	−.00068	.03492	.163	57	.00159	.05788	.064
33	−.00009	.03483	.191	58	.00147	.05935	.046
34	.00040	.03523	.212	59	.00129	.06064	.030
35	.0078	.03601	.226	60	.00047	.06111	.016
36	.00100	.03701	.235	61	.00028	.06139	.010
37	.00102	.03803	.239	62	.00046	.06185	.007
38	.00136	.03939	.242	63	.00010	.06195	.002
39	.00194	.04133	.239	64	.00021	.06216	.001
40	.00229	.04362	.224				
41	.00338	.04700	.202				
42	.00372	.05072	.160				
43	.00383	.05455	.108				
44	.00318	.05773	.051				

While the above table seems to indicate that the basic data have been slightly distorted in the process of graduation, it should be borne in mind that by reason of this graduation according to the Makeham formula we have obtained very valuable property of uniform seniority which is even more important in the work for

which these tables are to be used than in life insurance computations, by reason of the large number of joint-life annuities which must be calculated.

Mr. Dawson's formulae, perhaps, do not call for special comment, although it should be noted that in the second equation on page 311 and the several equations on page 313 the symbol $a_{xy}^{(2)}$ has been used in a different sense than the standard notation, although following naturally from Mr. Dawson's separation of the l_x column. A change in notation in this regard might be desirable.

MR. J. H. WOODWARD:

Mr. Dawson's paper on mortality and remarriage tables is of great value to actuaries interested in workmen's compensation. It furnishes a striking example of the debt which one of the newer forms of insurance—one which is distinctively a branch of social insurance—owes to the actuarial methods which during the past half century have been developed to meet the requirements of life insurance. Had it not been for the discovery and development of Makeham's law of mortality, the problem of valuing workmen's compensation death benefits, involving, as it does, the valuation of annuities upon a very large number of joint lives, might still remain unsolved. Had it not been, also, for the idea brought forward by the late Sir George F. Hardy in connection with the graduation of the British Female Annuitants' Table, of dividing the l_x column into two separate series, each series being subject to the Makeham law with the same value of the constant c but with different values of the other constants, we might still be searching for a solution of the particular problems discussed by Mr. Dawson. Again, in order to write an expression for the valuation of the compensation death benefit under such a statute as that of New York State, it is necessary to use an extension of the "Z" method (so familiar to actuarial students), of expressing the value of joint life annuities on exactly or at least r out of n lives. Thus does workmen's compensation insurance call for assistance upon life insurance methods.

Mr. Dawson has sufficiently described in his paper the character of the death benefit which under the compensation act it is necessary to value. He has pointed out that for the purpose of valuing this benefit it is first necessary to adopt three standards: (1) A table of mortality, (2) a table of remarriage, and (3) a rate of interest. The table of mortality adopted by the New York Commission—the Danish Survivorship Annuitants'—is well known to members of this Society and appears to me to be well suited to the purpose. The rate of interest, $3\frac{1}{2}$ per cent., might at first appear somewhat low, in view of the increase in interest rates during recent years and more especially now that a further increase seems likely to occur for some years to come, owing to the immense

destruction of capital incident to the European war. Nevertheless, it seemed to the actuarial advisers of the Commission unwise to adopt a rate of interest higher than the highest rate at which life insurance companies are, in general, permitted to value currently issued policies, meaning, as it would, that should these standards be adopted for reserve purposes, those companies issuing both life insurance and workmen's compensation insurance contracts would be carrying reserves for voluntary annuities issued to individuals upon a more secure basis than the reserves to provide annuities to the widows of the victims of industrial accidents. Again, there are inevitable uncertainties attending the adoption for the State of New York of a remarriage rate based upon foreign experience, and it seemed wise to offset these uncertainties by a reasonable conservatism in the selection of the rate of interest.

The greatest amount of interest in Mr. Dawson's paper must center in the question of the rates of remarriage assumed and the highly ingenious methods of practically employing these rates which it is the main purpose of the paper to develop. As to the suitability of the remarriage table adopted, it may be noted that there being no other remarriage experience which seemed even remotely applicable, the selection was a case of Hobson's choice. The only alternative to the adoption of the experience of the Dutch Royal Insurance Institution would have been the adoption of some table manifestly less suitable or else largely empirical in its nature. The point which it seems to me should be strongly emphasized in connection with the adoption of this remarriage table is the fact that the Dutch Royal Insurance Institution, whose experience it represents, administers a compensation act which, for the purposes in hand, is practically identical with that of the State of New York. The death benefit under the Dutch act is, as in New York, 30 per cent. of the wages of the deceased to the widow until death or remarriage, with two years' compensation in a lump sum on remarriage. This means that the economic motives underlying remarriage are, so far as they may be affected by the terms of the benefit, in both cases the same, which must, it would seem, have a strong tendency to produce similar rates of remarriage. Such difference in the remarriage rate as may be due to differences in social customs can only be ascertained by the accumulation of actual experience in this country.

The introduction of the rate of remarriage into actuarial calculations is in itself a matter which calls for comment. The rate of remarriage differs essentially from the rate of mortality, because of the fact that remarriage is an event subject to the volitional control of the individual as distinguished from death which, for actuarial purposes, may be regarded as an event whose causes are beyond the control of the individual. It is this which makes it important that the experience upon which a remarriage table is based should be influenced by economic and social considerations

of a similar kind to those affecting the lives involved in the benefit to be valued. For purposes of establishing reserves there seems no good reason to doubt the validity of the introduction of this factor, particularly when it is borne in mind that in dealing with workmen's compensation we are dealing with a branch of social insurance which is compulsory, or virtually compulsory, in character. Where, however, we have to do with calculations made for the purpose of commuting compensation benefits into a lump sum, there is room for considerable doubt as to the applicability of a remarriage rate based upon the experience of the beneficiaries as a whole. The possibilities of securing a selection adverse to the insurance carrier where commutations are freely allowed seem to be unlimited. It appears self-evident that a commutation upon the basis of the standard tables is equitable only where there is satisfactory evidence (1) of the good health of the widow at the time of commutation and (2) of the fact that there is no present intent on the part of the widow to remarry. Even assuming that there were satisfactory evidence on these points, the pronounced self-selection shown among voluntary annuitants should place us on our guard as to the effect of such commutations upon the experience. It is not impossible that, even in the absence of any well-defined intent to remarry, widows with a latent consciousness of superior possibilities of remarriage may apply in disproportionately large numbers for commutations of compensation. Again, it should be considered that these calculations are upon the basis of an aggregate table of remarriage, whereas it is not unlikely that the rates of remarriage are largely influenced by the number of years elapsed since the death of the husband as well as by the age of the widow. It may also prove true that the rate of remarriage will prove to be strongly influenced by the number of children receiving compensation. The close analogy between the problems of surrender values for life insurance contracts and the commutation to a lump sum of benefits under a compensation law will immediately occur to you. In general, it would appear to be the course of wisdom to use the greatest caution in applying calculations involving the rate of remarriage to settlements or transactions in individual cases.

On page 312 of Mr. Dawson's paper is given a table showing certain functions involving remarriage and mortality combined. The first column of this table is headed l_x^{m+s}. It may assist in the understanding of this table if it is interpreted to mean that in this column we have the total number of widows living at age x, including all those who will eventually remarry, and also including all those who will remain widows. The next column of the table, headed $l_x^{(1)}$, may be interpreted to mean the number of widows living at age x who will remain widows. The third column, $l_x^{(2)}$, may be interpreted to mean the number of widows living at age x who will eventually remarry. Thus, in the subdivision of

the l_x column into two series, which from one point of view may be regarded simply as a mathematical expedient without any physical interpretation, it also appears that it is possible to give an objective significance to these series which, at least to some minds, may render the paper more easily understood.

In connection with this paper, it seems well to direct attention to a paper on the calculation of death benefits under the New York Workmen's Compensation Law written by Mr. W. W. Greene, assistant actuary of the New York Workmen's Compensation Commission, which appears in Number 1, Volume I, of the *Proceedings of the Casualty Actuarial and Statistical Society of America.* Mr. Dawson's paper and Mr. Greene's paper, read together, constitute an almost complete exposition of the methods now employed by the New York Commission in computing the present value of death benefits under the compensation law.

MR. M. M. DAWSON:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

I am naturally much gratified by the close and careful criticism of my paper, "Mortality and Remarriage Tables for Valuation of Compensation to Widows and Other Dependents," which Messrs. Mowbray and Woodward have provided and the interest which the paper itself has elicited.

It would have been very agreeable to have supplied fuller particulars concerning the Dutch Remarriage Table, but we had merely the ungraduated table to work from and time for communicating further about the matter and obtaining this information was wanting.

The condition to which Mr. Mowbray calls attention, that the graduated table is uniformly such as to produce slightly higher annuities than the ungraduated at most of the ages was one which was planned for. It was considered that to adopt a graduation which at many of the ages or most of them showed annuities smaller than by the ungraduated table would be in effect to discard the Dutch Remarriage Table itself. Differences are not large but they are, as Mr. Mowbray has said, pretty uniformly on the same side and that the safe side.

Some slight alteration of the ungraduated curve in order to fit it to the Makeham formula was unavoidable, and the sole question was as to whether this departure should be such as to make the resulting annuities safe or to take chances in that regard. It was, and is, my judgment that the proper departure was by a very small margin to insure their entire safety as compared with annuities by the ungraduated table, and this was accomplished by modification of one of the constants used in graduation.

It is true that the symbol signifying that the second column is

that from which the value is to be taken might conceivably be confused with the symbol signifying an annuity payable semi-annually, but in view of its uniformity with other symbols in which the use of this second table is indicated in exactly the same manner, I think that it cannot create, perhaps, any confusion in practice.

It is, perhaps, desirable to note here the relationship which Mr. Olifiers of my office force and myself have had to the production of this table. The Dutch Remarriage Table was furnished me by my Holland correspondents with whom I am in touch because of personal acquaintance and mutual interest and also because of my work as Secretary of the American Section of the International Permanent Committee on Social Insurance and as Secretary General of the International Congress on Social Insurance, which would have been held in Washington this year had it not been for the war. It appearing, after comparison by Mr. Woodward, Actuary of the State Workmen's Compensation Commission, and his assistants, of this table with other remarriage tables, that it was more suitable for this purpose than any other, and it also appearing from assurances sent me from Holland that the ungraduated table was actually derived from their experience, I undertook, without asking for compensation therefor—my period of employment with the State Workmen's Compensation Commission having expired—to see whether it would not be possible so to graduate this table as to unite it with the Survivorship Annuitants' Mortality Table. The work of doing this was entrusted by me to Mr. Olifiers, with instruction to apply the method employed by the late Sir George F. Hardy in the graduation of the British Female Annuitants' Table, of superimposing a curve to represent the special departure of the table, the series representing both curves being according to the Makeham law with the same value for the constant c but with different values for other constants, so that equal age formulas could be used.

Proceeding on this basis and referring the matter to me from time to time as to the suitability of the graduation, in consequence of which references a modification of constants was made so as to produce annuity values at least equal on the whole to those by the ungraduated table, Mr. Olifiers produced the graduation which was reported in my paper, using the methods described by him in the note appended to the paper.

It seems clear that Mr. Woodward's conclusions as stated in this criticism, that this table combined with the Survivorship Annuitants' Table is entirely suitable for reserve computations and that it is only reasonably conservative to employ the same rate of interest in these computations as in computing life insurance reserves, are correct.

It is true, also, however, that as stated by Mr. Woodward, it would be undesirable to make use of the table in precisely this way perhaps for the purpose of arriving at commuted values of the

benefits, in view not only of the uncertainties in general but also and peculiarly of the moral hazard involved in the widow's knowing far better what the prospects of her remarriage are than the insurance company or state fund can possibly know.

An important addition to the literature on this subject, viz., my paper and Mr. Greene's paper published in No. 1 of Volume 1 of the *Proceedings of the Casualty Actuarial and Statistical Society of America* is a paper by Mr. Olifiers contributed to this meeting of the Actuarial Society and printed in this issue of the *Transactions.*

A TABLE OF MORTALITY ACCORDING TO HEIGHT AND WEIGHT—JAMES F. LITTLE.

VOLUME XV, PAGE 315.

WRITTEN DISCUSSION.

MR. A. A. WELCH:

Whenever a mortality experience is tabulated as the result of an extended investigation its value is rarely recognized until it is illuminated by the criticism of many minds. Even though the technical value may be appreciated we instinctively refrain from applying the new tables in correction of confessedly faulty rules of underwriting until others have tested them, criticized them, and publicly asserted their practical value.

Of all the information which the Medico-Actuarial Mortality Investigation has given to underwriters that regarding the unexpectedly high mortality existing among overweights and underweights at certain ages was the most clearly demonstrated, and yet I dare say that very few of the companies have as yet made practical use of this valuable information. We are all waiting for some one to show how the experience should be used; we are each waiting for our neighbor to point out how he has formulated new standards of underwriting from the tabulated results of the investigation. For this reason we owe a debt of gratitude to Mr. Little in that he has moulded some of the information contained in these tables into a form more adaptable for daily use.

My criticism of Mr. Little's paper will consist in accenting some of the statements it contains. Especially would I call attention to what is said on page 321, where Mr. Little, first by the table wherein the mortality by policy years is given, and secondly by a description of the manner in which the tables in this experience were formulated, has clearly demonstrated the fact that his table given on page 324 and 325 shows a more favorable mortality than may be expected from the completed experience of any similar class of overweights.

If we accept this table as a fair statement of what may be again experienced among overweights we have forced upon us the necessity for considerably greater care in the selection among applicants in this class; for if one company is known to be more free in the acceptance of overweights than another, that company is sure to get such an unusual amount of overweights at the upper limit of acceptance as will result in a perceptible raising of the general mortality of the company.

My criticism, then, of Mr. Little's paper is confined to praise for what he has done and an appeal to those who have not allowed this new experience to guide them in their rules for underwriting, that they exercise that same care in the acceptance of this class of risks as they would exercise in accepting risks in an occupation where statistics show a mortality 25 per cent., 50 per cent., or even 75 per cent. in excess of the normal.

MR. A. G. PORTCH:

Mr. Little has good reason to be satisfied with the smoothness and adequacy of his table, especially in view of the numerous difficulties he must have encountered in its preparation. His basic assumption is that the general mortality rates for men of all heights, as tabulated on page 23 of the M. A. Volume II, are applicable to the section consisting of men of medium height. This would mean that the influence on the general experience of the mortality among tall men must be equal and opposite to that of short men. Under so broad an assumption, the total deviation of —120.9 finally brought out for the medium group will not seem unduly large, even though this deviation would be increased to —142.5 if Mr. Little had not included the two extreme weight groups that were omitted in the M. A. tabulation because of lack of data.

The table gives evidence of skilful interpolation and adjustment. For example, Mr. Little employed only three weight groups in each age section, and from these three groups by interpolation deduced a series covering fifteen weight groups in each section and running smoothly with the similar series for ten age sections. This process must have been difficult. The broad grouping was doubtless adopted only because of the irregularities in the data.

From a practical point of view, Mr. Little has produced an adequate table that is much more serviceable in form than the table given in the M. A. report. This new table, if not interpreted too literally, will be of valuable assistance in office work, especially in the medical department.

Mr. Little's analysis of the influence of early policy years on the mortality percentage, is extremely interesting. It brings home to us, more sharply than ever, the truth that neither his table nor that in the M. A. report, should be used as accurately representing the probable mortality for the entire course of policies issued in the various groups. The tables may be considered as showing the mortality experience from commencement up to a fixed date, of a company continuously doing an active business. The tables do not represent the mortality in groups of policies closed to further issues.

It should also be noted that a similar statement may be applied to all the mortality ratios deduced from the M. A. investigation where the mortality is dependent to any important extent on the duration. Such tables, therefore, cannot be used directly for the

purpose of computing extra premiums or ratings. In practical work, we must consider them as indicating primarily the degree of severity required in selecting risks from certain classes, always keeping in mind the underlying features. The great danger in the widespread circulation of tables obtained from the M. A. investigation, is that the tables will be taken too literally, without regard for the qualifications so carefully and so frequently pointed out, and that companies, especially small companies, will accept sub-standard business under an impression of security that is not fully justified.

There is an interesting speculation arising in connection with Mr. Little's conclusion that mortality among overweights grows progressively worse up to age 70. The very favorable mortality among overweights entering at the young ages when compared with older entrants, is clearly shown by his table as in the preceding table. Since the bulk of the business issued in this class consisted of life policies, how can we reconcile this favorable mortality with Mr. Little's conclusion? It will be noted that his adjustment for the undue influence of early policy years does not materially change the tabular figures below age 30.

This point has been previously referred to. Dr. Rogers and Mr. Hunter in their admirable paper (*T. A. S. A.*, XV, 74) state that this favorable mortality is probably due largely to natural causes. Further down on the same page, the effect of the short average period of exposure in lowering the mortality ratios is noted, this short exposure being due to the fact that the investigation covered a period of only twenty-four years, and that during a time when the lapse rate among new issues was very high.

In view of Mr. Little's conclusion, it is interesting to consider also the effect of the incidence of mortality from tuberculosis upon the mortality rate among overweights. Two points regarding tuberculosis seem to be established in medical opinion. First, overweights are much less subject to the disease than are people of average weight or underweights. Second, the relative mortality from tuberculosis decreases as age increases. These points are substantiated by the M. A. results.

May we not, therefore, say that possibly the apparently low mortality experience among overweights entering at early ages, is favorable only in relation to the mortality caused by tuberculosis from which these young overweights are comparatively immune? This view is corroborated in a rough way by an examination of the causes of death among men (Table L, Vol. II). By excluding the deaths from tuberculosis, we increase the ratio of mortality in the overweight group as compared with the standard group from 115 per cent. to 142 per cent. for ages 15–29, from 163 per cent. to 182 per cent. for ages 30–44 and from 139 per cent. to 144 per cent. for ages over 45. The exclusion of tuberculosis causes a proportionately larger increase in the early age group than in the others. It would therefore appear probable that the favorable mortality

among young overweights is to some extent due to the opposing tendencies of the mortality from tuberculosis and the mortality from the diseases of which overweight is an indication.

This brings us to the thought that if the mortality from tuberculosis is materially reduced as time goes on, we may expect a higher relative mortality among young overweights.

Finally, we would thank Mr. Little for giving us the benefit of a work that was probably intended primarily for use in his own office. I know at least one medical department that is devoutly thankful for his success in releasing them from the bondage of a table that did not proceed by regular intervals. I know, too, that very sincere praise will be the lot of that medico-actuarial genius who will now tell us how we may best apply Mr. Little's table in practical work.

MR. P. C. H. PAPPS:

Mr. Little undertook a difficult task in preparing the table shown on pages 324 and 325 of Volume XV of the *Transactions*, and I venture to say that the difficulties can only be thoroughly appreciated by one who has attempted to, in any way, duplicate Mr. Little's work. We are told that for the middle section for men of medium height, the rates were taken from the table on page 23 of the second volume of the Medico-Actuarial Report, with some slight modifications. This was made up from the data of men of all heights combined. Whether or not the percentages derived from men of all heights properly represent the percentages for men of medium height, depends upon whether or not the mortality for men of medium height is equal to that for short and tall men combined. The relative amount of data from the short, medium and tall men entering into the preparation of the table on page 23 of the second volume of the Report also affects the question.

If we are to assume that the percentages shown by Mr. Little in Table I on pages 324 and 325 reasonably represent the mortality for the ages and weights shown, and if for the moment we assume that there is an equal amount of data for short, medium and tall men, we can then test the wisdom of assuming that the mortality for all heights is a fair measure of the mortality for the medium heights. It will be found that in a good many cases the percentage for the medium heights is nearly a mean of the percentages for the short and tall men, but in many cases this is not so. As an illustration, we may refer to age group 40 to 44, where it will be found that for all the overweights the mortality for short and medium heights is identical or nearly so, whereas the mortality for tall men is considerably greater. Furthermore, as I understand it, Mr. Little endeavored to give as close an agreement as the data would permit between the mortality percentages for the different height groups, and it may, therefore, be found that the ungraduated

data will cast further doubts on the wisdom of using the percentages for all heights as the percentages for the medium heights.

I spent some considerable time in an attempt to make an independent graduation and went back to the original data shown in Table IV of Volume II of the Medico-Actuarial Investigation. Mr. Little has described the manner in which he grouped the expected and actual deaths in three weight groups for each age group, in order to arrive at the general trend of the mortality percentages. I proceeded in a somewhat different manner. For example, I grouped the expected and actual deaths for the three age groups 25 to 29, 30 to 34 and 35 to 39, for each of the three weight groups, —25 to —30, —15 to —20 and —10 lbs. I took the resulting percentages as applicable to the center of the nine values. Since the ages used were from 25 to 39 and the weight groups from —30 to —10, I assumed that the percentage derived from the nine values represented that for central age 32 and central weight —20 lbs. In a similar manner the percentages for each age group and for the weight groups having central weights of —35, —30, —20, —12½ and —5, average, +5, +12½, +20, +30, +42½, +57½, +65 lbs. were obtained; the only variation being that on the four borders of the table groups of six instead of nine values were used.

It was not expected that the resulting percentages would be smoothly graduated, but the result of this first grouping was, of course, to give a roughly graduated set of percentages. While in many cases these roughly graduated percentages progressed with a fair degree of smoothness between the short, medium and tall men, there are many indications that the percentages for the medium height do not lie midway between those for short and tall men. In some cases there is an indication that the percentages for the short and tall are higher than for the medium heights in the same weight and age groups.

A further graduation of the roughly graduated percentages affords considerable difficulties. My first thought was to attempt to use all the roughly graduated percentages, but finally decided to graduate the weight groups —35, —12½, +5, +30 and +65 lbs.; these being the 1st, 4th, 7th, 10th and 13th groups, ranging from underweights to overweights. It will be remembered that Mr. Little used three values to obtain his final graduated figures, and it was my thought to use five values in an endeavor to make the final figures fit as nearly as possible to the unadjusted or roughly adjusted data.

The roughly adjusted percentages for each of the weight groups mentioned were first graduated graphically. It was not intended to ascertain the values for all ages by this graphic graduation, but merely five points on the curve, namely: ages 20, 30, 40, 50 and 60. The data for short men, medium heights and tall men for each weight group were set out on the same diagram in order to keep in view, if possible, the fact that there might be some con-

sistency between the percentages for the various height groups. The values for the five ages, 20, 30, 40, 50 and 60 were then used to obtain the values for all ages by means of the formula, $u_x = a + bx + cx^2 + dx^3 + ex^4$, where the constants a, b, c, d and e have to be determined from the values u_{20}, u_{30}, u_{40}, u_{50} and u_{60}. The values thus ascertained for weight groups — 35, + 5, + 30 and + 65 may be directly compared with Mr. Little's values, and to do so I have used ages 22, 27, 32, 37, 42, 47, 51½, 55, 58 and 61 as the central ages of Mr. Little's age groups. The comparison is as follows:

TABLE *A*.

5 Feet 3 Inches to 5 Feet 6 Inches.

Ages.	—35 Pounds.		+5 Pounds.		+30 Pounds.		+65 Pounds.	
	Little.	Papps.	Little.	Papps.	Little.	Papps.	Little.	Papps.
22	116	111	92	84	94	94	113	105
27	107	106	93	91	100	102	121	124
32	102	101	94	100	109	112	141	147
37	97	97	95	108	117	120	162	165
42	93	93	96	113	121	125	170	174
47	91	90	98	115	122	126	168	172
51½	89	89	100	111	120	122	163	161
55	89	90	104	104	120	118	154	150
58	92	92	107	95	121	114	145	138
61	95	96	109	83	122	109	142	128
5 Feet 7 Inches to 5 Feet 10 Inches.								
22	131	117	102	97	102	94	118	110
27	119	111	99	95	105	102	125	123
32	111	106	96	94	111	113	141	137
37	103	102	96	94	117	124	163	147
42	97	98	96	94	121	130	170	152
47	92	94	98	94	122	131	168	152
51½	88	91	99	94	120	126	163	148
55	86	88	99	93	117	121	154	145
58	86	86	98	93	116	116	145	141
61	86	84	98	91	112	112	140	139
5 Feet 11 Inches to 6 Feet 2 Inches.								
22	141	129	110	112	110	111	130	114
27	126	121	106	108	112	114	137	126
32	115	113	102	105	119	122	156	148
37	105	105	100	103	125	130	175	165
42	99	98	98	102	128	133	181	171
47	92	90	98	103	127	131	175	164
51½	86	85	98	104	125	125	169	151
55	83	81	98	103	117	120	153	141
58	81	79	97	102	113	118	142	136
61	80	78	95	99	109	118	135	139

In comparing the figures in the above table it must be borne in mind that while my figures for each weight group are smoothly

graduated, there has been nothing but a rough graduation of the percentages for each central age. The comparison of Mr. Little's percentages with mine is very interesting, as much for the close agreement which occurs in many cases, as for the differences. In graduating my percentages horizontally some of my percentages will necessarily be changed as will appear hereafter. The greatest difference between Mr. Little's percentages and mine appears to be in the weight group + 65 lbs. and it will be noticed that, according to my graduation, percentages for short and tall men, for ages 27 to 51½, are greater than for men of medium height.

I next attempted to graduate the age groups using the percentages for the weight groups — 35, — 12½, + 5, + 30 and + 65 lbs. I found that since these points were not evenly distributed over the curve, the formula I was using produced some peculiar results, particularly near each end of the curve. It seemed apparent that these results were due entirely to the mathematical formula used, and did not reflect any similar tendencies in the ungraduated data. To overcome this difficulty I resorted once more to a graphic graduation.

TABLE *B*.

5 Feet 3 Inches to 5 Feet 6 Inches.

Ages.	+ 5 pounds.		+ 30 pounds.	
	Weight.	Ages.	Weight.	Ages.
20	81	88	93	87
30	97	97	108	109
40	112	105	124	126
50	113	109	124	127
60	87	93	111	107
5 Feet 7 Inches to 5 Feet 10 Inches.				
20	98	99	94	94
30	94	97	109	106
40	94	101	128	119
50	94	98	128	119
60	92	93	113	111
5 Feet 11 Inches to 6 Feet 2 Inches.				
20	113	115	114	113
30	106	108	119	117
40	102	106	132	129
50	103	103	127	125
60	100	98	118	115

Taking each height group separately, I set out on a diagram the points for weight groups — 35, —12½, + 5, + 30 and + 65. Joining these points, I found that in many cases it was a simple matter to draw a smooth running curve fairly close to the points.

In certain cases it was a little more difficult to do so. From the smoothly drawn curves I took the percentages for weight groups —35, —10, +15, +40 and +65 lbs., retaining the original values for —35 and +65 lbs. It will be noticed that the weight groups chosen have a constant difference of 25 lbs. From the values thus ascertained, the values for each variation of 5 lbs. were ascertained by interpolation, using the same formula as was used in graduating each weight group. In this manner it was possible to obtain smoothly running values for each of the ages 20, 30, 40, 50 and 60.

It will be noticed that this second graduation gave different values for weight groups +5 and +30 lbs. from those obtained

TABLE *C*.

5 Feet 3 Inches to 5 Feet 6 Inches.

Ages.	—10 Pounds.		+15 Pounds.		+40 Pounds.	
	Little.	Papps.	Little.	Papps.	Little.	Papps.
22	97	96	92	89	98	95
27	94	96	95	96	105	108
32	93	95	97	103	118	121
37	91	95	100	110	131	131
42	89	97	104	114	138	137
47	90	99	107	117	137	138
51½	90	100	109	115	132	134
55	92	98	111	110	126	128
58	94	95	114	103	125	121
61	97	89	116	93	125	112
5 Feet 7 Inches to 5 Feet 10 Inches.						
22	107	103	98	94	104	98
27	102	100	97	96	108	107
32	98	99	97	100	119	117
37	95	98	100	105	133	126
42	92	96	104	107	140	130
47	90	92	108	107	139	130
51½	88	89	108	104	132	127
55	88	87	107	102	123	123
58	88	85	106	100	118	120
61	88	86	105	99	115	119
5 Feet 11 Inches to 6 Feet 2 Inches.						
22	118	117	108	111	114	111
27	109	111	107	109	117	116
32	104	105	105	111	130	127
37	98	100	105	113	144	137
42	93	96	106	114	148	141
47	89	94	108	113	145	138
51½	87	93	108	110	137	131
55	84	92	107	108	123	125
58	83	91	105	106	117	122
61	82	89	103	104	112	122

TABLE *D*.

5 Feet 3 Inches to 5 Feet 6 Inches.

Ages.	−35 Pounds.	−30 Pounds.	−25 Pounds.	−20 Pounds.	−15 Pounds.	−10 Pounds.	−5 Pounds.
20	113	110	107	103	100	96	93
25	108	105	102	100	98	96	94
30	103	100	98	97	95	95	95
35	98	96	94	94	94	95	97
40	94	93	92	93	94	96	99
45	91	91	92	93	96	98	101
50	89	90	92	94	97	100	103
55	90	92	93	95	96	98	100
60	94	94	93	92	91	91	91
Ages.	**0 Pounds.**	**+5 Pounds.**	**+10 Pounds.**	**+15 Pounds.**	**+20 Pounds.**	**+25 Pounds.**	**+30 Pounds.**
20	90	88	87	86	85	86	87
25	93	93	93	93	94	96	98
30	96	97	98	100	103	106	109
35	99	101	104	107	111	114	119
40	102	105	109	113	117	121	126
45	105	108	112	116	120	124	129
50	106	109	112	116	119	123	127
55	102	105	107	110	113	116	120
60	92	93	94	97	100	103	107
Ages.	**+35 Pounds.**	**+40 Pounds.**	**+45 Pounds.**	**+50 Pounds.**	**+55 Pounds.**	**+60 Pounds.**	**+65 Pounds.**
20	88	90	92	95	97	99	100
25	100	103	105	108	111	113	116
30	112	116	120	124	129	133	138
35	123	128	133	138	145	151	159
40	131	136	141	148	155	163	172
45	133	139	144	150	157	165	174
50	131	136	141	146	152	159	166
55	124	128	132	136	141	145	150
60	111	115	119	123	127	130	131

5 Feet 7 Inches to 5 Feet 10 Inches.

Ages.	−35 Pounds.	−30 Pounds.	−25 Pounds.	−20 Pounds.	−15 Pounds.	−10 Pounds.	−5 Pounds.
20	119	117	114	111	108	106	103
25	113	111	109	106	104	101	99
30	108	107	105	103	101	99	98
35	104	103	101	100	99	98	98
40	100	98	97	97	96	97	97
45	96	94	93	93	93	94	95
50	92	90	88	88	89	90	92
55	88	86	85	85	85	87	89
60	85	83	83	83	84	86	87
Ages.	**0 Pounds.**	**+5 Pounds.**	**+10 Pounds.**	**+15 Pounds.**	**+20 Pounds.**	**+25 Pounds.**	**+30 Pounds.**
20	101	99	97	96	95	94	94
25	97	95	95	95	95	96	98
30	97	97	97	98	100	103	106
35	98	99	101	103	106	109	113
40	99	101	103	106	110	114	119
45	97	100	103	107	111	116	121
50	95	98	101	105	109	114	119
55	91	94	98	102	106	110	114
60	90	93	96	99	103	107	111

TABLE *D.—Continued.*

Ages.	+35 Pounds.	+40 Pounds.	+45 Pounds.	+50 Pounds.	+55 Pounds.	+60 Pounds.	+65 Pounds.
20	95	95	97	98	100	102	105
25	100	103	106	109	112	115	118
30	109	113	117	121	125	129	132
35	118	123	128	132	137	140	143
40	124	129	134	139	143	147	151
45	126	131	136	141	145	149	153
50	123	128	133	138	142	146	150
55	119	123	128	132	137	141	145
60	115	119	123	128	132	136	140
5 Feet 11 Inches to 6 Feet 2 Inches.							
Ages.	−35 Pounds.	−30 Pounds.	−25 Pounds.	−20 Pounds.	−15 Pounds.	−10 Pounds.	−5 Pounds.
20	132	129	126	124	121	119	118
25	124	121	119	117	115	113	112
30	116	113	111	109	108	107	107
35	108	105	103	102	101	102	103
40	101	98	97	96	96	98	100
45	93	92	91	92	93	95	98
50	87	87	88	89	91	94	96
55	81	83	85	87	89	92	95
60	78	80	82	84	87	89	92
Ages.	0 Pounds.	+5 Pounds.	+10 Pounds.	+15 Pounds.	+20 Pounds.	+25 Pounds.	+30 Pounds.
20	116	115	114	114	113	113	113
25	111	110	110	110	110	110	111
30	107	108	109	110	112	114	117
35	104	106	109	112	116	120	124
40	102	106	110	114	118	123	129
45	101	105	109	114	118	123	129
50	100	103	107	111	116	120	125
55	98	101	104	108	111	115	118
60	95	98	102	105	108	111	115
Ages.	+35 Pounds.	+40 Pounds.	+45 Pounds.	+50 Pounds.	+55 Pounds.	+60 Pounds.	+65 Pounds.
20	113	113	113	114	114	115	115
25	112	113	114	116	117	118	119
30	120	123	126	129	132	136	139
35	128	133	138	143	148	154	159
40	134	140	146	152	158	164	170
45	134	140	145	151	157	163	168
50	129	134	138	143	147	152	156
55	122	125	129	132	135	138	141
60	118	121	125	128	131	134	137

by the first graduation of these weight groups. A comparison of the graduation by weights with that by ages for these two weight groups is shown in Table B, page 190.

In this Table B the percentages in the columns headed "Weights" were obtained by graduating the data vertically. The

percentages in the columns headed "Ages" were obtained by graduating horizontally. A comparison of Mr. Little's percentages with my final percentages for weight groups — 10 lbs., + 15 lbs. and + 40 lbs. is shown in Table *C*. A comparison of weight groups — 35 and + 65 is shown in Table *A*.

It is interesting to notice that in weight group + 15 lbs. for central ages 32 to 58, my percentages for short and tall men are in excess of those for men of medium height. In weight group + 40 lbs. from central ages 27 to 58, the same thing holds true. Attention has already been directed to the fact that this peculiarity is also apparent in weight group + 65 lbs. as shown in Table *A*. It seems to me that for the overweights, Mr. Little's assumption that the percentages for medium heights lie between those of short and tall men, is contrary to the facts. I am not prepared to say that this tendency would be found in all future investigations of this character, nor am I prepared to advance any reason for the peculiarity, but I do not think that the same tendency would be shown in each of the three overweight groups were it not contained in the unadjusted data.

The figures that I showed for age 61 were really outside the limit of the curve, which ranged from age 20 to 60. I found that no reliance could be placed upon the direction of the curve beyond age 60, nor on the values derived from the weight group curves for values less than — 35 or more than + 65 lbs. Mr. Little gives values from — 50 to + 90 lbs. I have not attempted to do so for the reason that the data is so scanty for some of the age groups that it did not seem to me that we had sufficient material from which to derive any reliable results. I have prepared a table of percentages for quinquennial ages 20 to 60 and for each 5 lbs. in weight from — 35 to + 65 lbs. This Table D is on pages 192–3.

MR. J. F. LITTLE:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

There is very little in the remarks of the gentlemen who have been kind enough to discuss my paper that calls for reply. I may point out, however, that the adoption of the mortality percentages of the table derived from the data for all heights combined, to represent the ratios corresponding to men of medium height, was not the result of an *a priori* assumption as Messrs. Portch and Papps have supposed. What I actually did was to find the expected deaths in the three height sections according to the general table, when it appeared that in the middle section the actual and expected deaths agreed so closely that the general table needed no modification in order to give an entirely satisfactory practical set of ratios.

With regard to the table which Mr. Papps has taken the trouble to prepare, and from which he concludes that, among overweights, men of medium height are better than either short or tall men, I wish

to point out that did this appear consistently throughout the table, very wide differences between actual and expected deaths would result. Mr. Papps gets over this difficulty by making overweight entrants at medium heights worse than short men at the very young and very old entry ages, and reversing the condition at the intervening entry ages. At 35 pounds overweight, at age 20, short men show a ratio of 88 and men of medium height one of 95. At age 25 the corresponding ratios are 100 in each case. Now, if men just selected at age 25, showing a uniform extent of overweight, also experience a uniform mortality for the remainder of life whether short or of medium height, it is evident that men of the same age, but having entered five years earlier, at age 20, must also show practically identical mortality in the two height-groups. If this be accepted, it would follow that, as men of medium height entering at age 20, in the weight-group in question, show a mortality in excess of that of short men equal to 8 per cent. of the mortality of the latter for the whole of life, and as the actual excess mortality is confined to the first five policy years, the men of medium height must be regarded as showing a mortality during these years running to something like 50 per cent. in excess of that of the short men—an entirely inadmissible conclusion.

It was because I could not believe that, in a given weight-group, the short men could run, as Mr. Papps' table would imply, from seven points better than men of medium height at age 20 to seven points worse at age 40 and again to four points better at age 60, that I decided to take a larger view, and use wide groupings. There is no distinct evidence that, over any considerable part of the table, the men of medium height were better risks than both short and tall men, and I, therefore, treated isolated instances of this condition as accidental. I do not know whether Mr. Papps' table follows more strictly the original data than mine does, but during my examination of the material I was impressed with the desirability of remembering a remark, due, I believe, to the late Sir George Hardy, to the effect that very close agreement of a graduated table with the ungraduated data might indicate an unsatisfactory graduation. The point was, as I remember it, that in a series of actual results there would be expected deviations of a certain average extent as well as a number of relatively large deviations, and if the adjusted table produce deviations decidedly less than those expected, it would be probable that close agreement with the original data was secured at the expense of failing to obtain the truest representation of the facts afforded by the data. I am inclined to think that something of this kind has happened in regard to Mr. Papps' table, in view of the very improbable progression of the ratios already referred to.

Mr. Papps is quite correct in stating that the data are too meager to supply dependable ratios for weights more than 60 or 65 pounds above the normal. If, however, the final table had stopped at this

point, in practice no better way of handling still heavier weights would be available than by means of extrapolation. This would mean trouble and delay and possibility of error in making the calculations, all of which are avoided by extending the table in the first instance. It must be remembered that these added figures, and, indeed, the whole table, are not set forth as rigidly correct, but merely as the best available approximation to the facts.

Mr. Portch doubts my conclusion as to overweights becoming progressively worse up to about age 70, because, as indicated in the adjusted experiences given on page 322, the ratios for the younger entry ages are but little affected by giving proper weight to the longer durations. For entry ages 30–34 and 35–39 there are very decided changes, while for ages 20–24 and 25–29 the changes are trifling. In this connection I pointed out that the latter results are probably accidental. Data at entry ages 20–24 are too limited to be of value, but for ages 25–29, which showed a change from 110 per cent. to 113 per cent. only, in the weight group + 50 to + 60 pounds, I find that in the preceding weight group, + 35 to + 45 pounds, the change is from 128 per cent. to 174 per cent. The latter result probably exaggerates the facts about as much as the former minimizes them.

LIFE INSURANCE BOOKKEEPING AND ANNUAL STATEMENTS—R. D. MURPHY.

VOLUME XV, PAGE 328.

WRITTEN DISCUSSION.

MR. S. E. STILWELL:

When I received Mr. Craig's invitation to prepare a criticism of Mr. Murphy's paper on "Life Insurance Bookkeeping and Annual Statements," I read the paper with care for a critical review. Further readings, however, of this meritorious production caused me to change my thoughts entirely, and, instead of presenting a criticism, I am about to offer a resolution to make some of this material a permanent possession for the insurance fraternity. There are some things in the paper that might be justly criticized, but I will leave them for others to discuss. My main purpose in writing this memorandum will be to suggest to this Society the propriety of having two text-books written under the auspices of this Society and by members of this Society, one to be called Insurance Bookkeeping and the other to be called Annual Statements of Insurance Companies.

I find myself having the same thought that Mr. Murphy had when he wrote that he could not find any modern work on bookkeeping which deals with special life insurance problems. It is a third of a century ago that the writer completed a course in Bryant and Stratton's double-entry bookkeeping. Naturally, in those days, bookkeeping dealt with mercantile accounts. Many times since then I have examined texts on bookkeeping, only to find, as Mr. Murphy says he finds, the texts still deal with industrial enterprises, with little, if any, reference to insurance matters. It is possible that books on bookkeeping exist dealing with insurance matters, but I have not seen them. It seems to me that it is time that this condition of things should be remedied.

The title of the first book proposed is taken from the first part of the title of Mr. Murphy's paper, enlarged, however, to cover all kinds of insurance. Mr. Murphy deals only with life insurance. The book might well begin with a chapter on the general principles of bookkeeping, and then a good portion of the book should be devoted to the application of these principles to all the many and varied ledger accounts of the largest life insurance companies; a smaller and final portion should be devoted to the special ledger accounts for each kind of insurance other than life, but only deal-

ing with such as do not enter the ledger accounts of a life insurance company.

The title of the other book is taken from the second part of the title of Mr. Murphy's paper, but is made general, with respect to insurance companies, and not limited to the annual statements of life insurance companies. The larger portion should be for life insurance companies, and the smaller portion for all kinds other than life.

Several years ago, a professor of economics in the State University, asked permission to bring his students in insurance economics to my office that they might see and study the annual statements of insurance companies in the form in which they are actually filed with state insurance departments. There being no co-operation between the State University and the State Insurance Department, I was compelled to refuse the request, but suggested that if he could bring his class after office hours, i. e., after five P. M., I would be glad to give him, and his class, an hour (five to six P. M.) on different days at their convenience; but it seemed impossible for him to get his class together at that hour, and the matter was dropped. In his discomfiture, he said that somebody should write a book on the subject of annual statements of insurance companies, in order that college professors might give practical instruction to their classes. He did not want blank annual statements, but statements completely filled up with actual figures. And so I would suggest that if such a text-book should be produced, it should deal with concrete illustrations, and not merely with abstract presentations.

It has seemed to me that two such text-books as I have outlined would be most desirable, and would command a very large sale.

MR. E. E. RHODES:

Mr. Murphy's paper deals almost entirely with the basis on which the annual reports required to be made by life insurance companies to state insurance departments are prepared, and a careful study of it is recommended to students and others interested in the subject.

Mr. Murphy does not refer to the Gain and Loss Exhibit, which offers more difficulties to the actuary than the statement of income and disbursements, and assets and liabilities, and which might well be made the subject of a supplementary paper.

I do not agree with Mr. Murphy's comments under Item 6 of Income, regarding the desirability of a change in the nomenclature of certain bookkeeping items. The phraseology now used shows very clearly that these items do not affect either the income or the disbursements. A good deal might be said in favor of eliminating such bookkeeping items altogether from the statement blank. The rule followed by the insurance departments whenever they have

desired information regarding any particular phase of the companies' business has been to insert an item in the financial statement which would give the desired information, regardless of whether the item affected the income or disbursements. The present form of statement has thus become encumbered with a lot of extraneous matters which, in my opinion, might much better appear in some other part of the return than in the financial statement.

In connection with Item 8 of Income Mr. Murphy speaks of "the payment of the dividend by the company and its return to the company." It has been judicially established that in the cases referred to by Mr. Murphy there is no payment of the dividend by the company, and its return to the company. This, of course, has no bearing upon Mr. Murphy's purpose, but in a review of his paper I would not like, by silence, to seem to give assent to his statement. What I have said with regard to purely bookkeeping items in connection with Item 6 of Income applies with even greater force to Item 11, "Consideration for supplementary contracts involving life contingencies," for the reason that this item is really treated now upon a "revenue" principle, whereas the financial statement as a whole is based upon a "cash" principle.

Referring to Mr. Murphy's remarks regarding Item 21 of Income, "Consideration for supplementary contracts not involving life contingencies," in which he speaks of that form of supplementary contract which provides for payments for a term of years certain and thereafter continued payments only in event of the beneficiary being alive, I would say that I do not agree with him that such a supplementary contract should be considered as a whole, and the total value entered under Item 11. To my mind it would be better to divide the total value into its two component parts and enter one under Item 11 and the other under Item 21.

From Mr. Murphy's remarks regarding Item 41 of Income, "Agents' balances previously charged off," one might infer that the item relates only to advances made to agents on the security of first year commissions. I do not think the item is so limited.

Mr. Murphy's remarks under Item 46 of Disbursements, however, make it clear that any agent's debit balance, regardless of the manner in which it arose, previously charged off as uncollectible would appear in Item 41 of Income when recovered.

What I have said with reference to Mr. Murphy's remarks under Item 21 of Income applies equally to his remarks under Item 5 of Disbursements.

With regard to Item 16 of Disbursements, I question whether it is a general practice to maintain two accounts for legal expenses.

I would like to supplement Mr. Murphy's statement of the rule that every transaction affecting the amount of the ledger assets must appear in the Income or Disbursements items, by saying that transactions which do not affect the amount of the ledger assets involve neither income nor disbursements, strictly speaking.

There is considerable variance in the method of computing the "loading" on uncollected and deferred premiums. In some cases the actual loading appears to be used. In other cases a flat percentage is used. Item 26 of Liabilities, "Cost of collection on uncollected and deferred premiums in excess of the loading thereon," must be handled consistently with Item 29 in the Assets. One way of ascertaining the "cost of collections" is to ascertain the ratio of first year's commissions to new premiums, and the ratio of renewal commissions to renewal premiums on the whole business, and apply such ratios to the uncollected and deferred premiums. If the cost of collection so ascertained is in excess of the loading deducted from the uncollected and deferred premiums, the difference should go in Item 26 of Liabilities.

MR. J. H. NITCHIE:

There are several items in Mr. Murphy's paper in regard to which, while I agree with him, yet I question if there is uniformity in the practice of companies or in the requirements of state departments. I think that it might be well to bring such matters to the attention of department officials or of the Committee on Blanks, so as to secure such uniformity and specific instructions, perhaps to go with the statement blanks upon items of doubtful interpretation.

For instance, on page 330, referring to Item No. 9, Mr. Murphy would exclude paid-up insurance granted as surrender values. I approve his suggestion, indeed I favored such elimination at the period when padding was common, yet I have lately interpreted this item as a requirement that such surrender values be reported as income and disbursement.

On page 348, in regard to Item No. 41, his conclusion would probably not be questioned if valuation tables in the earlier form were still in use, with mid-monthly interpolations closely approximating true values. Mid-year values were later adopted as being more convenient, but only because in the aggregate they approximate the aggregates of true values.

But this does not justify individual comparisons which result in disallowing well secured assets. I have in the past urged state officials to accept true values as the standard of security, but have found them insisting on measurement by mean reserves.

MR. R. D. MURPHY:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

Mr. Rhodes disapproves of certain suggestions I made for a possible change of title in items appearing in both Income and Disbursements. My object in suggesting such changes was merely to point out the distinct credit and debit character of the opposing items. I agree with Mr. Rhodes that even then the difficulty would

not be satisfactorily adjusted, and, as he points out, the bookkeeping would belie the law. In my opinion, however, the best alternative is not the exclusion of such items from the principal financial statement, but rather a change of that statement to the revenue basis. I would question the wisdom of relegating to a supplementary exhibit such important items as those affecting the amount of dividends apportioned and the death claims incurred during the year.

Mr. Rhodes prefers to split the value of a supplementary contract representing an annuity for a term of years certain, and thereafter for life, into its two component parts. At least one insurance department has ruled against such splitting, and I think that ruling is a very practicable and reasonable one.

I wish to correct any impression I gave in discussing Item 41 of Income that advances to agents included therein are limited to those granted against first year's commissions. I did not intend so to limit it, and I would call attention to the fact that throughout the paper I have attempted to point out only some of the commonest transactions which involve the various accounts.

PARTICIPATING ANNUITIES—COLIN C. FERGUSON.

VOLUME XV, PAGE 354.

WRITTEN DISCUSSION.

MR. EDWARD W. MARSHALL:

Mr. Ferguson, in his paper on "Participating Annuities," correctly summed up the situation when he wrote as follows concerning the yield on participating annuities as compared with non-participating contracts: "If there were not a considerable increase, it would be doubtful whether the participating feature would have much effect in popularizing annuities."

Pursuing this thought further, there seems to be a practical objection to those methods of annuity participation which produce an increasing dividend. Unless the estimated return under participating contracts compares favorably with the non-participating yield, *almost from the beginning,* the prospective annuitant might turn from the company offering but a small contractual return together with vague promises of future large dividends, and buy a non-participating contract guaranteeing a tangible payment of moderate size.

While this objection applies more strongly to the so-called compound bonus plan under which the initial return would be but little larger than the contract payment, and in all probability well under the non-participating yield, it also applies under certain circumstances to the "semi-deferred" plan proposed by Mr. Ferguson. This method is put forward to eliminate an objectionable feature of the "uniform cash" or level dividend plan. To ascertain this level dividend, some assumptions must be made as to the future interest and mortality rates to be expected. Hence if unfavorable fluctuations should occur therein, it might be necessary to reduce the dividend. In order to avoid this unpleasant possibility, Mr. Ferguson proposes to defer a portion of the profits for a few years, and then if the basic assumptions prove conservative, to pay these withheld profits in the form of an annuity. Whether this plan would be satisfactory would seem to depend on how large a portion of the profits would be deferred. If too great an amount were withheld at first, then the objection contained in the last paragraph would apply.

But the question of methods of dividend distribution is secondary. The prime consideration is the *ability* of a company to pay dividends to annuitants. Could dividends be paid at the

younger ages on present rates? If the rates were raised would the provisional promise of future dividends offset the disadvantage of a smaller contract payment and popularize annuities to the extent that the present adverse selection would be reduced? What would be the effect on the situation if the legal reserve basis were changed to conform more closely to actual conditions? These questions are intimately connected with the subject of annuity participation.

The following annuity experience of the Provident Life & Trust Co. may be of some use in discussing these problems. The data include all single life, complete, immediate annuities issued since July 1, 1899, the date on which the schedule of annuity rates in use today went into effect.

TABLE I.

Year.	Premiums Less Commissions.	Annuity Payments.	Net Receipts.	Net Receipts Accumulated at 4½ %.	McClintock 3½ % Reserve.	Surplus.
1908	$ 57,956	$42,120	$15,836	$438,037	$437,816	$ 221
1909	116,926	47,956	68,970	528,271	531,790	−3,519
1910	82,983	55,660	27,323	579,981	574,764	5,217
1911	112,934	60,926	52,008	659,257	644,686	14,571
1912	120,361	70,156	50,205	740,258	733,883	6,375
1913	41,586	76,497	−34,911	737,873	724,411	13,462
1914	144,394	77,555	66,839	839,421	833,619	5,802

The annuity payments made in each year (1899 to 1914 inclusive) were deducted from the purchase money less commissions, and the successive differences accumulated at 4½ per cent. compound interest. As the McClintock 3½ per cent. reserve was not readily available for the years prior to 1908, all information relating to those years was omitted from the above exhibit. The average gross interest rate for the period 1899–1914 was about 4.67 per cent., and if due allowance is made for investment and other expenses exclusive of commissions, the average of 4½ per cent. is very close to the net rate realized.

The above results seem to indicate two facts: (1) The aggregate annuity fund is barely solvent after the McClintock 3½ per cent. reserve has been set up, (2) Participation is not advisable with such a small fluctuating surplus.

In view of the composite character of the material employed in this investigation it was thought best to subdivide the experience into

1. Male and female.
2. Entrants aged 60 and over, and under 60 respectively.

The following table which is self-explanatory presents the results.

TABLE II.

Year.	Males. Age at Entry under 60.			Females. Age at Entry under 60.		
	Net Receipts Accumulated at 4½ %.	McClintock 3½ % Reserve.	Surplus or Deficit.	Net Receipts Accumulated at 4½ %.	McClintock 3½ % Reserve.	Surplus.
1908	$101,875	$87,187	$14,688	$117,869	$124,306	−$6,437
1909	103,574	89,239	14,335	128,716	132,411	− 3,695
1910	141,217	128,703	12,514	126,855	131,477	− 4,622
1911	150,741	137,258	13,483	135,270	141,153	− 5,883
1912	151,474	138,228	13,246	178,364	187,442	− 9,078
1913	153,854	134,392	19,462	174,856	183,954	− 9,098
1914	197,481	179,316	18,165	193,440	204,524	−11,084
	Males. Age at Entry 60 or Over.			Females. Age at Entry 60 or Over.		
1908	$58,497	$61,239	−$2,742	$159,796	$165,084	−$5,288
1909	85,084	88,120	− 3,036	210,897	222,020	−11,123
1910	89,390	77,333	12,057	222,519	237,251	−14,732
1911	99,432	81,698	17,734	273,814	284,577	−10,763
1912	103,188	77,446	25,742	307,232	330,767	−23,535
1913	108,177	85,000	23,177	300,986	321,065	−20,079
1914	116,866	88,640	28,226	331,634	361,139	−29,505

The most striking feature of this exhibit is the large deficit arising in both the female groups, and the very large surplus in the male sections, especially at the older ages. While these results are partly due to excessive survivorship in the female groups, and to a favorable mortality in the male groups, yet an important cause is loss or gain from loading as is revealed by the following percentages.

TABLE III.

RATIO OF "PROVIDENT" ANNUITY RATES LESS COMMISSIONS TO $\mathring{a}_x$

(McCL. 3½ PER CENT.)

Age x.	Males.	Females.	Age x.	Males.	Females.
40	.972	.975	65	1.004	.971
50	.993	.981	70	1.013	.951
55	1.003	.983	75	1.069	.989
60	1.003	.980	80	1.193	1.076

These three tables compel the belief that if participation is to be granted, the "Provident" annuity rates for females must be raised very materially even at the younger ages, and possibly a slight increase made in the rates for males. Would such a course popularize annuities?

In the above calculations the McClintock 3½ per cent. reserve was the standard by which the various surpluses and deficits were ascertained. Hence it might be well to consider the effect thereon of employing another reserve based on rates of interest and mortality

conforming more closely to actual conditions. For the purpose of investigation it was assumed that the future net rate of interest would be 4½ per cent. and that the mortality to be experienced would follow the Mutual Life Domestic Annuity Table (*T. A. S. A.*, XIII, 273). By means of an approximate method the 1914 reserves were estimated for the four groups as follows:

TABLE IV.

1914.

	McClintock 3½% Reserve.	Surplus or Deficit.	Mutual Life 4½% Reserve.	Surplus or Deficit.
Males—under 60	179.316	+18.165	179.300	+18.181
Males—60 and over	88.640	+28.226	92.200	+24.666
Females—under 60	204.524	−11.084	202.500	− 9.060
Females—60 and over	361.139	−29.505	368.400	−36.766
Total	833.619	+ 5.802	842.400	− 2.979

This table shows that the reserves are not greatly affected by the change, and that unless the mortality grows much heavier than that of the Mutual Life Domestic Experience (which is a little lighter than the average), the McClintock 3½ per cent. reserve is not too heavy for an age distribution such as above. Hence the above results are useful as general indications of the conditions obtaining in the "Provident" annuity fund.

An inspection of one of the standard agents' manuals for 1914 reveals the fact that the "Provident" annuity rates are close to an average of the rates charged by other companies, and it seems not unlikely that conditions somewhat similar to those shown above exist in the annuity funds of several of these companies realizing a net rate of interest close to 4½ per cent. after making allowance for all annuity expenses exclusive of commissions.

It is a question whether the rather indifferent attitude of many company officers to the growth of the annuity business has not been a large factor in the adverse selection made by annuitants who in many cases buy annuities without much if any solicitation. While the solution of the question may possibly lie in annuity participation it would be interesting to watch the combined effect of these two actions by a company issuing non-participating annuities:

1. The adjustment of the non-participating rates in order to reduce any such inequities as appear in the above tables, no provision, however, being made for participation.

2. An energetic educational advertising campaign for annuities coupled with increased efforts by the agency force to solicit these contracts.

MR. C. R. FITZGERALD:

Before making any detailed comment upon Mr. Ferguson's paper, I have thought it would be of interest to investigate to just what extent an interest gain on an annuity would offset a vitality loss and in so doing, I have developed a method by which, from a standard annuity table with a certain rate of interest, the annuity values at a different rate of interest and by a mortality table whose rates of mortality are a constant or varying percentage of the rates by the standard table may be deduced. I have by this method deduced from the annuity values by the Actuaries' 3 per cent. table, those by the Actuaries' $66\frac{2}{3}$ per cent. table and 4 per cent. interest. I have chosen this table for the reason that I had the annuity values available and have thus been able to judge the correctness of the computations involved.

In our life insurance work, we are familiar with the equation

$$(V+P)(1+i)=q(1-{}_{+1}V)+{}_{+1}V.$$

In connection with the life annuities the analogous equation is

$$a(1+i)=-q\mathrm{a}_{+1}+\mathrm{a}_{+1} \quad \text{or} \quad a(1+i)-\mathrm{a}_{+1}=-q\mathrm{a}_{+1},$$

by Actuaries' 3 per cent. table we have at age 10,

$$24.220269-(1.03)\ (23.335864)=q\mathrm{a}_{11}=.163729,$$

which represents the accrual to each annuity from survivorship in the group. An interest margin of 1 per cent. will give \$23.36 on basis of an annuity of \$100 and a vitality loss of $33\frac{1}{3}$ will give \$5.46 or a net gain of \$17.90, dividing this by $(1-\frac{2}{3}q)$ we have \$17.98.

Following this method we have at the ages given below:

Age.	(1) Interest Gain.	(2) Vitality Loss.	(3) = (1) − (2) Net Gain or Loss.	(4) = (3) ÷ $1\frac{2}{3}q_x$
10	\$23.36	\$ 5.46	\$17.90	\$17.98
55	12.02	9.14	2.88	2.92
59	10.56	10.53	.03	.03
80	3.80	21.31	−17.51	− 19.32
98	.24	25.00	−24.76	− 49.52
99	.00	33.33	−33.33	−100.00

Now starting at age 99, let us discount the net losses or gains with interest at 4 per cent. and mortality at $66\frac{2}{3}$ per cent. Actuaries' table in the following manner:

Age x.	(1) Net Gain or Loss.	(2) = (3) × (4) for Age One Higher.	(3) (1) + (2).	(4) v 4 Per Cent. $(1-\frac{2}{3}q_x)$.
99	−$100.00	$——	−$100.00	.320512
98	−49.52	−32.05	− 79.57	.480769
97	−51.33	−38.25	− 89.78	.517751
96	−51.41	−46.38	− 97.79	.545737
95	−45.85	−53.37	− 99.22	.587353
94	−40.64	−58.28	− 98.92	.630574
93	−37.00	−62.38	− 99.38	.668444
92	−34.04	−66.43	−100.47	.701755
91	−31.70	−70.51	−102.21	.730137
90	−29.83	−74.63	−104.46	.754019
89	——	−78.76	——	——

Column (2) for an age one younger represents the difference between the annuity values by the Actuaries' 3 per cent. and the $66\frac{2}{3}$ per cent. Actuaries' 4 per cent. tables, for example by Actuaries' 3 per cent. a_{90} = $251.57 and by $66\frac{2}{3}$ per cent. Actuaries 4 per cent. a_{90} = $330.46, a difference of $78.89 as compared with $78.76 computed by the method above given; by Actuaries' 3 per cent. a_{10} = $2435.59 and by $66\frac{2}{3}$ per cent. Actuaries' 4 per cent. a_{10} = $2,162.41 or difference of $273.18 as compared with $273.25 computed.

Column (2) increases negatively to age 72 and commencing at age 71, decreases continuously and changes from negative to positive between ages 46 and 45. Hence the annuity values from ages 99 to 47, will be greater by $66\frac{2}{3}$ per cent. Actuaries' 4 per cent. than by Actuaries' 3 per cent. and for ages 46 and less, the contrary will be the case. This we see to be the case since by Actuaries' $66\frac{2}{3}$ per cent. table and 4 per cent. interest a_{47} = $1,589.00 and by Actuaries' 3 per cent. table a_{47} = $1,586.37, a difference of $2.63, while by Actuaries' $66\frac{2}{3}$ per cent. table, 4 per cent. interest a_{46} = $1,614.82 and by Actuaries' 3 per cent. a_{46} = $1,620.39, a difference of $5.57.

I have used the method above indicated for determining to what age interest gains of 1 per cent. and $1\frac{1}{2}$ per cent. will offset vitality losses reckoned as percentages of the tabular rates according to the McClintock Male and Female Tables with the following results:

Table.	Interest Margin.	Will Offset a Vitality Loss of				
		5%	10%	15%	20%	25%
		of the Tabular Mortality up to and Including Age				
Male.....$3\frac{1}{2}$ per cent.	1 per cent.	79	70	63	58	54
Male.....3 per cent.	$1\frac{1}{2}$ per cent.	84	75	70	65	61
Female.....$3\frac{1}{2}$ per cent.	1 per cent.	83	73	67	63	59
Female.....3 per cent.	$1\frac{1}{2}$ per cent.	87	79	73	69	65

This table tells us for example, that if we write annuities based on McClintock's annuity tables and $3\frac{1}{2}$ per cent. interest and we experience a mortality of only 80 per cent. of the tabular, our basis for annuity rates will be a safe one for all ages at issue up to and including 58 for male lives and 63 for female lives, and, for the higher ages at issue there will be a loss due to the fact that the interest gains are more than offset by the vitality losses. The above table amplifies the general statement by Mr. Ferguson, page 363, No. 3: "At the younger ages at entry a very small interest margin would compensate for any deviation in the rate of mortality that might reasonably be expected."

On pages 355–358, Mr. Ferguson deals with the question of annuity surplus from interest, but has not dealt with the case where there is a variation in the rate of mortality from the tabular rates beyond the general statement on page 358, and his treatment throughout would allow the annuity surplus which consists of interests gains less vitality losses to be distributed among the survivors, who are necessarily those who cause the vitality losses. This situation has been referred to in the discussion of Mr. Fackler's paper and indeed furnished the chief argument against the justice or equity of participating annuities.

It is quite practicable for us to deal with the annuity surplus on lines similar to those followed in the distribution of our insurance surplus. Determine the interest gain as a percentage of the annuity and the vitality loss, if any, as a percentage of the tabular rate of mortality multiplied by the annuity due at an age one year in advance. The difference would be the basis of the annual dividend. Discard the principle that the survivors are to benefit from the contributions of those who die by using an interest factor only for discount or accumulation in all computations with the annual annuity surplus contributions. This would be analogous to the dividend accumulation plan in reference to annual dividends in life insurance practice.

For example, let us assume that the annuities are based on McClintock's 3 per cent. Male and Female Tables with interest margin of $1\frac{1}{2}$ per cent. and vitality loss 20 per cent. of tabular rate from age 70 to end of table, 19 per cent. at age 69, 18 per cent. at age 68, etc., with 1 per cent. at age 51. We should have to hold back from the annuity surplus at the younger ages, amounts which would be allowed to accumulate a fund to meet the vitality losses at the higher ages. For accumulation let us use a somewhat lower rate of interest, say 4 per cent., than the interest margin assumes is being earned, let us determine the annual dividend to be paid as an annuity certain for the term from the age of the annuitant one year in advance of the age of issue to the limit of life by the mortality table used. This would allow us to give a uniform dividend in accordance with the conclusion (1) on page 353 of Mr. Ferguson's paper.

I have made some calculations along the lines indicated which are given in the table following. On the basis of an annuity of $100, the value, at age 21, of the accumulations from the limit of life to age 21 would be for the male table $562.44; dividing this by $a_{\overline{85}|}$ at 4 per cent., we would have an annual dividend of $22.45. Here at age 20, male, the annuity surplus at end of first year would be $32.76; deducting the dividend would leave $10.31 to accumulate at interest, which at end of second year would amount to $10.78, etc. In the following table I give the accumulations for a contract issued at age 20:

Age at Issue.	Dividend.		At End of Years	Accumulation. Age 20.	
	Male.	Female.		Male.	Female.
10	$26.29	$29.31	10	$122.08	$128.02
20	22.45	25.18	20	247.52	258.59
30	17.79	20.38	30	382.12	397.82
40	12.27	14.69	40	503.10	526.78
50	5.78	8.12	50	564.46	604.18

By following this method we should allow as a death benefit the fund which has accumulated, and we should thereby avoid the chief objection which has been raised against participating annuities. In the practical operation of the plan, we should credit each particular annuity with interest gain and debit with vitality loss, if any, and with the dividend payment made, the balance would be credited with interest and this account carried along year by year. It would not be advisable to exhibit to the annuitant the detail of the accounting or even inform him as to the amount of the accumulated funds. The sum of these accumulated funds would form a special annuity reserve, which might be simply designated as such or desgnated as "dividend annuity equalization fund," or else need not be considered as a liability at all, if we keep in mind that a part of the company's general surplus is for the purpose of insuring the payment of the uniform annuity dividend. It would seem advisable, however, if we pay a death benefit of the accumulated fund that the total fund should be considered as a liability.

Now suppose we have been operating on the basis of a scheme such as I have outlined and our experience shows that there has been a vitality loss of 10 per cent. at all ages to 61, increasing 1 per cent. each year to age 70, from which ages upward it is 20 per cent. With the same margin of interest, the dividends and accumulated funds would be as shown in the table on page 210.

Suppose we have a contract issued at age 20, and we have paid the dividends under scale (1) for twenty years, male $22.45, female $25.18, and we find it advisable to adopt the second scale, we should change the dividends payable to $20.97 and $24.36 respectively and reduce the accumulated funds from $247.52 male

Age at Issue.	Dividend.		At End of Years.	Accumulations. Age 20.	
	Male.	Female.		Male.	Female.
10	$24.72	$28.41	10	$118.08	$126.74
20	20.97	24.36	20	237.42	254.77
30	16.47	19.55	30	359.86	385.45
40	11.20	13.98	40	473.16	508.38
50	5.24	7.74	50	538.20	587.51

and $258.59 female to $237.42 male and $254.27 female, and the difference can go into the general surplus to help make up for the vitality losses which were not provided for under the first scale. This would be right and proper, for we are causing the survivors, who have caused the vitality loss to assume the burden, imposed by the same. If we have been operating under the second scale and after twenty years we propose to change to the first, we should increase the dividends from $20.97 and $24.36 respectively, to $22.45 and $25.18 respectively, and the accumulated funds from $237.42 and $254.27 respectively, to $247.52 and $258.54 respectively, doing this by making up the difference from the company's general surplus. This procedure would be justified on the ground that the experience of the period of twenty years had shown that the second scale was not liberal enough and that, by distributing on that basis, there had been a profit made which had gone into the general surplus.

In this case the beneficiaries of those annuitants who have died within the period would have received the death benefit represented by the accumulated funds under the second scale, whereas experience has shown that they should have been under the first scale. We can adopt either of two courses, (1) make no re-adjustment, which would be analogous to the treatment of our policyholders under life contracts or (2) make a re-adjustment by paying the difference between the dividends on scale (2) and scale (1) with interest, in addition to the accumulated fund by scale (2). In this case if we were to re-adjust dividends in connection with annuitants who have died, we should do the same with those who live. Our objection to (2) is that we perhaps would be making a payment for a trivial amount to an estate which may have long since been closed. I think that in conformity with our insurance practice, we should not attempt to make any re-adjustment either with those who die or those who live.

If in the adjustment of the annual dividends on our life business we recognize the characteristic differences in the mortality rates experienced as compared with the tabular rates, we can, on the average, produce entirely equitable results. Similarly in the operation of a participating annuity experience we may obtain equitable results by observing the same principles, but we must see to it that the annuity rates adopted by the company (1) are based on mor-

tality rates at the older ages which may be as light as we can conceivably expect the actual mortality experience to be, or even somewhat lighter than this, and (2) are computed at a rate of interest which will insure an ample interest margin.

I have computed some annuity rates which appear on page 212 and which may be of interest. These rates are based on the McClintock mortality rates to age 50, 99 per cent. of same at age 51, 98 per cent. at age 52, etc., and 80 per cent. from ages 70 and over. For the non-participating rates, I have used 4½ per cent. interest and for the participating 3 per cent. The annuity values have been deduced according to the method herein explained. The loading provided allows an acquisition expense of 2½ per cent. of the consideration and a general expense of 1½ per cent. of the amount of the annual payment, and the annuities are complete annuities, i. e., proportionate part payable to time of death. I do not make any claim as to the adequacy of the rates which are merely intended to be for the purposes of comparison.

On the participating annuities, I have calculated dividends based on a return of 1½ per cent. interest with accumulation of credits at 1 per cent. and adjustment so that a uniform dividend may be paid from one year from time the annuity is issued to limit of life. I have also deduced average annuity rates of the companies whose annuity rates appear in the statistical publications, *The Handy Guide, The Life Insurance Manual* and *The Unique Manual.* The averages have been taken in two ways: (1) A simple average giving the rates of all companies the same weight and (2) an average based upon the amount received by the individual companies in 1913 on life annuity contracts. Except at the higher ages (1) and (2) give results which differ by only a few cents. I therefore give the average only by method (2).

As an explanation of the difference between the results under the non-participating rates and the participating rates with the dividend, it will be noted that with the participating annuity there would be the death benefit which consists of the accumulated annuity fund.

Now let us consider Mr. Ferguson's summary, page 363, in connection with the table on page 212.

Conclusion four: "as compared with present non-participating annuity contracts, participating annuities would give a larger yield at the younger ages but not materially greater at the older ages."

This conclusion is correct but it will be noted that the "guaranteed" annuities under the participating contract will be less at practically all ages than the amounts now paid on all annuities at present issued and they would therefore be safer to the companies who might issue the same. In reference to conclusion (5), "annuities payable for a term certain and for the remainder, if any, of the payee's life should be permitted to participate for at least the term certain in order to preserve consistency

ANNUITY RATES FOR CONSIDERATION OF $1,000.

Male Lives.

Age.	Non-Part. 4½ Per Cent.	Participating Annuity.			One Company.	Average Rates.	Three Large Companies.
		Guaranteed.	Dividend.	Total.			
10	$53.53	$40.83	$11.48	$52.31	$47.25	$43.25	$39.75
5	54.12	41.91	11.17	53.08	49.00	44.72	41.04
20	55.58	43.26	10.84	54.10	50.65	46.46	42.63
5	57.08	44.95	10.49	55.44	52.65	48.43	44.55
30	59.02	47.07	10.15	57.22	55.25	50.89	46.97
5	61.35	49.76	9.79	59.55	58.55	54.02	50.04
40	64.83	52.88	9.40	62.28	62.85	58.05	53.97
5	69.10	57.56	9.08	66.64	68.40	63.28	59.07
50	74.65	63.18	8.73	71.91	75.75	70.16	65.72
5	81.95	70.49	8.38	78.87	85.45	79.29	74.51
60	91.71	80.31	8.06	88.37	98.70	91.80	86.37
5	105.31	93.82	7.75	101.57	115.35	108.62	102.54
70	124.75	113.04	7.54	120.58	138.50	131.19	124.82
5	153.29	141.19	7.41	148.60	170.05	160.10	153.46
80	194.61	181.94	7.41	189.35	214.10	193.38	182.88
5	255.01	241.62	7.34	248.96	239.30	216.99	204.29
Female Lives.							
10	$49.60	$37.08	$11.53	$48.61	$46.80	$42.58	$39.39
5	50.42	38.08	11.22	49.30	48.50	44.12	40.97
20	51.46	39.27	10.91	50.18	50.10	45.67	42.61
5	52.78	40.82	10.58	51.40	52.00	47.38	44.42
30	54.49	42.71	10.24	52.95	54.35	49.61	46.68
5	56.69	45.07	9.89	54.96	57.25	52.54	49.46
40	59.52	48.05	9.55	57.60	60.80	55.74	52.85
5	63.18	51.85	9.28	61.13	65.15	59.96	56.99
50	67.91	56.66	8.83	65.49	70.65	65.39	62.20
5	74.08	62.89	8.48	71.37	77.95	72.64	69.09
60	82.35	71.16	8.15	79.31	88.25	82.88	78.89
5	93.65	82.39	7.85	90.24	103.50	97.72	93.38
70	109.58	98.15	7.59	105.74	124.70	118.52	113.85
5	132.60	120.84	7.43	128.27	155.25	144.46	137.19
80	165.34	153.39	7.33	160.72	196.45	175.86	165.70
5	213.65	200.82	7.35	208.17	215.50	203.28	198.09

with similar contracts issued in settlement of death claims." This is right and proper if we are considering guaranteed annuities issued as such, where the deferred annuity is based upon a proper annuity table; but if we are dealing with settlements under (*a*) continuous instalment policies, where the extra annuity premiums have been based on such a table as the American Table for the annuitant or (*b*) under the continuous instalment option on a policy contract, where the American Table is used as a basis of the deferred annuity, I believe we should be justified in discriminating between the annuity certain part of such contracts and regular annuity certain settlements, with a view to the providing of a fund from surplus interest during the period in which the annuity cer-

tain is being paid to offset a possible loss later on when the life annuity would be payable.

I personally believe that annuity participation can be planned upon principles which will deal equitably with annuities irrespective of age, be absolutely safe for the office issuing the same, give general satisfaction to the annuitants enjoying the same and be free from the objection which hitherto has been urged against annuity participation.

MR. C. C. FERGUSON:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

I feel very grateful to the gentlemen who have expended so much time and thought in their discussions of my paper. They have added very materially to our information on the subject, but unfortunately for lack of time I find myself unable to review their contributions at all fully.

The suggestion has been made that there is an objection to participating annuities because those who die early are the real contributors of the profit, should there be any, from mortality, while, nevertheless, the survivors would ordinarily receive the whole of that profit. I do not agree with this suggestion.

The purchaser of an annuity usually wishes to obtain a uniform life income for the largest amount which his investment will procure. This may be given to him in either of two ways. In the first place we may issue a non-participating contract based upon assumed rates of interest and mortality. Or we may issue a participating annuity applying the profits as far as possible as a constant addition to the guaranteed annual payment. The result in the latter instance is to substitute for the assumed rates of interest and mortality the actual rates as experienced, thus equating more closely the benefit with the cost.

In the latter instance the so-called profits are applied in purchasing an additional annuity and I can see no hardship in that to the annuitant since his original purchase of the annuity was a clear intimation that he wanted only an income in return for his investment. He would see no reason for our insisting on combining with the annuity a small insurance which the death benefit suggested by Mr. Fitzgerald would really amount to.

For this reason I am strongly inclined to adhere to my original proposal of applying all the surplus as an addition to the annual payment.

A METHOD OF DETERMINING PURE PREMIUMS FOR WORKMEN'S COMPENSATION INSURANCE—MR. HARWOOD E. RYAN.

VOLUME XV, PAGE 364.

WRITTEN DISCUSSION.

MR. A. W. WHITNEY:

Mr. Ryan's paper indicates some important considerations which must form the foundation for an intelligent and effective analysis of the cost of workmen's compensation. Some further points may be more specifically brought out even though they are implicitly contained in the paper. Strictly speaking there is only one equation for a rate, but this takes different forms according to the logical basis of classification that is used in each case to bring into prominence the elements that are desired for the purpose for which the formula is to be used and particularly in view of the nature of the experience that is available. There is nothing so simple and nothing so good, if the experience is sufficient, as the formula L/P where L is the incurred losses and P is the payroll exposure. It will be years, however, before, in most states, rates can be accurately determined in this way, except for a few of the leading classifications. For the smaller classifications and for individual risks this method is not sufficient.

Mr. Ryan's analysis of the rate into the product of an accident frequency and an expectation of loss based upon injury instead of exposure is a response to the necessity of finding some immediately practical way of dealing with the problem, and to prepare the way for certain simplifying assumptions.

For the sake of definiteness and also, as I believe, as a matter of general interest, I may be allowed to state just how the workmen's compensation rates for the different states are at present being got by the Workmen's Compensation Service Bureau. I will connect this later with the formulas that have been given by Mr. Ryan.

The first assumption that has been made is that, provided the basic pure premiums for one given state are known, these may be extended to the same classifications for other states provided the relative beneficence of the compensation laws of the several states are known. I advisedly use the vague term beneficence until I can define this more exactly. Valid criticism may be directed against this principle; in fact the Bureau has recognized that it is not in itself a sufficiently exact statement of the facts by providing for the individual treatment of certain classifications in the various

states. The importance of this assumption however is enormous, not only in simplifying the problem so as to bring it within the bounds of what is practicable, but what is perhaps even more important in this formative period of the science, in centralizing the treatment of compensation rating so that it may receive every possible contribution from every source instead of having the science separately developed for each state by itself.

The most trustworthy body of statistical experience on workmen's compensation in this country at the present time is the experience of Massachusetts. On the basis of this, supplemented by the experience of some other states and fortified by such statistical expedients as the grouping of allied classifications and, it must be admitted, a liberal admixture of judgment, the basic pure premiums for Massachusetts were determined. It goes without saying that these are subjected to revision as new experience develops.

The next problem was to determine the relative cost of the laws of the different states. The basis of this was, of course, the fundamental rate formula but so developed as to fit the immediate needs. The second form in Mr. Ryan's paper was used in which the accident frequency was developed as one of the factors. In view of the fact that the basic pure premiums were now known for one state, the formula could be modified so as to show only relative accident frequencies. These are certainly hard enough to determine but the difficulty does not compare with that of determining the absolute accident frequency.

The other factor, the relative cost in two given states of compensating a given number of injuries, was treated as an actuarial problem rather than as one of pure statistics; both numerator and denominator were of the same form and consisted of the sum of expectations each of which was the product of the probability of a given injury multiplied by the compensation allowed for that given injury by the law of the particular state. The scale of compensation differs from state to state, but it was assumed that the distribution of the injuries both as to nature and severity would be the same not only for different states but for different countries. This was an exceedingly important assumption, for, if it was valid, it made it possible to make use of foreign experience which was ample while our own experience was still very deficient. Repeated tests of the truth of this hypothesis have been made and seem amply to confirm its validity as a practical working method provided it is used with a reasonable degree of care.

The detail of the preparation of a table or set of tables giving the distribution of accidents, on a radix of 100,000, as to severity and nature of injury is given in a paper by Dr. I. M. Rubinow in the last number of the *Transactions of the American Statistical Society*. This table stands in much the same relation to workmen's compensation insurance that the mortality table does to life insurance. It has been based upon the best available experience

wherever it could be had both in this country and in Europe and, while later developments may doubtless show that it should be modified in some details, there is every indication to show that it can safely be used for computing costs and reserves.

I might say that the Bureau is not to be given the entire credit for the development of these general methods as they are due to the cumulative work of a number of persons.

The application of the Bureau plan gives us, first, a set of basic pure premiums for some state such as Massachusetts, and, second, a set of differentials giving the relative cost of the laws of the other states as compared with that of the given state. The multiplication of the basic pure premiums by the differential for a given state gives the pure premiums for that state.

A word about schedule-rating. If the same fundamental equation for the rate is developed and analyzed on the logical basis of *causes,* it becomes the ground work for the establishment of a system of schedule-rating in which the endeavor is to project the basis rate so as to fit the peculiarities of the individual risk. To make this clearer let me remind you that L, the expression for the incurred losses, breaks up into a sum of expectations, differing one from the other, not only as to the nature of the injury but as to the cause as well, so that there will be, for instance, terms of this sort: expectation of payment because of loss of arm due to being caught in gears, expectation of payment because of loss of eye due to chips from an emery wheel. Now these may be arranged according to nature of injury, which is the usual way, or they may be rearranged according to cause so that all the expectations of payment arising out of the various injuries due to unguarded gears, for instance, stand in one group, and in another the expectations of the various injuries due to the use of emery wheels, &c. This, however, is no place to go further into details. My purpose here, as in the paper as a whole, is to call attention to the fact that the same fundamental equation by means of an analysis in these various ways can be adapted both to the particular characteristics of the experience that is available and to the particular purpose that is in view.

MR. A. H. MOWBRAY:

An uncritical reading of Mr. Ryan's paper would leave the impression that there was no relation between the method of computing pure premiums by the use of accident frequency data and that based upon insurance experience. As the two methods are presented in his paper the relation can hardly be traced.

It will be noted, however, that he has used the formula quoted from my paper (*T. A. S. A.,* XV, 91) in an entirely correct but somewhat different sense from that given it in my paper. As quoted by Mr. Ryan the formula would lead to a premium rate in

terms of full time workers per year, instead of the usual rate of current practice, per unit of payroll, for which I intended it. The difference is in the definition of p, the frequency of any given type of injury. Mr. Ryan defines p as equal to "the number of persons sustaining this particular form of injury divided by the number exposed," while in my paper it is defined as "the probable number of such disabilities arising out of the operations covered by a unit of payroll exposure," so that the denominator would be payroll rather than full time workers. It is true that this is not the usual way in which frequency statistics are published, but unless the elusive average wage factor is to be brought in, it is the way such statistics must be gathered and published to be of use for premium computation work as long as the premiums are quoted on a payroll basis.

Taking my formula as quoted by Mr. Ryan and substituting i_k/e for p_k we get

$$\pi = a_1 \cdot i_1/e + a_2 \cdot i_2/e + a_3 \cdot i_3/e + \cdots + a_k \cdot i_k/e. \qquad (1)$$

It is not practicable within a classification to separate the exposure to injury according to the type of injury so that e the denominator is common to all the fractions, hence

$$\pi = \frac{a_1 \cdot i_1 + a_2 \cdot i_2 + a_3 \cdot i_3 + \cdots + a_k \cdot i_k}{e}, \qquad (2)$$

where e is measured in terms of payroll exposure. But, using Mr. Ryan's notation $l_1 = a_1 \cdot i_1$, $l_2 = a_2 \cdot i_2$, $\cdots k = a_k \cdot i_k$, and substituting in (2)

$$\pi = \frac{\Sigma l_n}{e} = \frac{L}{P}.$$

It is obvious that to follow out Mr. Ryan's methods in detail would require almost if not fully as complete data as would be required for a direct computation by a purely frequency method.

The principles of his method, however, can be used to greatly simplify practical work. For example, though theoretically L/I should be found for each classification or group of cognate classifications, it can with reasonable approximation be determined for an entire state and period or even determined with reference to a general standard, say 100,000 accidents, for the compensation laws of several states, and then the pure premium for all of them computed by using I/P for classifications as determined by actual experience in some one, modifying its value for each of the others if there is evidence that such modification is necessary. This, I understand, is the method now used by the Workmen's Compensation Service Bureau in preparing its rates for the several states.

MR. H. E. RYAN:

(AUTHOR'S REVIEW OF DISCUSSION.)

First, as to Mr. Mowbray's comments,—I am glad to have my attention called to the slight misunderstanding which I had at the time of citing his formula for the pure premium. I inadvertently assumed that Mr. Mowbray had developed the cost per unit in terms of persons exposed, whereas he related the probability of loss to the payroll exposure, a procedure which is much more practical even though certain theoretical errors are introduced. Employers cannot be expected to do the rather elaborate statistical work necessary to establishing the exposure in terms of full-time workers. Their payroll records, however, are available and these furnish a very fair approximation to the risk exposure. Moreover, the policies issued in connection with this class of insurance provide for an audit of the employer's books by the carrying company so that reasonable accuracy may be expected. By using the payroll as the basis, it becomes unnecessary to introduce into the calculation the average wage, for the reason that the payroll is a function both of wages and of the duration of exposure.

Professor Whitney has referred to Dr. Rubinow's statistical distribution of 100,000 accidents. A table of this kind has great practical value in the absence of pure premium results. It gives the data for a complete analysis of the expression

$$i_1 + i_2 + i_3 + \cdots + i_k$$

and makes possible the assignment of appropriate weights to the several types of injury according to any desired scale of benefits and the establishment of differentials as Prof. Whitney has explained. It should be mentioned, however, that the use of such a table does not clear up all of the difficulties in the way of ascertaining true differentials. I believe within a very short time,—two or three years,—we shall be able to get along with a much simpler procedure. The accumulation of data upon some uniform plan by the several states and by the rating associations maintained by the casualty insurance companies, will have a large influence in that direction. The tendency has been toward great refinement of the data, but the expense of such refinement is scarcely justified by the results. The important considerations are: *First,* to relate the amount of loss in the aggregate, by industrial classification, to the earned payroll. This will yield the information of greatest value, i. e., the pure premium for each classification. *Second,* a broad division of the losses (both number and amount) into the groups: death; dismemberment; other permanent partial disability; permanent total disability; temporary disability; medical and hospital services.

These two sets of data (which should be obtained at the same time so that they may be properly related) will give practically all the needed information, once we have established the ultimate cost of those benefits which, under most workmen's compensation acts run for a period of years or else terminate by the death or remarriage of the beneficiary. The determination of differentials by a theoretical method will then give way to a comparison of actual results. The theoretical method, however, has served a very important purpose in the early days of workmen's compensation in this country.

ON THE PRINCIPLES GOVERNING THE VALUATION OF DISABILITY BENEFITS BY A REGULAR LIFE INSURANCE COMPANY—GEORGE BOHLMANN.

(VOLUME XV, PAGE 77.)

REVIEW OF THE DISCUSSION (VOLUME XV, PAGE 431) BY THE AUTHOR.*

This review is in response to the invitation appearing on page 438 of Volume XV. I shall chiefly follow Mr. Franklin B. Mead's discussion which will give me an opportunity to consider at the same time the criticisms raised by the other gentlemen, viz., Mr. Thompson and Mr. J. F. Little.

The equality $d_x = d_x^{aa} + d_x^{ii}$ has been written down by me as outlining the whole process of Mr. Hamza's scheme, when applied to the American Table. I agree with Mr. Mead when he says: "The assumption . . . is . . . in the fact that the r_x and q_x^i which we employ are based upon the same, or at least equivalent, data as the q_x of the American Table," excepting that this is not the only assumption, but merely one of several assumptions involved in Mr. Hamza's process. If it does not appear correct to Mr. Mead to look upon any mortality table as a fictitious generation, inasmuch as the l_x and d_x are merely functions of the q_x employed, we come to a mere difference as to the words to be used. The expression "fictitious generation" is not my invention, but has been—almost universally and in my opinion luckily—adopted in life insurance and mathematical statistics in order to distinguish between those mere functions of the q_x and between the exposed to risk from which the q_x were derived. In this connection it appears very proper to call attention to the difference between the two formulae:

$$q_x(l_x^{aa} + l_x^{ii}) = l_x^{aa} q_x^{aa} + l_x^{ii} q_x^i$$

and

$$q_x(E_x^{aa} + E_x^{ii}) = E_x^{aa} q_x^{aa} + E_x^{ii} q_x^i,$$

where E_x^{aa} and E_x^{ii} may designate the number of active and disabled lives respectively into which we should separate the number living exposed to risk at the age x. In both equations q_x and q_x^i may be

* The "Review" of the Discussions appears in the present number of the *Transactions* because the time was not sufficient for the Discussions to be transmitted to the author in Germany and a "Review" returned before the previous *Transactions* went to press.

considered as the given functions and q_x^{aa} as the residual. The difference is only that the distribution in the first equation is determined by the Hamza process when the commencing age is given, but unknown in the second equation. The two series of q_x^{aa} would coincide only if the ratios $l_x^{aa} : l_x^{ii}$ and $E_x^{aa} : E_x^{ii}$ agreed for all ages x. That will never be the case. Consequently the two series of q_x^{aa} will always differ. The q_x^{aa} derived from the E are the true values and will be certainly adopted whenever the separation of the E is possible. The q_x^{aa} found by the Hamza process are merely *conventional* and will only be taken, if we can not split the E into the active and disabled lives. I can never hope to come to an understanding with any one who does not acknowledge that conventional nature of the Hamza process.

In my paper I did not develop at length these considerations, because I thought they were sufficiently intimated by the two words "fictitious generation," and I did not anticipate any controversy about that expression. I therefore mentioned particularly only a third more hidden assumption involved in the equation $d_x = d_x^{aa} + d_x^{ii}$, viz., the possible influence of the withdrawals. We could conceive that among the holders of life insurance contracts without disability benefits there would be a number who, being in bad financial situation and though impaired in health were obliged to discontinue their policies before they died. Had they been insured with disability benefits, their premiums might have been waived, so that their deaths had been lost in the American experience, but would now come under observation. In that event, other factors being unchanged, future experience could show higher death rates among the living policyholders, because the ratio $E_x^{ii} : E_x^{aa}$ might be increased by the mere introduction of disability benefits.

Mr. Mead rightly surmises that I am in this connection an advocate of a single set of commutation columns as regards the q_x^{aa}.

If, as in certain German experiences (see for instance O. Dietrichkeit, Fundamentalzahlen für Invaliditätsversicherung, Elberfeld, 1894) all probabilities have been derived from the actual distribution $E_x^{aa} : E_x^{ii}$, there will be—so I hope—no question about it. If the actual distribution is not known, the fact remains nevertheless that such a distribution and only one distribution exists. To assume a commencing age in the Hamza process is to presuppose a distribution. Assuming several commencing ages means assuming several distributions which is a contradiction, as only one distribution exists.

In the first sentence of his comments Mr. Mead makes the principle under discussion chiefly responsible for the fact that he must disagree with me on many important points. I appreciate that remark. The reader will have noticed that we come to a critical phase of the discussion, when asking, how the American mortality

and net premium shall be introduced. Mr. Mead's principle has the advantage that π_x and his π_x^a are always identical, so that no artificial steps are necessary to introduce the π_x. In my paper, on the contrary, there is no room whatsoever for π_x, as long as we remain within the company B which shall be the true standard in theory, and the π_x comes in only later as a sort of appendix by way of a compromise, by a practical principle.

If nevertheless I cannot accept the solution proposed by Mr. Mead and also by Mr. Little, it is for the reasons stated above. The theoretical principles thus set forth in my paper are, therefore, by no means innovations, but the same on which the theory of disability insurance has always been based. Mr. Mead compares the distinction of active and mixed lives, of π_x^a and π_x, with the difference between select and ultimate mortality tables in life insurance. That is a good simile. But in disability insurance we have no choice, we must consider the different mortality of active and disabled persons, the theory must result in the use of the π_x^a and not the π_x.

I now enter into Mr. Mead's and Mr. Little's discussion of my valuation formulae. In order to clear the ground, I ask them to start, as required in my paper, from the theoretical company B, disregarding at first "practical principles." In that company the net premiums agree exactly with the reserves. The corresponding formulae of my paper are those which do not bear [like (6′), (7′), (8′)] an accent. So far I hope there is no difference in opinion between Mr. Mead and myself, for he confirms "that the use of A_x^a and A_x^i is the most theoretically correct and flexible method of valuation." In proceeding to the practical principle and to that part of the contract which consists exclusively of the waiver of premium clause (formulae 6′, 7′, 8′ of my paper), I introduce the American net premium like a quantity known in advance, say like the amount insured in ordinary life insurance. But the mortality remains, as in company B, the mortality of the active lives, when the policyholder is active, and the mortality of the disabled lives, when the policyholder is disabled. In that regard I appear to be in harmony with Mr. Little. If I rightly understand his further comments, we also agree in the view that the net extra premium (6′) and the extra reserve (7′) of my paper correspond with each other. His reference to the identity of formula (7′) with a formula recommended by Mr. Hunter has been instructive to me.

Now, as to the reserve for a disabled life, the formula (8′) suggested in my paper for use in practice, really involves a difficulty. The net extra premium (6′) supplies for a disabled life only the reserve $\pi_x a_{x+k}^i$ as stated by Mr. Little, while my formula (8′) requires to hold a greater reserve, namely the excess of A_{x+k}^i over the American reserve. Theoretically the proper reserve is therefore $\pi_x a_{x+k}^i$ and not (8′). In practice, however, I consider it vital to be

fully protected against the heavy mortality expected on disabled lives and to hold for that reason a reserve which accounts for the fact that—apart from any theoretical constructions—the entire contract is in reality nothing else than a paid-up policy subject to the mortality of the disabled lives. I therefore advocate formula (8′), sacrificing harmony to safety, but suggest that the excess reserve, thus required in addition to $\pi_x a'_{x+k}$, be covered by an extra net premium exclusively constructed for that purpose.

With this suggestion I hope I have removed the only real difficulty which I can find in my paper. In all other respects I can only repeat that I have, as far as I know, simply followed the usual theory of disability insurance, particularly, when stating the principles, and especially, when requiring to base the valuation of disability benefits of disabled persons on the mortality of disabled lives, and not of mixed lives. To deviate from that demand and not to adopt that requirement, involves in my opinion a radical change.

Book Notices.

Supplement to the Seventy-Fifth Annual Report of the Registrar-General of Births, Deaths and Marriages in England and Wales. Part I.—Life Tables. London, T. Fisher Unwin, 1914. Pp. iv + 85.

The main body of this supplement consists of the report by Mr. George King on the construction of the English Life Tables Nos. 7 and 8 and the other tables for sections of the population which were constructed at the same time.

English Life Tables No. 7 are based on the censuses of 1901 and 1911 and the deaths of the ten years 1901 to 1910. The tables for males and females, respectively, were prepared on identical lines, the mean population for quinquennial groups being determined in each case by the method of Mr. A. C. Waters described by Mr. King in *J. I. A.*, Volume 42. Mr. King's construction A method described in Volume 43 is then followed to determine quinquennial pivotal values for the rate of mortality at ages 12 to 97, inclusive, and the interpolated values from 17 to 92 inclusive, log $(q_x + \cdot 1)$ being the function operated on in the interpolation. At the extreme old ages no assumption is made with regard to the termination of the table at any definite age but a fourth difference series of values for log p_x is determined from the values at ages 89, 90, 91, 92 and 97. The rates of mortality at ages 0 to 4 inclusive are determined from the death returns at individual ages and the readjusted average population.

This readjustment was found necessary because the census returns indicated a deficiency of population at ages less than two last birthday. The theoretical population for each attained age was accordingly calculated from the birth and death returns and each age was then reduced in the proportion necessary to bring the total exposed to risk under age five down to the total derived from the census returns. The assumption that the total population at ages under 5 was correctly given in the census returns was made because a similar assumption had been made in the construction of English Life Tables No. 6 and it was desired that the two should

be comparable. The birth and death returns, however, indicate that the deficiency at the two younger ages was an absolute one and not a transfer to older ages. The rates of mortality at ages 5 to 16 inclusive were then inserted by Lagrange's formula from the values for ages 3, 4, 12, 17 and 18.

English Life Tables No. 8 were based on the census of 1911 adjusted to July 1 of that year on the assumption of a geometrical increase for each age group and the deaths of the three years 1910 to 1912. In the construction of these tables the same methods were followed as in the case of No. 7 with the following exceptions:

First, population returns being available for every individual age, a different quinquennial grouping was adopted as giving more accurate results. The groups commenced at ages 4, 9 and so on instead of at exact quinquennial ages, the result being the obtaining of pivotal values one less throughout. Second, the rates of mortality at ages zero to five inclusive were calculated from the birth and death returns without reference to the census returns.

Probably the most interesting part of this report is the comparison of the mortality shown by the various tables, the rate being lower at all ages and for both sexes by the English Life Tables No. 7 than by No. 6, and lower at practically all ages in No. 8 than in annum in the population age 0 to 80 inclusive shown by the census returns of 1911 with the following result:

EXPECTED DEATHS PER ANNUM.

	Ages Last Birthday.	No. 6.	No. 7.	No. 8.
Males.....	0– 4	106,677	92,197	79,357
	5–59	122,830	101,995	91,628
	60–89	94,369	87,114	84,216
	0–89	323,876	281,306	255,201
Females.....	0– 4	89,394	76,879	65,980
	5–59	112,806	93,667	81,497
	60–89	108,344	94,472	91,091
	0–89	310,544	265,018	238,568

The fourth part of the report is devoted to the description of a short method of constructing mortality tables which consists in finding quinquennial values of q_x and hence of $\log p_x$ then, by a finite difference formula, quinquennial values of $\log {}_5p_x$, then quinquennial values of l_x and finally quinquennial values of $\mathring{e}_x$.

Two appendices of the report contain, respectively, the data upon which the tables are based and the mathematical formulas used. At the end are given fifteen mortality tables which have been constructed. Four of these have already been referred to, three others represent the mortality of spinsters, wives and widows, while the remaining eight represent the mortality among males and females separately in the county of London, the county boroughs, urban districts and rural districts of England and Wales.

Medico-Actuarial Mortality Investigation. Volume V. Compiled and published by the Association of Life Insurance Medical Directors and the Actuarial Society of America, New York, 1914. Pp. 185 + xii.

This is the concluding volume of a series giving the results of the recent mortality investigation. It contains the experience of insured lives of those classes which showed a family history of tuberculosis, also the experience of certain companies in seven southern states and the experience in three classes involving a personal history of malarial fever and in four classes of joint life insurance.

The volume concludes with a general index of the five volumes and a list of tables.

The Total Disability Provision in American Life Insurance Contracts. Bruce D. Mudgett, Ph.D. Philadelphia, The Annals of the American Academy of Political and Social Science, 1915. Pp. ix + 83.

Dr. Mudgett is instructor in insurance in the Wharton School of the University of Pennsylvania and has attacked the subject from a somewhat different point of view from that of previous writers to whose work he has added an interesting supplement.

The book is divided into two parts. The first deals with an historical sketch of disability insurance with an extended outline of its statistical basis in which he has quoted freely from the papers in the *Transactions* by Messrs. Jackson, Pipe, Hunter and Mead. The second part consists of an exhaustive analysis of the disability clause as it is granted by American life insurance companies. Dr. Mudgett's treatment is almost entirely from a sociological viewpoint and after careful analysis he reaches the conclusion that a

proper total and permanent disability provision affords a valuable form of protection in connection with life insurance policies and has, therefore, a valid *raison d'être.*

Chapter 2 part 1 deals with the reasons for and objections to the disability clause, and the author in this chapter seems to successfully combat the five principal arguments which have been advanced against the disability provision, although from his analysis he finds that its immediate motive with most insurance companies was to furnish a talking point to increase the saleability of policies. After in the first part establishing its right to exist, Dr. Mudgett then proceeds in the six chapters of the second part to deal exhaustively in a critical way with the restrictions, privileges and benefits as found in the provisions of the various companies. Dr. Mudgett demonstrates that with many companies the disability provision has been used as a talking point without having in mind affording the insured the proper protection to which he is entitled without prejudice to the interests or safety of the company. Dr. Mudgett contends that the benefit as granted by some companies is a travesty upon life insurance but highly commends that granted by a large proportion of the companies. Although he expresses a realization of the necessity of conservatism on the part of the companies owing to the lack of a well-established statistical basis, he set forth three fundament criticisms against a large portion of the disability clauses offered, namely, the definition of disability, the benefit given and the restrictions under which the clause is granted.

The officers of life insurance companies, as well as all students of insurance, will be amply repaid by reading Dr. Mudgett's book which seems to confirm the idea that the disability provision has established a permanent place in American life insurance.

The following letter is recommended by the Council as offering fruitful suggestions of subjects for future papers.

STATE OF NEW YORK.

INSURANCE DEPARTMENT.

NEW YORK, APRIL 9, 1915.

THE ACTUARIAL SOCIETY OF AMERICA,
346 Broadway,
New York City.

Dear Sirs: The New York Insurance Department recently proposed to the Legislature an amendment to Section 97 of the Insurance Law in relation to the limitation of total insurance expenses. Section 97 as originally enacted provided for a limitation upon total insurance expenses equivalent to the total loadings upon premiums collected in any calendar year, plus the assumed mortality gains for the first five years of insurance. The department measure sought to establish a new test in addition to that of total loadings, by providing that either such total loadings, or 25 per cent. of the net premiums—whichever might be the greater—should, in addition to the assumed mortality gains, measure the total expenses which might be incurred by a mutual company. The purpose of this amendment, as is well known, was to afford relief to certain companies transacting business in this state which have mutualized, and hence become subject to the statutory expense limitation.

Depending largely upon the principle enunciated in the report of the Armstrong Committee with respect to the inadvisability of enacting legislation which would require any company to charge higher premium rates, the department believed that the mutualized companies should be permitted to continue their existing rates on the new business transacted, notwithstanding that such rates primarily were calculated for non-participating business. It was felt that these companies, on their own records, should continue to have the privilege of offering insurance at present rates—which are undoubtedly adequate for their own needs—without statutory restriction of a more or less arbitrary character.

The officials, among them several actuaries, of certain mutual companies which charge high gross premiums, urged the unwisdom of the proposed amendment mainly on the ground that an important principle was involved—namely, whether a mutual company should be permitted to draw upon its gains from mortality, inter-

est and miscellaneous sources for the purpose of defraying insurance expenses. In this, however, there seems to be involved the question whether a mutual company should not be required to defray its insurance expenses entirely out of its loadings, without resort to other funds, such as the assumed mortality gains now permitted by the statute.

Another important principle which came under discussion developed an opinion expressed by some actuaries that a company should have the right to base its premiums upon its own mortality experience, or upon some other mortality experience approved by proper authority.

From what has been said on both sides, it would seem that at least three very important questions are deserving of the fullest possible actuarial discussion:

First: Should any mutual company be limited as to total insurance expenses in such a manner that it must increase its premiums in order to comply with the limitation, when its own experience demonstrates that it can safely proceed without increasing its rates?

Second: Should mutual companies be required by statute to charge loadings upon their net premiums which will be adequate to cover all insurance expenses?

Third: Should a new mortality table be recognized by statute for the calculation of premiums and reserves?

In view of the widespread interest aroused by the proposed amendment, and of its evident importance from the actuarial standpoint, it is hoped that the members of your Society will be moved to a serious consideration thereof, to the end that those who are vitally concerned—insurance companies, the legislature of this state, and the public generally—may find available, when these questions are again presented for determination, a fund of information which can be relied upon as the best expert testimony upon the subject.

Respectfully yours,

FRANK HASBROUCK,
Superintendent of Insurance.

JESSE J. BARKER.

Jesse J. Barker, a charter member of the Actuarial Society of America, died in Philadelphia on May 20, 1915.

He was born in Western Pennsylvania in 1849. He was educated at the Philadelphia High School and at Waynesburg College, Pa. After graduation he studied law and was admitted to the Philadelphia Bar, but soon turned to actuarial pursuits and was assistant actuary of the Insurance Department of Pennsylvania for some years. He became connected with the Penn Mutual Life Insurance Company in the middle 70's, was elected its assistant actuary on August 1, 1879, and actuary in January, 1880, serving actively in this capacity until January, 1906, when he resigned because of ill health, and was subsequently made consulting actuary.

Mr. Barker was a man of strong, vigorous but gracious personality; an indefatigable worker and devoted to his chosen calling.

He took a keen and active interest in the affairs of the Acutarial Society from its inception, and will be remembered with appreciation, esteem and affection by all the members who had the privilege of knowing him.

JAMES CAMP CRAWFORD.

On January 8, 1915, occurred the death of James Camp Crawford, associate actuary of the Northwestern Mutual Life Insurance Company. Mr. Crawford was born in Herrick, Penn., January 15, 1844. Educated in the public schools of Herrick and the Collegiate Institute of Towanda, he was graduated from Lafayette College in 1871, receiving the degree of M.A. in 1874. Until 1888 Mr. Crawford was engaged in educational work as principal of high schools and school superintendent, both in Pennsylvania and in Wisconsin. In 1888, however, he made an agency contract with the Northwestern Mutual Life Insurance Company; on July 8, 1892, he was appointed assistant actuary of that company, and in 1905 received the title of associate actuary, which position he held until his death.

Mr. Crawford was a Civil War veteran, having served in the 141st Pennsylvania Volunteers, and his experience with men in the various activities which engaged his attention in his earlier years gave him peculiar qualities for the successful prosecution of his profession.

Mr. Crawford was elected a member of the Actuarial Society of America in October, 1892, and has been a constant attendant at its meetings, a loyal member in undertaking its responsibilities, and a delightful companion to all his brother members. His loss will be keenly felt not only in his own office but by the many friends he had made in his profession.

MORRIS FOX.

Morris Fox, Fellow of the Actuarial Society of America and Associate of the Institute of Actuaries of Great Britain, the actuary of the Government Life Insurance Department, Wellington, New Zealand, died on February 5, 1915.

Mr. Fox was born in Leicestershire, England, on May 24, 1858. In 1880 he went to New Zealand and entered the Government Insurance Department as an actuarial clerk. In 1888 he became Assistant Actuary, and two years later, on the retirement of Mr. F. W. Frankland, he was appointed to the position of Actuary, which position he held to the date of his death. In 1886 he represented the department at a conference of actuaries in London. He was the Secretary to the Royal Commission on Federation in 1901, and in 1903 he was the delegate of the New Zealand government to the Fourth International Congress of Actuaries in New York.

It was on this visit to New York that many of us had the opportunity of becoming acquainted with Mr. Fox and of appreciating the personality of one whom we had already become acquainted with as a member of the profession.

During the later years of his life as government actuary he was called upon to advise his government on many lines outside of pure life insurance.

In the death of Mr. Fox the country of his adoption has lost a talented officer, and the actuarial societies of Great Britain and America a valued member.

HENRY WILLIAM MANLY.

By the death of Mr. Henry W. Manly on December 29, 1914, the actuarial profession throughout the world has lost one of its commanding figures, and many of us have lost a very dear friend.

Mr. Manly was born on April 12, 1844, and was educated at the City of London School, where his father was a master.

His business career commenced in 1859, when, a lad of fifteen years, he became a clerk in the London Discount Company. About a year later he entered the insurance business, taking a position in the London and Provincial Life Assurance Society. In 1870 he entered the services of the Mutual Life Assurance Company and in 1875 he was appointed its Assistant Actuary. One year later he became its Actuary and chief officer, and he remained with this company until 1893, when he was appointed Actuary of the old Equitable Life Assurance Society. He retired from the Equitable in 1905 because of ill health, but happily, after one or two years of ill health, he recovered sufficiently to be able to pursue the practice of consulting actuary.

Shortly after joining the London and Provincial Society he became attached to the Institute of Actuaries, and passed his second and third examinations in 1862 and 1865, in both cases taking first place in his examinations. It was not until 1870, however, that he was transferred to the class of Fellow. In 1868 he had made a name for himself as an actuary by winning the Messenger Prize Essay, the subject of his paper being "A Comparison of the Values of Policies as Found by Means of the Various Tables of Mortality and the Different Methods of Valuation in Use Among Actuaries." He was always a popular member of the actuarial fraternity, and filled all the positions of honor in the Institute, ultimately becoming its president in 1898.

Mr. Manly was the first to publish any serious essay on the subject of staff pensions and much of the work of his later years was on this subject. His papers which appeared in the *Journal* of the Institute of Actuaries are invaluable to the student of pension funds. To Mr. Manly belongs the credit in so far as the public is concerned, of practically creating the mathematics of staff pension funds.

He was elected a member of the Actuarial Society of America in 1895. He regularly attended the international actuarial congresses, and it was during his visit to New York in 1903 that many of us made his acquaintance. His geniality and unostentatious manner endeared him to all who attended this congress, and in his death, many of us have lost a friend.

Abstract from the Minutes of the Annual Meeting of the Actuarial Society of America, Held in New York City on Thursday and Friday, May 20 and 21, 1915.

Hotel Astor, New York City,
May 20, 1915.

FIRST DAY.

The meeting was called to order by the President, Mr. J. M. Craig, at 11 o'clock A. M. The following members were present:

FELLOWS.

Alsop,
Blehl,
Brinkerhoff,
Cammack,
Carpenter,
Cole,
Cooper,
Coburn,
Craig, J. D.,
Craig, J. M.,
Dow,
Fackler, D. P.,
Fackler, E. B.,
File,
Fitzgerald,
Flynn,
Gaylord,
Gore, J. K.,
Graham, W. J.,
Grow,
Hall, S. S.,
Hallman,
Hardcastle,
Henderson,
Hunter, A.,
Hunter, R. G.,
Hutcheson,
Jackson,
Joffe,
Kilgour,
Kime,
Laird,
Linton,
Linzmeyer,
Little,
Macaulay,
Macfarlane,
Maclean, A. T.,
Marshall, E. W.,
Marshall, E. P.,
Mead,
Moir,
Morris,
Murphy,
Papps,
Perrin,
Pipe,
Portch,
Reiter,
Rhodes,
Rose,
St. John,
Sanders,
Salter,
Sheppard,
Stilwell,
Strong,
Thompson,
Torrey,
Walker,
Watt,
Welch,
Wood, A. B.,
Wood, W. A.,
Woodward, J. H.,
Young.

ASSOCIATES.

Allison,
Allstrom,
Angell,
Bain,
Bliss,
Breiby,
Brown,
Bulkley,
Cameron, W. J.,
Davenport, J. S.,
Fairlie,
Flanigan,

FORSTER,	KING,	STANLEY,
FRANKS,	MATHESON,	STRONG,
GIBB,	MILLIGAN,	TEBBETTS,
GOULD,	OLIFIERS,	THOMAS,
GRAHAM,	PARKER,	WASHBURN,
HOLMES,	PHILLIPS,	WASHBURNE,
HOMANS,	RYAN,	WHITNEY,
HOWE,	RYDGREN,	WILLIAMS,
HUGHES,	SMITH, V. R.,	WOLFENDEN.
KAUFMAN,	SPEERS,	

After the calling of the roll the President read his address to the Society.

The Minutes of the semi-annual meeting held in Milwaukee October, 1914, were approved as printed in the *Transactions*.

The Secretary reported that the Council had been considering the subject of a new mortality table and had on April 8 appointed a committee of seven to consider the question and report thereon at the May meeting; that the committee had reported but, that in order to have the advantage of the discussion on Mr. Moir's paper, the Council had postponed final action on the report until Friday morning. He also reported that the Council had appropriated the sum of $150 to be expended during the current year in the purchase of further new books for the library.

The report of the Treasurer was then read and on motion approved.

The Society then proceeded to the election of officers and members of council with the following results:

President	JAMES M. CRAIG.
Vice-Presidents	WILLIAM A. HUTCHESON, HENRY MOIR.
Secretary	ROBERT HENDERSON.
Treasurer	DAVID G. ALSOP.
Editor of the Transactions..	WENDELL M. STRONG.
Members of Council for three years	HERBERT B. DOW, BENEDICT D. FLYNN, DAVID E. KILGOUR.

The reading of Mr. Moir's paper was then proceeded with and at its conclusion the Society adjourned.

AFTERNOON SESSION.

When the Society reassembled at 2 P. M., the discussion on Mr. Moir's paper was proceeded with and at the conclusion of that discussion the reading of the other papers prepared for this meeting was taken up.

At 5 o'clock the Society adjourned to meet at dinner at 7 and reassemble for business on the following morning at 10:30 A. M.

SECOND DAY.

FRIDAY, *May* 21, 1915.

The meeting was called to order by the Vice-President, Mr. Moir.

The Secretary reported that the Council had given further considerations to the report of the special committee on the subject of a new mortality table and adopted the following resolution:

Resolved: That the Council is of the opinion that a new Mortality Table or Tables should be prepared under the direction of the Actuarial Society; that the Committee of seven be continued for the purpose of conferring with the proper Committee of the National Convention of Insurance Commissioners and with the companies whose statistics may be desired; that, if the way be clear, the Committee proceed with the work of preparing such new table or tables and that the Committee report the result of its conclusions to the Council.

Also that the Council had considered a letter received from the Superintendent of Insurance of New York State and had ordered it printed in the Transactions as a suggestion to the members of subjects suitable for papers.

Minutes were then read regarding the deaths of Messrs. Crawford, Fox and Manly.

The discussion of papers presented at the fall meeting was then proceeded with.

At one o'clock the Society finally adjourned.

Examinations of the Society

Held on May 26 and 27, 1915.

Examination Committee.

Chairman, - - - - ARTHUR B. WOOD.
Vice-Chairmen, - - - { PERCY C. H. PAPPS. JOSEPH H. WOODWARD.

J. J. COOPER.
JAMES D. CRAIG.
B. D. FLYNN.
J. M. LAIRD.
M. A. LINTON.
J. B. McKECHNIE.
A. T. MACLEAN.
RAY D. MURPHY.
JOHN S. THOMPSON.

EXAMINATION FOR ENROLMENT AS ASSOCIATE.

Section A—Part I.

1. A, B and C, whose rates of walking are $3\frac{1}{2}$, 4 and 5 miles an hour respectively, walk on circular tracks, the circumferences of which are $\frac{1}{5}$, $\frac{1}{4}$ and $\frac{3}{8}$ of a mile respectively, and the centers of which are on the same straight line. At the same instant they start from points on this line. Find (*a*) when they will all be on the line at the same time, and (*b*) when they will all be at the same time at the points from which they started.

2. (*a*) A crew can row a certain course up stream in $8\frac{4}{7}$ minutes, and if there were no current they could row it in 7 minutes less than it takes them to drift down the stream; how long would they take to row down with the stream?

(*b*) Find the cube root of $1.\dot{2}$ to three decimal places.

3. Solve the equations:—

(*a*) $\frac{x}{a}+\frac{a}{x}=\frac{y}{b}+\frac{b}{y}=\frac{y}{x}+\frac{x}{y}$.

(*b*) $(x^2+7x+5)^2-3x^2-21x=19$.

(*c*) $\frac{x^2}{y}+\frac{y^2}{x}=4$.

$\frac{1}{x}+\frac{1}{y}=1$.

4. If $(ax^2+bx+c)\,y+a'x^2+b'x+c'=0$, express in simplest terms the condition that x may be a rational function of y.

5. (*a*) Show that if a, b, c be in Arithmetical Progression, and a, c, b be in Geometrical Progression; then will b, a, c be in Harmonical Progression.

(*b*) Simplify: (1) $\dfrac{3}{\sqrt{5}}-\sqrt{2}+\sqrt{7+2\sqrt{10}}.$

(2) $\dfrac{(1+\sqrt{-1})^2}{2\sqrt{-1}-1}$

6. (*a*) Prove that the perpendiculars from the vertices of a triangle upon the opposite sides meet in a point.

(*b*) Describe a circle of a given radius tangent to a given circle and passing through a given point.

7. Prove that in any quadrilateral the sum of the squares of the sides is equal to the sum of the squares of the diagonals, increased by four times the square of the line joining the middle points of the diagonals.

8. (*a*) Given $\tan\dfrac{A}{2}=2-\sqrt{3}$. Find $\sin A$.

(*b*) Solve for x the equation

$$\sin\frac{n+1}{2}x+\sin\frac{n-1}{2}x=\sin x.$$

9. (*a*) Having given two sides and the included angle of a triangle, how would you determine the other side and angles? How would you check your work?

(*b*) Find $\log_{125}1800$.

Given $\log_{10}2=.30103$.

$\log_{10}3=.47712$.

10. (*a*) A point moves so that the line connecting it with the origin makes an angle with the X-axis whose tangent is one half the tangent of the angle made with the X-axis by the line connecting the moving point with a fixed point. Find the equation of the locus of the moving point.

(*b*) Find the condition that the line $y=mx+c$ may be tangent to the parabola $y^2=4ax$.

11. Find the equation of the circle whose diameter is the common chord of the circles

$$x^2+y^2+2x+3y+1=0$$
$$\text{and } x^2+y^2+4x+3y+2=0.$$

12. Show that the equation $Ax^2+2Hxy+By^2+C=0$ represents a conic section whose axes are inclined at an angle a to the given coordinate axes, where $a=\frac{1}{2}\tan^{-1}\dfrac{2H}{A-B}$.

13. Show the entries to be made on the books of a new company for the following transactions:—

(*a*) $10,000 par value of the company's stock is sold for $11,000 in cash.

(*b*) $1,000 is paid for merchandise.

(*c*) Half the above merchandise is sold to A on credit for $600.

(*d*) Wages of $50 are paid.

(*e*) A pays his bill of $600 less 5% discount for prompt payment.

14. What are the relations between Real, Personal and Nominal Accounts, and what is the relation of each to the Balance Sheet and Profit and Loss account of a trading company? What is the function of an inventory and how are its results introduced into the accounts of the company?

SECTION A—PART II.

1. If p things be distributed at random among p people, find the chance that at least one of the persons gets nothing. Find the chance that exactly three persons get nothing.

2. (*a*) A writes to B and does not receive an answer. Assuming that one letter in n is lost in the post, find the chance that B received the letter, it being considered certain that B would have answered the letter had he received it.

(*b*) How many odd numbers taken at random must be multiplied together that there may be at least an even chance that the last figure of the product will be 5?

Given $\log_{10}2=.30103$.

3. A coin is tossed $m+n$ times $(m>n)$; prove that the chance of at least m consecutive heads appearing is $\frac{n+2}{2^{m+1}}$.

4. (*a*) Show that the number of combinations of $3n$ things taken n together when exactly n are alike is $2^{2n-1}+\frac{\underline{|2n}}{2(\underline{|n})^2}$.

(*b*) In how many different ways can $2n$ persons be seated at two round tables, n persons being seated at each?

5. (*a*) Prove the Binomial Theorem for any positive fractional index.

(*b*) If $c_0, c_1, c_2, \ldots\ldots c_n$ be the coefficients of the terms of the expansion $(1+x)^n$, show that

$$c_0-\frac{c_1}{2}+\frac{c_2}{3}-\ldots\ldots\frac{(-1)^n c_n}{n+1}=\frac{1}{n+1}.$$

6. (*a*) Show that

$$\log_e(n+1)=\log_e n+2\left\{\frac{1}{2n+1}+\frac{1}{3(2n+1)^3}+\frac{1}{5(2n+1)^5}+\ldots\ldots\right\}$$

(*b*) Show that

$$\frac{e^2+1}{e^2-1}=\frac{1+\frac{1}{\underline{|2}}+\frac{1}{\underline{|4}}+\frac{1}{\underline{|6}}+\ldots\ldots}{1+\frac{1}{\underline{|3}}+\frac{1}{\underline{|5}}+\frac{1}{\underline{|7}}+\ldots\ldots}$$

7. Test for convergence the following series:—

(*a*) $\left(\frac{3^2}{2^2}-\frac{3}{2}\right)^{-2}+\left(\frac{4^3}{3^3}-\frac{4}{3}\right)^{-3}+\left(\frac{5^4}{4^4}-\frac{5}{4}\right)^{-4}+\ldots\ldots$

(*b*) $\frac{3}{4}+\frac{x}{2}+\frac{x^2}{12}+\ldots\ldots+\frac{2n-5}{n^3-5n}x^{n-1}+\ldots\ldots$

8. Sum the following series:—

(*a*) $(1^2+1)\,\underline{|1}+(2^2+1)\,\underline{|2}+\ldots\ldots+(n^2+1)\,\underline{|n}.$

(*b*) $1.2.3+3.4.5x+5.6.7x^2+7.8.9x^3+\ldots\ldots$ when $x<1$.

9. (*a*) If δu_0, $\delta^2 u_0$, etc., be the differences of u_0, $u_{\frac{1}{2}}$, $u_{\frac{2}{2}}$, etc., and Δu_0, $\Delta^2 u_0$, etc., be the differences of u_0, u_1, u_2, etc., find the values of δu_0, $\delta^2 u_0$, $\delta^3 u_0$, in terms of Δu_0, $\Delta^2 u_0$, $\Delta^3 u_0$, assuming third differences constant.

(*b*) If fourth differences are constant, prove that

$$\frac{d^3 u_x}{d_x^{\,3}} = \Delta^3 u_{x-\frac{3}{2}}$$

10. (*a*) State Lagrange's interpolation formula and employ it to determine from three observations of a quantity taken near its time of maximum or minimum, the time of the maximum or minimum.

(*b*) In the series 1, *a*, 49, *b*, 313, *c*, 985, find by interpolation the values of *a*, *b* and *c*.

11. (*a*) Prove $\Delta^m 0^n = m(\Delta^{m-1} 0^{n-1} + \Delta^m 0^{n-1})$.

(*b*) Using the method of Finite Differences, sum the first 10 terms of the series whose general term is n^3+2n^2+3n.

12. (*a*) The first differences of the first differential coefficient of $\log f(x)$ are in geometrical progression. Determine the form of $f(x)$.

(*b*) If $\varphi(y)$ be a continuous function, show that the first derivative with regard to x of $\int_0^x \varphi(y)\,dy$ is $\varphi(x)$.

13. (*a*) Find the first differential coefficient of $(x)^{x^n} + \dfrac{x^{2n}}{(1+x^2)^n}$

(*b*) If $f(x)$ and its derivatives be continuous in the vicinity of $x=a$, obtain its expansion in powers of $(x-a)$.

(*c*) Find the limit of $\dfrac{a^n-1}{n}$ as n approaches zero.

14. (*a*) Evaluate

$$\int \sqrt{\frac{x+a}{x+b}}\,dx$$

$$\int_a^b \int_b^a \frac{xy}{\sqrt{x^2+y^2}}\,dy\,dx.$$

(*b*) Show that the process of integration is equivalent to summation.

SECTION B—PART I.

1. (*a*) Define:—

Discount, Nominal Rate of Discount, Effective Rate of Discount, Force of Discount.

(*b*) Prove $i = d + \delta^2$ approximately.

(*c*) In how many years will a sum of money treble itself at 8%?

Given Log. 2 = .3010.
Log. 3 = .4771.

2. (*a*) Prove by general reasoning: $a_{\overline{n}|}^{(p)} = \dfrac{1-v^n}{p\{(1+i)^{1/p}-1\}}$

What proportion of the periodic payment must be paid for $\dfrac{1}{mp}$ of a year in order that the above formula may hold when for n we substitute $n + \dfrac{1}{mp}$?

(*b*) On August 1, 1914, a Company borrows $10,000 at 6% (3% each half year), repayable in four equal semi-annual instalments, including capital and interest. Capital repayments, however, can earn only 5% and the Company has agreed that the lender shall receive 6% on the full $10,000 until the entire loan is repaid. Given $a_{\overline{4}|} = 3.762$ at $2\frac{1}{2}$% interest, what will be accrued capital payment and accrued interest payments on December 31st, 1915?

3. An issue of bonds of the nominal value of $100,000 is made on January 1, 1915. The bonds bear interest at 4% per annum payable half-yearly and are redeemable by drawings as follows:—

$10,000 on January 1, 1916 at 101
$10,000 on January 1, 1917 at 102
. . . . . .
$10,000 on January 1, 1925 at 110

Given $v^{20} = .61027$ at $2\frac{1}{2}$%, what must be the issue price to yield 5% compounded half-yearly?

4. (*a*) Prove that $\dfrac{1}{n}\left[1-(p_x)^n\right]$ represents the probability that out of n lives all aged x, one designated life A will die in the year and be the first to die.

(*b*) Find the probability that in the 10th year from the present time a life now age 30 will die, having survived a life now age 28 by at least 2 years and a life now age 26 by at least 4 years.

Given $l_{26}=8831$
$l_{28}=8688$
$l_{30}=8544$
$l_{40}=7811$
$d_{30}=72$
$d_{31}=72$
$d_{32}=72$
$d_{33}=73$
$d_{34}=73$
$d_{35}=73$
$d_{36}=74$
$d_{37}=74$
$d_{38}=75$

5. Calculate the value of the expression $e_{30:\overline{40}\,(\overline{10|})}$ assuming De Moivre's Law.

6. (*a*) Derive an expression for Q^1_{xyz} in terms of the probability of living and expectation of life.

(*b*) Out of l_x married couples, the husbands being all aged x and the wives aged y, find how many husbands will become widowers in the nth year.

7. (*a*) Give a formula for the net annual premium for an annuity of $100 a year payable until the death of the last survivor of a male now age 60 and a male now age 55, but deferred for 10 years, and state under what conditions you would issue the contract.

(*b*) The annual premium for a Whole Life Policy of unity is .04 for 5 years and .02 thereafter, or .0475 for 5 years and .0175 thereafter. Find the annual premium for an Endowment insurance payable at the end of 5 years or previous death in terms of ${}_5P_x$ and d.

8. (*a*) If a population consists of l_x persons of age x, l_{x+1} of age $x+1$, and so on down to the extremity of life, show that the total fund to be raised to provide an insurance of unity on the death of each person is $l_x(a_x+A_x)$.

(*b*) A loan of $a_{\overline{n}|}$ to a person aged x is to be repaid by equal annual amounts of $1 each, including principal and interest. Assuming the same rate of interest throughout, prove that the single premium for an insurance to secure the amount outstanding should x die during the n years is $a_{\overline{n}|}-a_{x\overline{n}|}$.

9. (*a*) Explain the rationale of the Approximate Formula

$$P_{xy\overline{n}|} = P_{x\overline{n}|} + P_{y\overline{n}|} - P_{\overline{n}|}$$

What is the error introduced by the use of this approximation?

(*b*) Transform the formulæ

$$1 = ia_x + (1+i)A_x$$

and

$$A_x = 1 - d(1+a_x)$$

from yearly to $m$thly intervals and give verbal interpretation of each result.

10. (*a*) A person aged x wishes to take, subject to a present medical examination only, n Whole Life policies. One is to be issued immediately, another one year from date, another two years from date and so on for $(n-1)$ years. Prove that the value of the Single Premium to provide this option is (assuming "Selection" to last for n years).

$$\frac{1}{D_{[x]}}\left[R_{[x]} - R_{[x]+n}\right] - \frac{1}{D_{[x]}}\left[\Sigma_{n=0}^{n=n-1} \frac{M_{[x+n]}}{N_{[x+n]}} N_{[x]+n}\right]$$

(*b*) Prove that the single premium for an assurance payable on a life now aged x years attaining the age of $x+t$ or dying previously may be expressed by the formula

$$\frac{1-i\,|_{t-1}\,a_x}{1+i}$$

and show that this is equal to

$$\frac{M_x - M_{x+t} + D_{x+t}}{D_x}$$

11. (*a*) Derive a formula for the annual premium for a policy to provide an annual Life Income to a child now age 4, income to commence either 20 years after the father's death, if that should occur within 20 years from the present date, or at the end of 40 years. Father is now age 25 and premiums are to cease at the first death or after 20 premium payments have been made.

(*b*) Find in commutation column form the annual premium for a Pure Endowment to x payable at age $(x+n)$; the premium to be payable only so long as another life aged y is alive jointly with (x) and to be returnable if (x) should die within the n years.

12. (*a*) How would you apply Simpson's Rule to calculate $\pi_{\overline{xy}:z}$?

(*b*) State with what function it would be necessary to enter the Conversion Table to derive the value of $\pi_{\overline{wxyz}}^{3}$ and express this function in terms of similar functions, each terminating at the first death.

(*c*) Express $A_{\overline{xy}:\underset{3}{z}}$ in terms of Single premiums for insurances which terminate at the first death.

13. (*a*) Prove: $A_x > v^{1+e_x}$

(*b*) Prove the proportion:—

$$(a_x - a_{xy}) : (A_y - A_{\overline{xy}}) :: (1+i) : i$$

14. Endowment assurances for n years are effected by l_x lives aged x. Prove that the amount payable to the survivors is equal to the accumulated premiums paid, less the difference between the accumulated claims and premiums, in respect of those who died before reaching $x+n$.

15. (*a*) A certain State makes it compulsory that all men of age 26 shall serve one year in the regular Army and two years in the Reserve. What average annual increase in the birth rate must take place so that 30 years hence in place of an army of 10,000 (maintained if the present birth rate be continued stationary) there shall be an army of 20,736, and how many would be in the Reserve at that time?

Given $T_{27} = 380865$, $T_{29} = 361072$, $p_{27} = .99396$, $d_{27} = 40$.

(*b*) By what should $\dfrac{2Y_0}{l_0}$ be divided to obtain an expression for the average age at death of the present members of a community?

16. Prepare a schedule for the construction of $M_{\overline{x+n}:\overline{y+n}}^{1}$ columns.

SECTION B—PART II.

1. (*a*) How would you explain to a jury the expression "Reserve" as used in connection with a Life Insurance Policy?

(*b*) Define "Initial Reserve" and state when it is greater than, equal to or less than the Terminal Reserve.

2. (*a*) If .01 is added to P_x at all ages and a new mortality table obtained, prove that the reserves on a Whole Life contract are less under the new table than under the original.

(*b*) Prove that two tables will give equal values on Whole Life policies when

$$p'_x = p_x\left(1 - \frac{k}{a_x}\right)$$

3. (*a*) Under a Whole Life policy taken out at age x which has been n years in force, the sum assured and bonuses amount to S. Prove that the value of the policy is equal to

$$\left(S + P_x + \frac{P_x}{i}\right) A_{x+n} - \left(P_x + \frac{P_x}{i}\right)$$

(*b*) Prove

$$A_{x+n+\frac{t}{m}} - P_x^{(m)}\left(\frac{m+1}{2m} + a_{x+n+\frac{t}{m}}\right) =$$

$$\left\{ {}_nV_x + \frac{t}{m}\left({}_{n+1}V_x - {}_nV_x\right)\right\}\left\{1 + \frac{m-1}{2m} P_x^{(m)}\right\}$$

and state how the practice usually adopted in regard to Industrial policies varies from the procedure indicated by the right hand member of this equation.

4. (*a*) Prove that $\bar{A}_x = 1 - \delta \bar{a}_x$.

(*b*) Prove that $l_x \bar{A}_x = -\frac{d}{d_x}(l_x \bar{a}_x)$.

5. (*a*) Prove by the use of Woolhouse's Formula of Approximate Summation that

$$\mathring{e}_x = e_x + \tfrac{1}{2} - \tfrac{1}{12}(\mu_x).$$

(*b*) Demonstrate the identity of Lubbock's Formula with Woolhouse's Approximate Summation Formula as applied to a function of the third degree.

6. If Makeham's Law holds, show that $\bar{A}^{\,1}_{xyz} = \dfrac{\log_e s}{3 \log_e s - \delta}$ where x, y and z are such that

$$\frac{\delta}{\log_e s} = \frac{2\mu_x - \mu_y - \mu_z}{\mu_x + \log_e s}.$$

7. (*a*) Without making any assumption as to the distribution of deaths at the successive ages in a mortality table, express L_x in terms of l_x. Hence if deaths are uniformly distributed, show $L_x = l_{x+\frac{1}{2}}$.

(*b*) Define and evaluate $\bar{a}_{x\,y\,z\,|\,w}^{\;2:3}$.

8. If the force of interest declines continuously in such a way that after time t its value is δr^t, prove that an assurance payable at death of (x), if he dies before (y), can be expressed in the form $K\bar{A}^{\,1}_{xy}$ where $\bar{A}^{\,1}_{xy}$ is calculated at a special constant rate of interest.

9. (*a*) Prove algebraically that

$$({}_nV_x + P_x)(1+i) = q_{x+n} + p_{x+n} \cdot {}_{n+1}V_x.$$

and demonstrate verbally the "accumulation formula" giving in detail the functions which should be calculated in advance.

(*b*) Give five conditions which must hold in order that the mean net level premium reserve may correctly represent a company's liability on December 31st of any year on its Non-Participating Policies.

10. A policy for a term of n years is granted at an annual premium to a life aged x, with the option of continuance at the end of the term as an endowment assurance, maturing at age $(x+n+t)$, on payment of the normal yearly premium at age $(x+n)$. Find expressions for the value of the policy on a select basis–

(*a*) At the end of the $(n-1)$th year.

(*b*) At the end of the $(n+1)$th year, assuming the option to have been exercised.

11. Show that:

$$P_{x\overline{n}|} + d_t V_{x\overline{n}|} = P^{\;1}_{\overline{x+t}:\overline{1}|}\,(1 - {}_tV_{x\overline{n}|}) + vp_{x+t}\,({}_{t+1}V_{x\overline{n}|} - {}_tV_{x\overline{n}|})$$

and give a verbal interpretation.

12. A Reversionary Annuity payable to (x) after the death of (y) with annual premiums payable during the joint lives is valued by the Makehamized American Experience Table. Show that the reserve at the end of the first year, both (x) and (y) being alive, will be positive if

$$-\Delta \log (1+a_{ww}) > -\Delta \log (1+a_x) \text{ when } 2\,\mu_w = \mu_x + \mu_y$$

13. (*a*) Define a_x^a and prove that it is equal to

$$\frac{D_x}{D_x^{aa}}(a_x - a_x^i) + a_x^i$$

(*b*) How would you prepare the commutation columns for use in the formula

$${}^iA_{x:\overline{y-x}|} = \frac{{}^rM_x^{ai} - {}^rM_y^{ai}}{D_x^{aa}}$$

and how would you find values for ${}^{ra}M_x^{ai}$?

(*c*) Give three methods which have been suggested for determining the extra reserve for the "Waiver of Premium" benefit after premium has been waived.

14. (*a*) In what important respects do the development and present condition of life insurance in the United States and Canada differ from those in England?

(*b*) Discuss briefly the more important changes in the New York Insurance Law that have been made since 1905.

15. (*a*) Draft a form of endorsement providing for the reduction of a policy and the conversion of the portion terminated into a guaranteed amount of paid-up non-participating insurance.

(*b*) Draft a war clause or rider providing an extra premium for military service outside the country in which the policy is issued.

16. (*a*) How does a policy of Life Insurance compare with a policy of Fire Insurance as a contract of indemnity and what are the various conditions precedent to the issuance of a Life policy?

(*b*) What principles would you adopt in drafting a policy of Life Insurance to insure under a single contract a large group of people?

EXAMINATION FOR ADMISSION AS FELLOW.

PART I.

1. (*a*) Define "Mean Population" as used in connection with population statistics and show how it may be derived for any given age.

(*b*) How may rates of mortality at ages under 5 years be obtained from census statistics of births and deaths alone, illustrating your statements from the following actual data?

Calendar Year.	Births.	Deaths.					
		0–6 mos.	6 mos. 1 year.	1–2 years.	2–3 years.	3–4 years.	4–5 years.
1900	47104	5843	2103	1942	713	449	337
1901	47394	5772	2077	1752	633	415	310
1902	47914	5172	1861	1685	651	421	309
1903	48223	5127	1845	1659	590	359	258
1904	48132	5617	2021	1878	650	382	275
1905	47289	4910	1767	1576	605	370	263

2. With reference to the question of a new mortality table for use in the United States and Canada,

(*a*) Do you consider such a table desirable? Discuss as fully as possible.

(*b*) Upon what data should such a table be based?

(*c*) How would you suppose that such a table, would compare with existing standard tables with special reference to (*i*) net premiums and (*ii*) reserves?

3. How would you proceed to test the effect of selection in a life office experience? Can apparent selection be evidenced simply through statistical treatment of data which otherwise displays no trace of selection? Explain. What, therefore, are the essential characteristics of data on which a true select table is to be based?

4. The mortality experience of a company is exhibited in the form of aggregate tables showing respectively the mortality under life, endowment and term policies. In comparing the rates of mortality shown in the three tables, what considerations are to be kept in mind? What are the advantages and disadvantages of aggregate tables?

5. (*a*) What special difficulties presented themselves in the graduation of the British Offices' Select Annuity Tables and how were they overcome? In the published volume of adjusted data, what facilities are offered for the calculation of joint annuities on two lives, one of which is male and one female, the interest basis being 5%?

(*b*) Give an account of any ONE of the following Mortality Tables:—

(*i*) The Three Japanese Offices' Experience, 1910.

(*ii*) National Fraternal Congress Table.

(*iii*) $O^{[NM]}$ Table.

(*iv*) English Life Tables, Nos. 7 and 8.

6. With reference to the summation method of graduation,

(*a*) What are its advantages and disadvantages?

(*b*) How would you deal with the ends of the series to be graduated?

(*c*) How would you estimate the error involved in the results arrived at by a specified formula?

(*d*) Discuss the relative suitability of the functions l_x, d_x, q_x, and colog p_x for graduation.

7. How may Makeham's Law be applied to the data furnished by a limited experience (*i*) by means of reference to a standard table, (*ii*) without such reference? What are possible objections to such a graduation? Mention any advantages of graduation (*i*) as compared with graduation (*ii*).

8. Under what conditions would you issue policies to Commissioned Officers of the Army and Navy, (*i*) in time of peace, (*ii*) in time of war (mentioning two methods in case of (*ii*)).

How would you determine the amount of the extra premiums, if any?

9. (*a*) What classes of medically impaired lives do you consider could be taken with (*i*) a constant lien, (*ii*) a decreasing lien? How do the reserves under such policies compare with normal reserves?

(*b*) To what extent and under what circumstances is a company protected by offering to under-average applicants (*i*) a change of plan, (*ii*) a special dividend classification?

10. (*a*) State briefly six salient points to be observed in general in the selection of risks.

(*b*) What information is given by a blood-pressure reading in connection with the physical examination of applicants?

(*c*) Discuss the points mentioned in (*a*) in reference to possible compensation for the following adverse circumstances of risk:—

(*i*) 10% overweight.

(*ii*) Family history of consumption.

(*iii*) Occupation as coal miner.

(*iv*) Residence or occupation in the Panama Canal Zone.

11. Mention two objections to the Full Preliminary Term method of valuation. What are the principal alternatives which have been suggested to meet these objections? Which of these alternatives do you consider most satisfactory, and why?

12. (*a*) Describe briefly any two methods (other than Lidstone's) of valuing endowment assurance policies in groups according to the term to run to maturity. What is a characteristic defect of these methods?

(*b*) With reference to Lidstone's "Z" Method, discuss briefly

(*i*) the rationale of the method,

(*ii*) the effect of basing the average age at valuation on (1) sums insured and (2) office premiums.

(*iii*) the precautions to be observed in practical application.

(*iv*) how far the method is applicable to other than endowment assurance policies.

13. A manufacturing firm has an Insurance Benefit Fund, guaranteeing $1,000.00 Ordinary Life insurance to each of its employees, on account of which each of the employees contributes annually the net premium according to the American Experience Table with 4% interest. For various reasons the Fund is in a state of insolvency. On what terms would you recommend a life company to take over the outstanding insurance, assuming no legal restrictions?

14. What change privileges would you incorporate in Term Insurance policies (*i*) without "Waiver of Premium" benefit on permanent and total disablement, (*ii*) with "Waiver of Premium" benefit. How would you calculate the "cost to change" arising under the various options offered?

15. The holder of an Ordinary Life policy which has been ten years in force advises the Company that he desires his premiums to be reduced by 20% for the next five years and that subsequent premiums be increased uniformly to a sufficient extent to provide for the reduction. The sum insured is to be paid in full on death during the five-year period.

(*i*) Assuming the change be approved, state briefly the basis of your calculations and your requirements;

(*ii*) Draft a letter to the policyholder declining to make the change and explaining your reasons therefor.

16. What bases as to interest, mortality and loading would you adopt for premium rates for the following forms of policies:—

(*i*) Reversionary Annuities, annual premiums.

(*ii*) Deferred Annuities, annual premiums.

(*iii*) Children's Deferred Assurances, annual premiums.

(*iv*) Group Assurances.

(*v*) Immediate Life Annuities, the contract providing for payment to annuitant's estate of the excess, if any, of the purchase-price over the annuity instalments paid.

What options would you allow on surrender or lapse of contracts (*i*), (*ii*) and (*iii*)?

PART II.

1. (*a*) Explain Homan's contribution plan for distribution of surplus. What were the weaknesses of this method? What was Sprague's modification of Homan's plan?

(*b*) Describe the modifications of Sprague's method, proposed by Lidstone. Give your opinion of the value of Lidstone's method.

(*c*) What showing does the "asset share" system make as to the surplus for four or five years after the issuance of the policies? Give the reason therefor.

2. (*a*) Outline briefly the plan of basing deferred dividends upon annual dividend accumulations.

(*b*) Discuss the advisability of the issuing of participating annuities by a life insurance company. If such annuities were issued, what various alternative methods of allotting dividends might be adopted?

3. A large non-participating company decides to place all its business, both new and old, on a participating basis without altering its scale of premiums. Provided the surplus is not to be allowed to drop below its present amount, how would you advise the Directors (1) as to the future dividend policy of the company, and (2) as to the method to be adopted for the apportionment of dividends (*a*) on new business, (*b*) on the old business.

4. (*a*) What misleading deductions are apt to be made in interpreting the gains or losses from various sources as disclosed by the Gain and Loss Exhibit of the Convention Blank?

(*b*) Describe how you would treat the following transactions in preparing the Annual Statement upon the Convention Blank:—

(1) The application of the surrender value of a policy to purchase paid-up insurance (*a*) under a new contract; (*b*) under the non-forfeiture provision of the original contract.

(2) A death claim under a policy payable in instalments.

(3) Repayment of an advance to an agent (*a*) which has not been previously charged off; (*b*) which has been previously charged off.

(4) Income tax withheld by the company from the payroll of its employees (*a*) paid by it to the Federal Government; (*b*) withheld by the Company, but not paid at date of making the statement.

5. Prepare a concise statement showing the items to be used in determining the profit made by a life insurance company during the year in one of its large agencies. State how you would obtain the amounts of any of these items not found in the general and chief subsidiary books of the company.

6. (*a*) Companies operating under the Assurance Companies' Act of Great Britain are required to furnish periodically to the Board of Trade a statement concerning their life assurance and annuity business (Fifth Schedule), and also a statement concerning the valuation of such business (Fourth Schedule). State the main points on which information is required in these two Schedules.

(*b*) Contrast briefly the bases and methods of valuation adopted in the statements referred to in (*a*) with those called for in the Annual Statements to Insurance Departments in the United States and Canada.

7. (*a*) State the various methods in which a life insurance company may let out a small portion of its funds for temporary investment. Comment briefly as to the desirability of each method.

(*b*) Discuss briefly the objections to and the advantages of amortization of bonds by a life insurance company.

8. (*a*) Give in detail the course you would pursue in investigating the value of a

(1) Railroad bond;
(2) Municipal bond.

(*b*) Discuss the desirability of foreign government loans from the standpoint of a life insurance company located in the United States or Canada.

9. State and comment on the main requisites for safety in loaning money on city property.

10. (*a*) Define each of the following terms:—

bimetallism, legal tender, token coin, mint price of gold.

(*b*) Discuss the subject of exchange rates between New York and London with reference to each of the following:—mint par, specie points, fluctuations during a normal year, fluctuations during the past eleven months.

11. (*a*) Explain the operation of the Federal Reserve Banking System, with particular reference to how it is formed, how it is controlled and the relation of member banks to the Federal Reserve Board.

(*b*) What are the advantages of the Federal Reserve Banking System over the old National Banking System?

12. (*a*) Give a definition of "insurable interest" as applied to life insurance and give four examples of such insurable interest as will form a proper basis for a life insurance contract.

(*b*) Discuss the incontestable clause in a life insurance contract as affected by the doctrine of "public policy." What is the trend of judicial decisions on this subject and what are the situations producing this problem?

13. (*a*) Name the standard policy provisions required by the New York State Insurance Law, and explain the actuarial features involved in them.

(*b*) How is an insurance company incorporated under the laws of a Canadian Province affected by the Dominion Insurance Act of 1910?

14. (*a*) State briefly the disadvantages of (1) the contributory system of pensions and (2) the non-contributory system.

(*b*) A large association of school teachers has for more than 25 years maintained an old age pension plan. The pension scale has been 2% of total salary for each year of service, maximum amount of pension 50% of total salary with no pension for less than 10 years of service. There has been no provision for repayment of contributions in case of withdrawal or death of active members.

It is apparent that the Fund is insolvent and it is proposed that a valuation of it be made. Outline the work which must be done in making such a valuation.

15. (*a*) A company proposes to issue a sickness contract which will pay $1 per week as long as sickness continues up to age 65. How would you calculate the one year term single premium at the age 35 for this policy, given the values of l_x, v^x, the rate of sickness, and the force of sickness.

(*b*) State the chief elements which are generally considered responsible for causing a variation from a normal standard of sickness claims in a friendly society.

16. (*a*) What is the "elective" form of Workmen's Compensation Acts? Why was this method of framing an act devised? What is the purpose of the "waiting period" as regards the benefits under these Acts?

(*b*) Discuss briefly the nature and the prospect of:—

(1) Group Life Insurance.

(2) State Health Insurance.

(3) State Old Age Pensions.

(*c*) What essential features are necessary for successfully carrying out a state system of insurance against unemployment?

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TRANSACTIONS

OCTOBER 28TH AND 29TH, 1915.

Address of the President, James M. Craig.

Life Extension.

In James, 3d chapter and 2d verse, we read:

"For in many things we offend all. If any man offend not in word, the same is a perfect man, and able also to bridle the whole body."

This perfect man, who by implication at least has a perfect body, is evidently a thing of the past if we accept as conclusive a report recently issued showing the result of the physical examination of one thousand men.

The report is as follows:

"Analysis of physical examination of industrial workmen, (foremen and skilled workers) in the Ford Motor Company, Detroit.

Employees	1,000
Average age	32.7 years
No physical impairment found and no advice for correction of living habits needed	0 per cent.
Advice needed regarding physical condition or living habits	100 per cent.

A report of this character is not calculated to inspire confidence in the findings of the examiner, for it is inconceivable that among 1,000 skilled workers, all actively engaged in their usual duties, none should be found free from physical impairment. Of course we look upon physical impairment in its relation to life insurance. Any man who is able to pass a satisfactory medical examination is

considered physically a standard risk, and it seems to me that this is the measure by which lives should be classified. It is claimed by many that all the people who pass through a crowded thoroughfare could be safely insured as a group without medical examination, and it is probable that even conservative companies would not hesitate to insure the 1,000 skilled workers mentioned above, as a group, without examination, at regular rates.

When the code of impairments is examined, it is not surprising that a perfect man was not found. If a man is a vegetarian, he bears an imperfect mark. If he drinks one or more cups of tea or coffee daily, he goes into impaired class. If he smokes a cigar occasionally, or a cigarette—not more than one a day—he carries a stain on his body. If he drinks water copiously or sparingly or exercises too much or too little, he goes into the dump heap of impaired lives. A trace of albumen or sugar necessarily condemns him, and if his teeth or gums are not in prime condition, he cannot pass muster.

In this connection we are reminded about the admonition on sleep.

"Don't sleep on your left side, for it causes too great a pressure on the heart.

"Don't sleep on your right side, for it interferes with the respiration of that lung.

"Don't sleep on your stomach, for that interferes with the respiration of both lungs and makes breathing difficult.

"Don't sleep on your back, for this method of getting rest is bad for the nervous system and makes you snore, causing great discomfort to all within hearing distance.

"Don't sleep sitting in a chair, for your body falls into an unnatural position, and you cannot get the necessary relaxation.

"Don't sleep standing up, for you may topple over and crack your skull.

"Don't sleep at all."

Another group accessible for inspection and not subject to such rigid rules, is presented in detail.

In this group there were 3,583 persons, composed of 1,227 males and 2,356 females.

The number of examinations in which one or more impairments were found:

Males	840
Females	1,682
Total	2,522

The number of examinations in which no impairments were found:

Males	387
Females	674
Total	1,061

The number of clerks referred to family physician:

Males	39
Females	33
Total	72

The number of clerks under medical treatment at time of examination:

Males	39
Females	59
Total	98

Referred to dispensary:

Males	62
Females	163
Total	225

Summarizing the things that concern us most, it is found that out of 1,227 males examined, 140, or 11.4 per cent., were found sufficiently impaired to require medical treatment, and out of 2,356 females examined, 255, or 10.8 per cent., were found to require medical treatment. This summary does not include those referred to the eye specialist or dentist.

Most of these clerks were subject to a medical examination before being employed, and the result of these re-examinations may serve as a rough indication of what might be found among a body of insured lives upon re-examination. The average age of persons examined ought to be taken into consideration.

The large number of females classified as underweight is probably due to the imperfection of the standard used, being the same as that used for males.

IMPAIRMENTS RECORDED AS A RESULT OF THE EXAMINATION OF 3,583 CLERICAL EMPLOYEES OF THE METROPOLITAN LIFE INSURANCE COMPANY, 1914–1915.

Nature of Impairment.	Number of Impairments.		
	Males.	Females.	Total.
Albumin	59	85	144
Albumin, with casts	42	29	71
Anaemia	1	40	41
Arrhythmia	14	24	38
Blood pressure, high	64	65	129
Bronchitis	52	72	124
Deformities	9	9	18
Dysmenorrhoea	0	347	347
Ear discharge	14	27	41
Epilepsy	0	4	4
Glands enlarged	71	132	203
Glycosuria	33	32	65
Goitre	4	61	65
Hearing impaired	167	155	322
Heart hypertrophy and organic heart murmurs	52	79	131
Heart murmurs, inorganic	130	130	260
Nervous disorders, including history	37	189	226
Overweight	54	98	152
Pleurisy (history)	27	45	72
Pneumonia (history)	76	109	185
Rheumatism, muscular or articular, including history	87	101	188
Rupture	10	0	10
Skin disease	42	30	72
Teeth poor	266	378	644
Tonsils enlarged	94	96	190
Tuberculosis, present	3	5	8
Tuberculosis, history	5	14	19
Tuberculosis, suspected	8	17	25
Tumors, including history	3	15	18
Underweight	34	523	557
Vision impaired	144	291	435
Venereal diseases, including history	19	2	21
Total impairments	1,621	3,204	4,825
Number of employees examined	1,227	2,356	3,583
Number disclosing no impairment	387	674	1,061
Average age of employees examined	31	28	29

RE-EXAMINATIONS.

The weak spot in the system of re-examinations as at present carried out is the small percentage of policyholders who take advantage of the privilege. In the Provident Savings Life it was stated to be about 5 per cent. In the Metropolitan it is about 8 per cent., in the Equitable Life less than 1 per cent., and even in the Southern Life and Trust Company, where Mr. Watt informs

us that the re-examinations were made by the assistant medical director in conjunction with the local examiners and every effort was made in advance to insure the co-operation of policyholders, only about 20 per cent. of those entitled to be re-examined presented themselves. Mr. Watt states, however, "The goodwill manifested towards the company as a result of the scheme, both on the part of the medical director and policyholders, was very gratifying."

The only way that satisfactory information could be obtained would be by having all the persons insured periodically examined. This would involve the hearty co-operation of the insured. The experience available indicates that this co-operation under present conditions cannot be secured. The nearest approach to it we know of is in the company with which Mr. Watt is connected, where it reached 20 per cent. It would make a pretty expensive proposition to re-examine the whole body of policyholders, and if it could be done, it is questionable if the money saved in claims would equal the expense incurred. If, however, life insurance companies should enlarge their sphere of usefulness and assume somewhat the functions of public institutions by reaching out a helping hand to policyholders without expectation of financial gain, an educational movement might be inaugurated and substantial results obtained in securing the active support of policyholders.

Two things, however, must be kept in mind: first, there must be no shadow of suspicion that the companies have any ulterior object in view, and second, that most men consider it a nuisance to undergo an examination and therefore it should be made as simple and with as little inconvenience to the insured as possible.

As the rate of mortality below age 40 is not giving anybody any anxious thought, except perhaps from tuberculosis, and the object in view would be to reduce the rate in middle life, better results might be accomplished if the effort were concentrated between ages 40 and 60 or even between 40 and 50.

If the movement were general among life insurance companies in the effort to secure a lower rate of mortality among insured lives at the older ages and this movement were persisted in with the same energy displayed in securing new business, it is not improbable that gradually the whole body of the insured would become more or less interested, thereby securing the maximum benefit for all concerned.

In this connection a curious fact has developed in a limited experience of one company.

Out of 210 requests for a re-examination received from October 9 to October 25, 1915, 175 cases, or 83 per cent. of the total, were eligible for re-examination last year but did not make request then, nor until at least two or more circulars had been received, and only 35 cases, or 17 per cent. of the total, made application on receipt of the first notice. In these cases notices are sent out with every premium notice, whether quarterly, semi-annually or annually.

Some companies do not warm up to this question of re-examination at all, and some question the right to use the policyholders' money for this purpose. To all such, an indirect method of securing examinations might be resorted to at very light expense and possibly with better results. That method would be by calling attention to all of the insured who are say forty years of age or upwards, to the diseases to which such persons are peculiarly liable, and suggesting that they have periodical examinations made on their own account.

For instance, here are three sample notices designed to go out with every premium notice. It is not unreasonable to believe that persistence along this line would suggest to very many men the propriety of being examined who otherwise would not think of such a thing.

Have a sample of your urine examined at least once a year. You may thus be able to learn of, and take proper remedies for, incipient diseases of the bladder or kidneys. You would be ignorant of any such disease without an analysis of the urine, and if it had made much progress before being discovered, it might be too late.

Have your blood pressure tested. You do not know whether your blood pressure is too low or too high, nor can a physician generally know without the use of an instrument specially designed for this purpose. A very low or a very high blood pressure is a sure indication that you need immediate medical attention.

Be examined by a doctor once a year. If you do not care to have the company you are insured with meddling in your affairs, have it done on your own account. It will cost you about five dollars a year and will prove a good investment whether you are found to be impaired or not. It may add five or ten years to your life.

It is claimed by many that by reason of the greater skill in the treatment of disease, by earlier apprehension of impaired conditions

and by more general diffusion of knowledge as to the correct modes of living, many lives have been carried over to and past middle life, whereas in former times they succumbed; and that as a result of this there is a much larger number of lives at about age 40 and and over who are less rugged and vigorous and more prone to the degenerative diseases. This, they claim, accounts for the absence of any decrease in mortality worth speaking of at the higher ages.

It seems to be a fact, however, so far as we are able to discover, that on female lives there has been a decrease in the rate of mortality all along the curve. On industrial risks this seems to be true also on male lives, but not to so great an extent as on female lives. The two tables (English and Metropolitan) show not only that the rate of mortality on the two earlier tables was lower on female lives than on male lives, but also that, taken as a whole, they show a greater decrease during the last twenty or twenty-five years than is shown by the male lives.

If it be true that the same conditions exist among women who would come in under the ordinary class of insurance as compared with men who carry ordinary insurance, then the question would arise, whether it is possible by any means within our power to reduce the rate of mortality on the lives of such men to the level of the rate prevailing among such women.

I do not know whether any investigation has ever been made to explain why the mortality on female annuitants is lower than on males. Wherever reliable data of sufficient magnitude is available, the same condition exists on lives other than annuitants, and the question may well be asked, whether the divergence of rates on the lives of annuitants is not really an incident of the general fact that the rate of mortality is less among females than among males.

The following tables and graphs are appended.

Table No 1, showing comparative reduction of male and female mortality as shown by English Life Table No. 5 and English Life Table No. 8,

Table No. 2, showing comparative reduction of Industrial white male and female mortality as shown by the experience of the Metropolitan Life Insurance Company during the years 1894 and 1914,

Graphs 1 and 2, corresponding to Table 1,

Graphs 3 and 4, corresponding to Table 2 (graduation by graphic method),

Graph 5, showing mortality curve of the Medico-Actuarial Table compared with the American Table,

Graph 6, showing mortality curve of a recent Metropolitan Life Table compared with the American Table.

TABLE NO. 1.

SCHEDULE SHOWING COMPARATIVE REDUCTION OF MALE AND FEMALE MORTALITY IN ENGLAND AND WALES, AS SHOWN BY ENGLISH LIFE TABLE NO. 5 (BASED ON DEATHS OF 1881–1890) AND ENGLISH LIFE TABLE NO. 8 (BASED ON DEATHS OF 1910–1912).

Age.	Males.				Females.				If Male Deaths Had Been Reduced in Same Proportion as Female, then, under Table No. 8, Instead of Each 100 Male Deaths, there Would Have Been:
	Mortality Rate per 1,000 (1,000 q_x).			Ratio of Reduction.	Mortality Rate per 1,000 (1,000 q_x).			Ratio of Reduction.	
	Table No. 5.	Table No. 8.	Reduction.		Table No. 5.	Table No. 8.	Reduction.		
0	161.04	120.44	40.60	.252	131.13	97.67	33.46	.255	100
5	8.32	4.89	3.43	.412	7.86	4.80	3.06	.389	104
10	1.95	1.93	.02	.010	1.67	1.96	*−.29	*−.174	119
15	2.87	2.35	.52	.181	2.95	2.39	.56	.190	99
20	4.80	3.48	1.32	.275	4.89	2.95	1.94	.397	83
25	6.37	4.00	2.37	.372	6.21	3.40	2.81	.452	87
30	8.29	4.78	3.51	.423	7.89	4.11	3.78	.479	90
35	10.19	6.24	3.95	.388	9.24	5.23	4.01	.434	92
40	12.59	8.11	4.48	.356	10.53	6.60	3.93	.373	97
45	15.64	10.89	4.75	.304	12.35	8.54	3.81	.309	99
50	19.78	14.82	4.96	.251	15.65	11.40	4.25	.272	97
55	26.02	21.11	4.91	.189	20.90	16.15	4.75	.227	95
60	35.92	30.42	5.50	.153	29.02	23.10	5.92	.204	94
65	50.56	43.75	6.81	.135	41.99	33.42	8.57	.204	92
70	72.00	64.70	7.30	.101	62.26	52.59	9.67	.155	94
75	104.57	97.51	7.06	.068	92.79	80.79	12.00	.129	93
80	153.01	142.99	10.02	.065	136.57	124.19	12.38	.091	97
85	221.56	199.11	22.45	.101	196.02	173.37	22.65	.116	98
90	312.26	273.95	38.31	.123	272.25	238.26	33.99	.125	100
95	424.20	315.64	108.56	.256	363.94	312.33	51.61	.142	115

The method of calculating the last column in the two tables may be shown by taking as an example age 30 under the comparison of English Life Tables No. 5 and No. 8. The mortality rate per 1,000 for females was 7.89 under Table No. 5 and 4.11 under Table No. 8, so the rate under the later table was 52.1 per cent. of that under the earlier table. Now if we assume that the death rate on males had decreased in the same proportion, the rate for males under Table No. 8 would have been 52.1 per cent. of 8.29

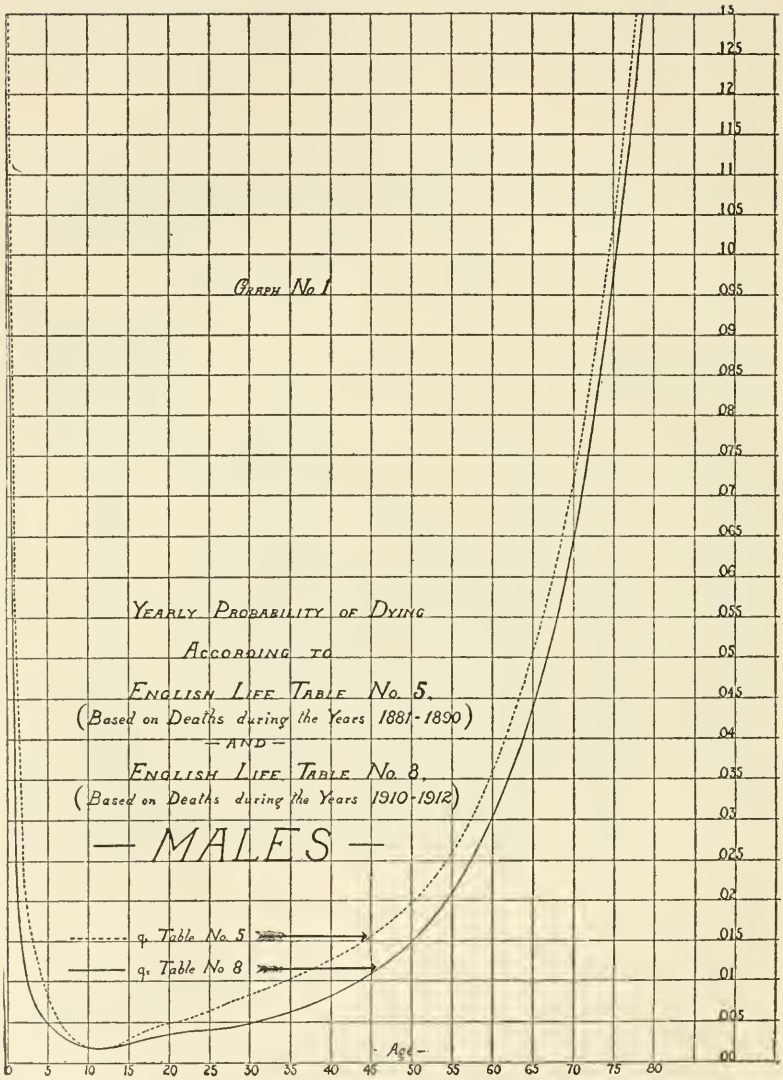
* Increase in mortality rate.

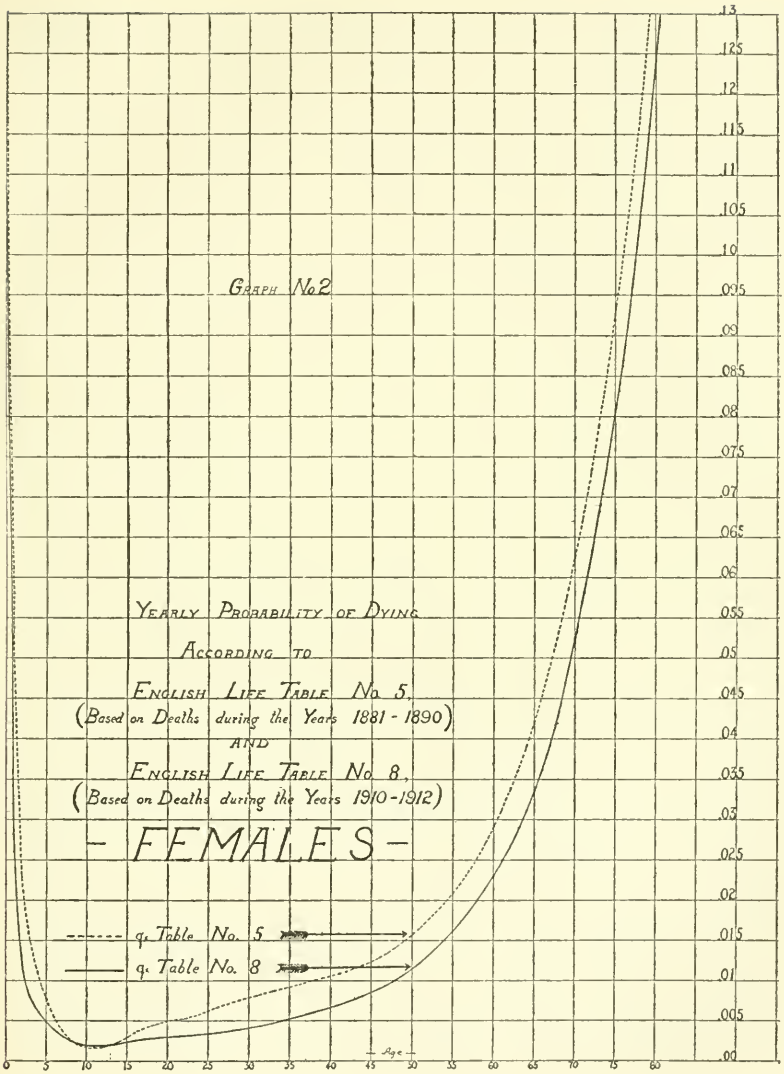
(the rate under Table No. 5), or 4.32 per 1,000. But the actual rate under Table No. 8, for males, was 4.78, so by simple proportion we find that if the reduction under the male table had been the same as under the female table, we would have had, under Table No. 8, ninety instead of each one hundred actual male deaths.

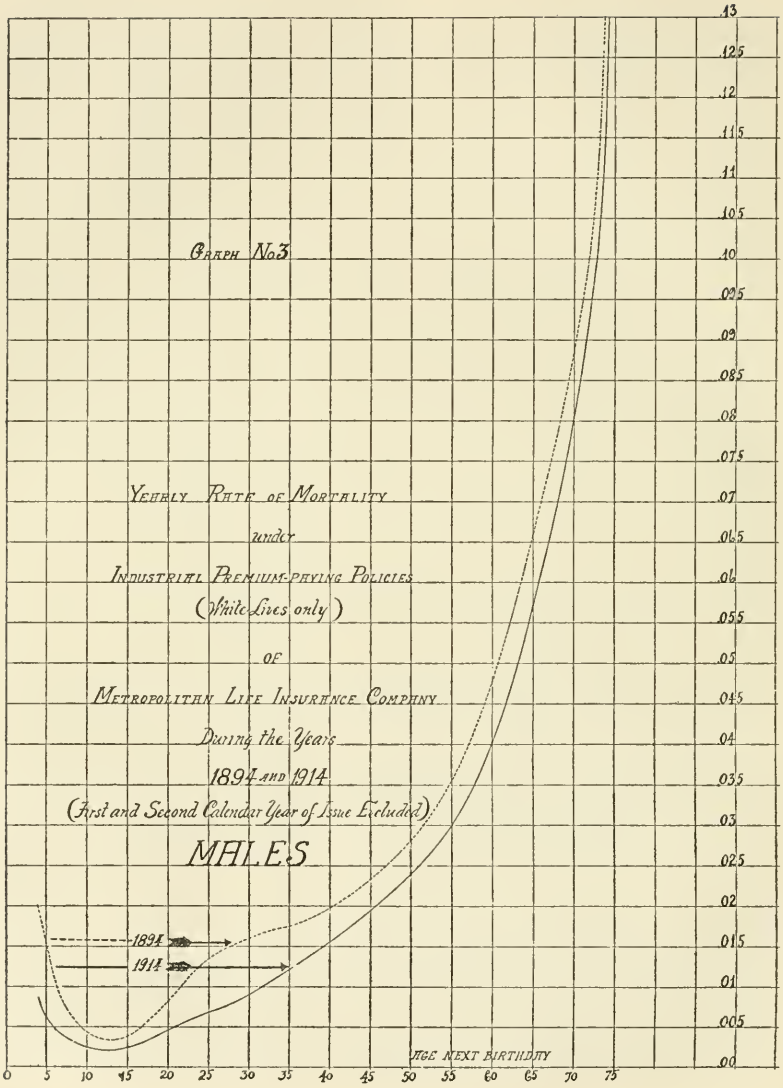
TABLE NO. 2.

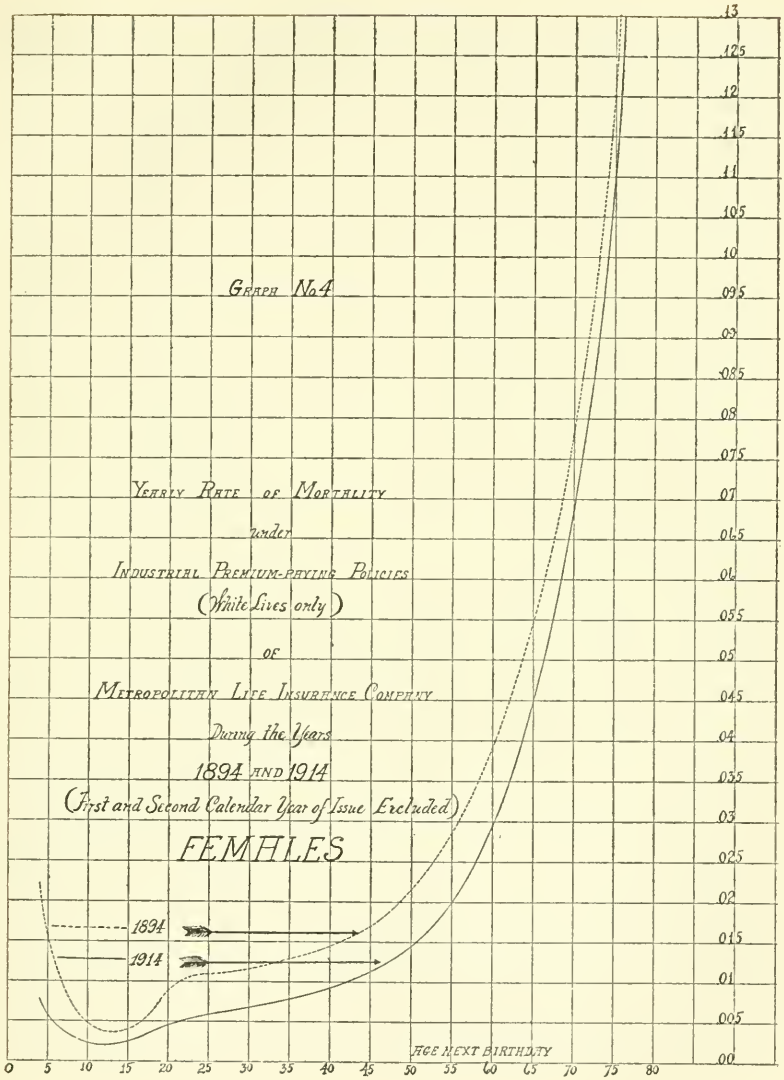
SCHEDULE SHOWING COMPARATIVE REDUCTION OF MALE AND FEMALE MORTALITY, AS SHOWN BY THE EXPERIENCE OF THE METROPOLITAN LIFE INSURANCE COMPANY ON INDUSTRIAL PREMIUM-PAYING POLICIES (WHITE LIVES ONLY) DURING THE YEARS OF 1894 AND 1914 RESPECTIVELY.
(First Two Calendar Years of Issue Excluded.)

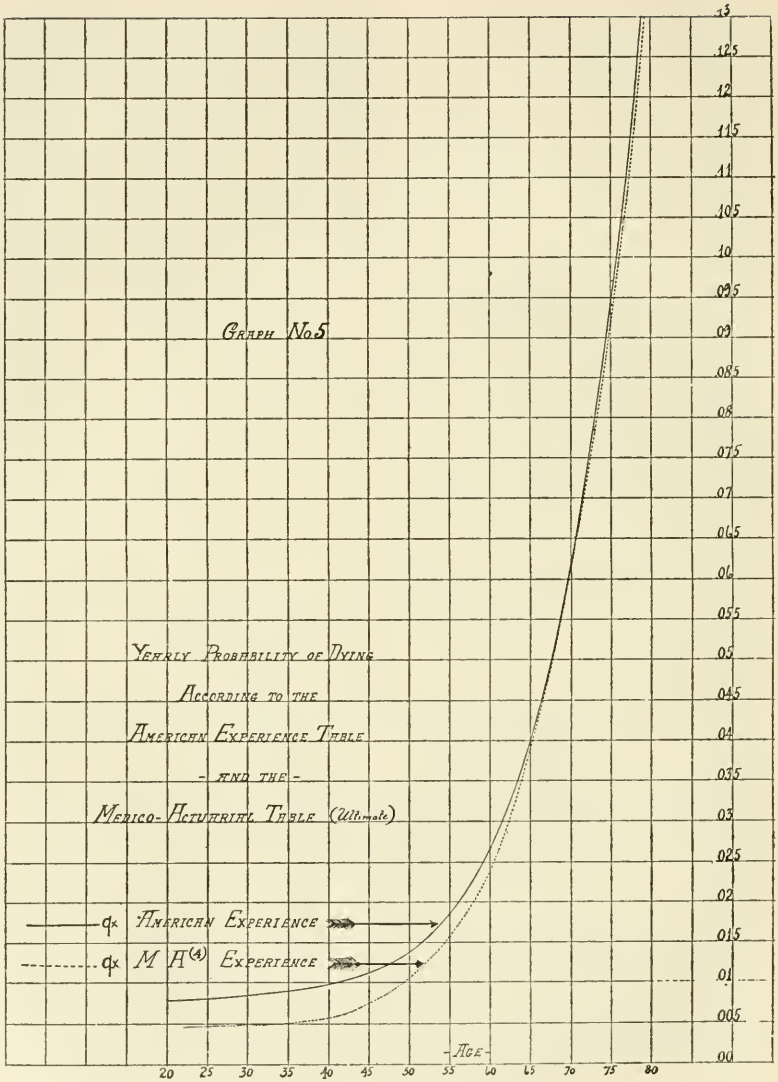
Age Next Birthday.	Males.				Females.				If Male Claims Had Been Reduced in Same Proportion as Female, then, in 1914, Instead of Each 100 Male Claims There Would Have Been:
	Claims per 1,000 Policies at Risk.			Ratio of Reduction.	Claims per 1,000 Policies at Risk.			Ratio of Reduction.	
	1894.	1914.	Reduction.		1894.	1914.	Reduction.		
4–9	10.80	4.96	5.84	.541	11.44	4.48	6.96	.608	85
10–14	3.96	2.32	1.64	.414	3.74	2.25	1.49	.398	103
15–19	5.55	3.32	2.23	.402	5.41	3.28	2.13	.394	101
20–24	10.10	5.46	4.64	.459	10.07	5.08	4.99	.496	93
25–29	14.41	7.31	7.10	.493	11.37	6.11	5.26	.463	106
30–34	16.42	10.49	5.93	.361	11.81	7.04	4.77	.404	93
35–39	19.06	13.99	5.07	.266	13.90	8.75	5.15	.371	86
40–44	21.18	17.14	4.04	.191	15.60	9.80	5.80	.372	78
45–49	24.32	21.53	2.79	.115	18.91	12.36	6.55	.346	74
50–54	31.31	25.98	5.33	.170	23.70	16.64	7.06	.298	85
55–59	38.96	34.12	4.84	.124	32.91	23.13	9.78	.297	80
60–64	56.43	46.84	9.59	.170	44.07	34.76	9.31	.211	95
65–69	73.04	65.10	7.94	.109	62.13	54.28	7.85	.126	98
70–74	101.30	92.38	8.92	.088	89.87	78.58	11.29	.126	96
75 and over			Data insufficient						

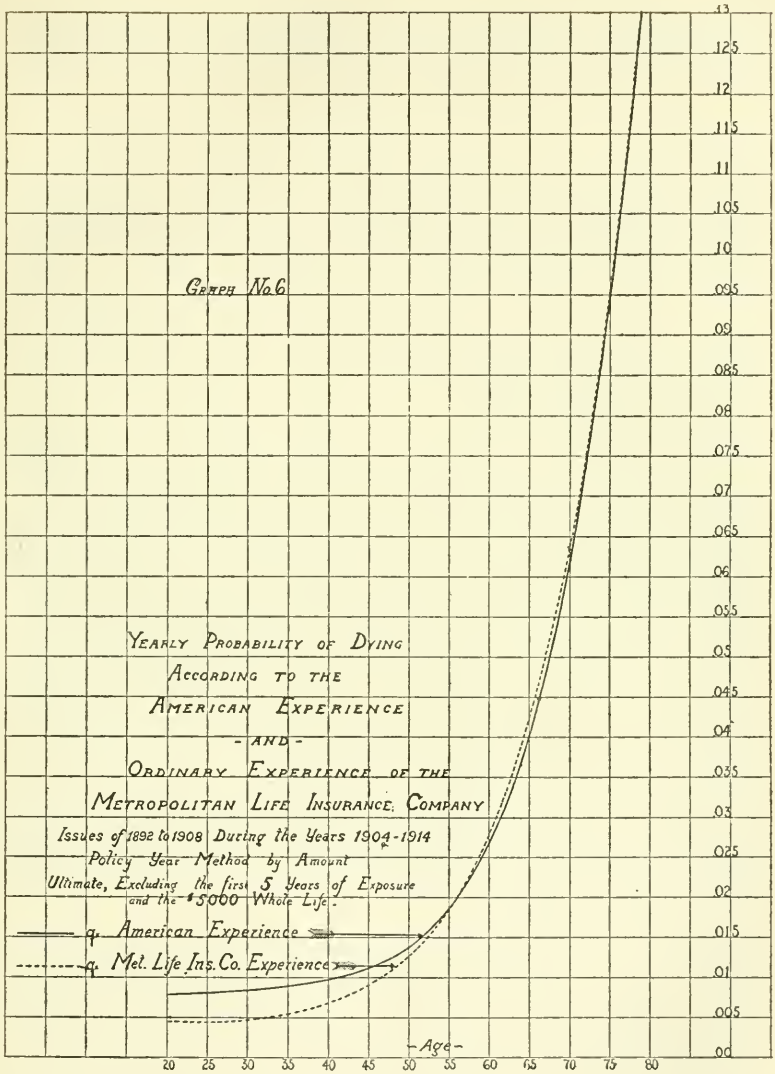

Graph No 1
Yearly Probability of Dying
According to
English Life Table No. 5.
(Based on Deaths during the Years 1881-1890)
— and —
English Life Table No. 8.
(Based on Deaths during the Years 1910-1912)
— MALES —
q. Table No. 5
q. Table No 8
Age
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80
.15 .125 .12 .115 .11 .105 .10 .095 .09 .085 .08 .075 .07 .065 .06 .055 .05 .045 .04 .035 .03 .025 .02 .015 .01 .005 .00


Graph No 2
Yearly Probability of Dying
According to
English Life Table No 5,
(Based on Deaths during the Years 1881 - 1890)
and
English Life Table No 8,
(Based on Deaths during the Years 1910-1912)
- FEMALES -
q. Table No. 5
q. Table No. 8
Age
.13
.125
.12
.115
.11
.105
.10
.095
.09
.085
.08
.075
.07
.065
.06
.055
.05
.045
.04
.035
.03
.025
.02
.015
.01
.005
.00
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80


Graph No 3
Yearly Rate of Mortality
under
Industrial Premium-paying Policies
(White Lives only)
of
Metropolitan Life Insurance Company
During the Years
1894 and 1914
(First and Second Calendar Year of Issue Excluded)
MALES
1894
1914
Age Next Birthday
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
.13 .125 .12 .115 .11 .105 .10 .095 .09 .085 .08 .075 .07 .065 .06 .055 .05 .045 .04 .035 .03 .025 .02 .015 .01 .005 .00


Graph No 4
Yearly Rate of Mortality
under
Industrial Premium-paying Policies
(White Lives only)
of
Metropolitan Life Insurance Company
During the Years
1894 and 1914
(First and Second Calendar Year of Issue Excluded)
FEMALES
1894
1914
Age Next Birthday
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80
13 125 12 115 11 105 10 095 09 085 08 075 07 065 06 055 05 045 04 035 03 025 02 015 01 005 00


Graph No. 5
Yearly Probability of Dying
According to the
American Experience Table
- and the -
Medico-Actuarial Table (Ultimate)
qx American Experience
qx M A(4) Experience
Age
20 25 30 35 40 45 50 55 60 65 70 75 80
.13 .125 .12 .115 .11 .105 .10 .095 .09 .085 .08 .075 .07 .065 .06 .055 .05 .045 .04 .035 .03 .025 .02 .015 .01 .005 .00


Graph No. 6
Yearly Probability of Dying
According to the
American Experience
- and -
Ordinary Experience of the
Metropolitan Life Insurance Company
Issues of 1892 to 1908 During the Years 1904-1914
Policy Year Method by Amount
Ultimate, Excluding the first 5 Years of Exposure
and the $5000 Whole Life.
q American Experience
q Met. Life Ins. Co. Experience
Age
.13
.125
.12
.115
.11
.105
.10
.095
.09
.085
.08
.075
.07
.065
.06
.055
.05
.045
.04
.035
.03
.025
.02
.015
.01
.005
.00
20
25
30
35
40
45
50
55
60
65
70
75
80

Premium Loadings and Expense Limitations.

BY

E. E. RHODES.

The letter of the Superintendent of Insurance of the State of New York printed on pages 228–229 of Volume XVI of the Society's *Transactions* presents some very interesting questions for discussion. In this paper I propose to discuss the true function of loadings, and, incidentally, the effect of the amendment to Section 97 of the New York insurance law proposed at the last session of the legislature of that state.

The law now limits the total insurance expenses of companies doing business in New York to the total loadings plus the present value of certain assumed mortality gains. The proposed amendment substituted for the loadings a percentage of the net premiums. The Superintendent of Insurance is mistaken in saying that this amendment was opposed on the ground that a mutual company should not be permitted to draw upon its gains from mortality, interest and miscellaneous sources, for the purpose of defraying insurance expenses. Practically all insurance companies draw temporarily upon one or the other of these sources of gain for this purpose, and the practice is directly recognized and validated by Section 97 as it now stands.

Before we can intelligently discuss the proposed amendment we must come to some agreement regarding our definitions. A search of insurance literature has brought to light the following definitions of the term "loading."

"Margin or Loading.—The amount added to, or subtracted from, the net rates or results, to provide for expenses and contingencies." —D. Parks Fackler, *"Agents' Monetary, Life and Valuation Tables,"* 1868.

"Possible deficiencies because of errors in the assumptions regarding mortality or interest, used in computing these (net) premiums, are not taken into account; neither are possible losses on investments, nor the expense of carrying on the business. All of these are provided for by means of an addition to the net premium

which is called the loading."—Dawson, "*Elements of Life Insurance,*" Third Edition, 1911, p. 76.

"The elements in the computation of ordinary office premiums are: (1) Rate of mortality, (2) Rate of interest, (3) Rate of expense, (4) Margin for possible fluctuations in these rates."—Moir, *Transactions of the Faculty of Actuaries,* Volume 2, p. 211.

"The amount by which such (net) premiums are loaded must provide for the expenses of administration and commission as well as for the profit; morcover, it may serve as a measure against the unfavorable consequence of possible inaccuracy in the basis of calculation."—Peek, *Proceedings of the Fourth International Congress of Actuaries,* Volume 1, p. 434.

"Loadings for expenses of management, whether 'initial' or 'renewal,' can only provide for normal expenditure and any expenditure in excess of the normal must come out of current profits."—Andras, *Seventh International Congress of Actuaries,* Volume 1, p. 769.

"It may be said of all branches of insurance business that the premium charged is made up of two distinct parts which either theory or practice has defined, viz.: (1) The charge necessary for undertaking the risk incurred. (2) Such a sum as will provide for the expenses incident to the carrying on of the business, having regard to its peculiar nature."—Walford, "*Insurance Cyclopedia,*" Vol. 3, p. 85.

"The margin added to the insurance portion of the premium should be sufficient, not only to provide for the necessary expense of conducting the insurance business, but should also be sufficient to meet unexpected or adverse insurance contingencies, such as excessive mortality and any undue exodus of sound lives."—Sheppard Homans, *Transactions of Actuarial Society of America,* Vol. 1, p. 144.

"The expenditures of a life insurance company may be classified under the following heads: (1) The death claims. (2) Surrender values, including endowments. (3) Cost of managing the reserve or self-insurance fund. (4) Cost of collecting the premiums. (5) All other working expenses, including the cost of procuring business. The first three heads are amply provided for by the net premiums and their interest. The last two only are to be provided for by added margin."—Elizur Wright, "*Savings Bank Life Insurance,*" 1872, p. 13.

"In participating insurance the loading should provide for dividends to policyholders as well as for the expenses of procuring and administering the business."—R. G. Hunter, *7th International Congress of Actuaries,* Volume 1, p. 727.

"The factors entering into the calculation of premiums for business other than 'industrial' business, which may for convenience be shortly termed 'Ordinary' office premiums, are: (1) Rate of mortality; (2) Rate of interest; (3) Rate of expense; and in the case of 'With Profit' policies; (4) A loading for future bonuses." —Bacon, *7th International Congress of Actuaries,* Volume 1, p. 784.

The above definitions comprise all that I have been able to find in my search. Unless others with a contrary meaning can be produced, the word "loading" must therefore be accepted as referring to that part of the gross premium which is intended to provide for expenses and contingencies. Some writers refer to the loading as providing also for dividends to policy holders; but it is evident that any provision therefor is in excess of the provision for expenses and contingencies. If it were otherwise a company would have to rely upon the more or less problematical gains from mortality and interest to meet the certain expenses of conducting the business. It is clear that there is no relation between net premiums and expenses, as the net premium for a life policy provides nothing more than the tabular cost of insurance. This fact was sufficient to condemn the proposed amendment.

The Superintendent asks "Should any mutual company be limited as to total insurance expenses in such a manner that it must increase its premiums in order to comply with the limitation, when its own experience demonstrates that it can safely proceed without increasing its rates?" The form of the question is perhaps open to criticism, as it seems fair to draw a distinction between a company organized and always conducted as a mutual company, and a company organized as a stock company, which, although mutualized, has had no experience as a mutual company. It will be agreed, doubtless, that in the case of a mutual company having consistent rates for all forms of insurance, whose experience as a mutual company has demonstrated such rates to be ample, the legislature should not impose any expense limitation which would have the effect of compelling such a company to increase its rates. If, however, the rates of a mutual company for various forms of policies are not consistent, the argument that a certain expense limita-

tion should be discarded because it would have the effect of increasing the rates on some forms in order to make them consistent with the rates on other forms, would not be valid. Mutuality certainly requires that premium rates for all plans should be consistent. If the rates for a particular plan do not provide the same margin as do the rates for other plans, any deficiency in the rates for the particular plan would have to be borne by other policyholders. If the rates for all plans are consistent, even if all prove to be insufficient, all members of a mutual company suffer alike in any necessary scaling down of the contracts.

It would seem to be a cardinal principle of mutual insurance that the rate for any particular form of policy should be sufficient for the following purposes: 1st, to meet all the policy obligations; 2d, to meet the necessary administrative expenses; 3d, to enable the company to maintain an adequate contingency reserve; and 4th, to maintain the agency organization, all on the assumption that *no other form of policy is issued by the company.* If other forms of policies are issued and the rates for a particular form of policy are insufficient for the purposes enumerated above, some part of the net cost which should be borne by policies issued under the insufficient rate is necessarily borne by policies issued under tables calling for higher premium loadings. If a mutual company has a high premium rate for one class of policy and a low premium rate for another (using the words "high" and "low" in their relative sense), the policyholders paying the high rates are guaranteeing something to the policyholders paying the low rates. In event of serious losses those paying the high rates lose first and lose most. If a company charges a smaller loading than is necessary to cover expenses, it is discounting unknown and uncertain future profits and applying such profits toward a reduction of the premiums which would otherwise be necessary. There is a violation of mutuality if a special form of policy be issued under rates so low as to preclude the possibility of their sustaining any part of the loss in event of the happening of some unforeseen contingency.

The second question of the Superintendent is "Should mutual companies be required by statute to charge loadings upon their net premiums which will be adequate to cover all insurance expenses?" Existing laws provide that if a premium rate be less than the net premium required to maintain the legal reserve a company shall be charged with a special reserve sufficient to meet such deficiency. If

a company's premium rates were all less than the net premium required to maintain the legal reserve, the company would become bankrupt, unless its gains from mortality, interest and miscellaneous sources were sufficient to pay expenses and put up the required special reserve. It is difficult to believe that any insurance department would license a company which adopted such rates. Furthermore, it is difficult to believe that an insurance department would license a company whose premium rates were only a trifle in excess of the required net premium. In such case the refusal to license might be properly based upon two grounds; first, that the company was discounting too heavily its problematical gains, and second, that inasmuch as the insurance public has been educated to regard the difference between the gross premium and the net premium as the expense provision, the adoption of an insufficient loading would be deceptive and misleading in that it would convey the idea that the company's business would be conducted much more economically than would be the case.

Whatever may have been our opinion regarding Section 97 of the New York insurance law when it was proposed, I think we may all agree that its effect upon the business of life insurance has been very wholesome, except possibly in the effect it has had of encouraging the issue of an undue proportion of term insurance. I would not defend the measure as a proper voluntary limitation of expense for all companies alike. Speaking from the standpoint of a company we may say that it is justified in paying for new business what the new business is worth to it. Probably in no two companies is new business worth the same amount. An established company with a well organized, efficient and loyal agency force and with a reputation for economical management and fair and liberal dealings with policyholders, can and should obtain business more cheaply than a company which lacks these things; and we may say that new business is worth more to the latter company than to the former. We may also say that new business is worth more to a moribund company than to a going concern. It is doubtful, however, whether a legislature would be justified in considering these facts. It must regard the question from the standpoint of the public, and it would seem good public policy for the legislature to say that a company which was unable to obtain new business within certain limits of expense should not be permitted to do business in the state. What these limits should be is a question upon

which we cannot expect entire unanimity; but we may perhaps be able to agree upon certain principles, the most important of which are that new companies should be able to establish themselves, and that companies which have suffered reverses should have a fair chance of retrieving themselves. The limitation, therefore, should not be based upon what the best managed companies have been able to do, but upon certain generally recognized principles which in my opinion are reflected in Section 97 as it now stands. That actuaries generally have recognized the loading as an expense provision is shown by the definitions incorporated in this paper, and the weight of evidence in this respect is so preponderant that it must be accepted. I think that actuaries will generally agree, also, that companies are justified in meeting the expense of new business out of the mortality gains arising from the effect of selection. Whether companies should be permitted by the state to expend the actual gains in this respect or be restricted to certain assumed gains, is a fair question, upon which there may well be a difference of opinion. If the American Table be retained as a legal standard, I think I would be inclined to favor an expense allowance based upon the actual mortality gains for the first five policy years in lieu of the present assumed gains, if the actual gains were more than the assumed gains. If a new select table be prepared, as now seems likely, and if such table shall be adopted as a legal standard, such table will probably afford very small mortality gains, and it will be necessary, unless Section 97 be retained in its present form, to find some new method of limiting expenses. Section 97 if retained would then become much more arbitrary than it is at present, as the mortality gains would be based upon a mortality table which was no longer used for the calculation of premiums and reserves, as well as upon certain arbitrary percentages of the tabular death rate. It might, nevertheless, continue to prove as good a practical measure as it has in the past. Section 97 as it now stands does not measure first year expenses by the actual loadings, although the total expenses are so measured. In general it may be said that Section 97 follows as closely as the legislature deemed practicable the idea that expenses should be limited to loadings and the mortality gains arising from the transaction of new business.

The true loading, or margin, as distinguished from the nominal loading, is the difference between the gross premium and what is actually required. This true loading may of course be averaged by

using a mortality table which represents a company's mortality experience, a rate of interest approximating that actually earned, and a nominal loading based upon actual expenses, and also a provision for the maintenance of a contingency reserve. The difference between the actual gross premium and a premium so computed would fairly represent the true loading.

The writer believes that so long as the states have an arbitrary and fixed reserve standard they should not provide by statute that the true loading, as distinguished from the nominal loading, should be available for expenses. In that case a company which availed itself of the statutory permission to expend the true loading would be in the position of issuing nonparticipating policies under the guise of participating policies. Moreover there would be danger of untoward losses which could not be met without embarrassment. A mutual company must constantly bear in mind that it has no means of recouping abnormal losses except by its redundant premiums.

The third question of the Superintendent, "Should a new mortality table be recognized by statute for the calculation of premiums and reserves?" will receive full consideration when the work of the Society's committee in charge of the preparation of the new table is completed.

Annuities with Participation Based Upon Select and Ultimate McClintock Tables.

BY

D. PARKS FACKLER.

In a former paper suggestions were made based upon the McClintock Annuitants' Tables in the shape in which they were adopted as the basis of legal reserves. Further study of the subject has developed the following points for consideration in connection with those already presented.

In presenting his annuity tables in May, 1899, Mr. McClintock stated:

"It has been questioned whether, for the valuation of annuities, select tables are not to be preferred, showing a rate of mortality quite small during the first policy year, somewhat larger the next year, and so on, until after say four years the effect of selection may be regarded as worn off. For my own part, I am inclined to prefer for this purpose the use of an average table showing a distinctly low mortality at all ages, rather than to take advantage of the somewhat higher mortality usually found to prevail after the self-selection among annuitants has worn away."

That his then views were generally approved at the time appears to be shown by the adoption of his tables for government use, but we may well doubt whether he would still hold the same opinion, now that his own company's recently published experience with domestic annuitants shows such a divergence from the standard tables, and also when none of the companies base their annuity premiums upon them for the older ages at which most of the business is done.

The extreme disparity between the legal reserves and the premiums charged at the advanced ages seems to show that the reserves are too low or else the companies' charges are utterly unreasonable. In other branches of insurance the rule is that there must be a certain relation between the premium and the reserve: thus if a company rates a risk high with a large premium it must reserve accordingly, which is a reasonable requirement. But for

annuities of $100 on males at ages 80 and 85, the three largest companies charge $546.80 and $489.50, respectively, while the tabular legal reserves at the close of the year are only $445.35 and $327.20. Even in life insurance there is no such disparity at any age between the single premiums and reserves for whole life policies.

In view of the above facts it seems clear that the legal basis for annuities should be modified to suit the facts, and the following suggestions are submitted:

Assuming that the McClintock tables are, as their author intended, conservative ultimate tables, we may apply the select and ultimate principle embodied in the New York law as to minimum reserves for insurances, and obtain tables of maximum reserves for annuities.

The New York law assumes for safety that the mortality in the first five years, despite medical selection, will be as high as 50, 65, 80, 90 and 95 per cent. of the rates by the ultimate table. These percentages are supposed to be higher than likely to be experienced, because the risk is mortality, but when the risk is the opposite, or survivorship, we should assume lower percentages than we think will be attained by the self-selection of the annuitants, or their friends, and thus it seems reasonable to make the percentages 30, 50, 65, 80 and 90.

Using these percentages we can make up Select and Ultimate Annuitant Tables with the McClintock Tables as bases and the single premiums at 3, 3½, 4 and 5 per cent., also the annuities bought by $1,000, are found to be as follows:

McCLINTOCK ANNUITY TABLES MODIFIED IN A MANNER ANALOGOUS TO THE SELECT & ULTIMATE PROVISIONS OF THE NEW YORK LAW.

The Percentages of the Mortality Rates of the Standard Tables during the First Five Years Being 30, 50, 65, 80 and 90.

Value of $100 Annuity.

Age.	3 %.		3½ %.		4 %.		5 %.	
	Male.	Female.	Male.	Female.	Male.	Female.	Male.	Female.
10*	2,355.95	2,579.27	2,142.61	2,328.77	1,960.32	2,116.68	1,667.33	1,780.53
20*	2,214.92	2,423.48	2,031.09	2,207.57	1,871.52	2,021.83	1,609.90	1,721.12
30*	2,023.70	2,217.25	1,873.83	2,040.50	1,741.43	1,885.71	1,519.48	1,629.21
40*	1,774.54	1,953.18	1,661.26	1,818.23	1,559.33	1,697.76	1,384.20	1,493.09
50	1,469.56	1,632.08	1,392.08	1,538.17	1,321.24	1,452.92	1,196.18	1,303.94
55	1,301.50	1,454.76	1,240.47	1,379.94	1,184.14	1,311.18	1,083.34	1,189.52
60	1,128.80	1,271.41	1,082.44	1,213.78	1,039.23	1,160.26	960.91	1,064.28
65	957.27	1,087.49	923.44	1,044.70	891.56	1,004.61	833.09	931.72
70	793.38	909.47	769.60	878.91	747.06	850.05	705.16	796.89
75	643.21	743.71	627.13	722.72	611.80	702.77	582.96	665.53
80	511.47	595.65	500.98	581.81	490.87	568.47	471.68	543.36
85	400.70	468.96	394.01	460.06	387.49	451.42	375.03	434.99

Annuity Purchased by $,1000.

Age.	3 %.		3½ %.		4 %.		5 %.	
	Male.	Female.	Male.	Female.	Male.	Female.	Male.	Female.
10*	42.45	38.77	46.67	42.94	51.01	47.24	59.98	56.16
20*	45.15	41.26	49.23	45.30	53.43	49.46	62.12	58.10
30*	49.41	45.10	53.37	49.01	57.42	53.03	65.81	61.38
40*	56.35	51.20	60.20	55.00	64.13	58.90	72.24	66.98
50	68.05	61.27	71.83	65.01	75.69	68.83	83.60	76.69
55	76.83	68.74	80.61	72.47	84.45	76.27	92.31	84.07
60	88.59	78.65	92.38	82.39	96.23	86.19	104.07	93.96
65	104.46	91.95	108.29	95.72	112.16	99.54	120.04	107.33
70	126.04	109.95	129.94	113.78	133.86	117.64	141.81	125.49
75	155.47	134.46	159.46	138.37	163.45	142.29	171.54	150.26
80	195.51	167.88	199.61	171.88	203.72	175.91	212.01	184.04
85	249.56	213.24	253.80	217.36	258.07	221.52	266.65	229.89

If we compare the net 3½ per cent. premiums with the actual charges of the companies we will find few considerable differences, and a moderate addition for expenses will make them fairly consistent in most cases.

In the discussion of the previous papers on this subject several remarked that it is very unreasonable to base annuity reserves on the same low rate of interest that is used in connection with insurances, for the time during which annuity reserves are largest is

not in the distant future, as is the case with insurances. To base annuity premiums and reserves upon 4 per cent. when companies are easily getting 5 per cent. or more would be safe, for the higher interest earned in the earlier years would much more than offset any possible loss on the diminished reserves of many years hence, if interest should ever fall below 4 per cent. It is to be hoped that the state legislatures will be shown these facts and led to reduce the reserves on annuities, so as to enable the companies to offer the public reasonable returns at all ages.

On the subject of probable future rates of interest experience has shown that the views of actuaries are more reliable than those of financiers, which appear to be influenced very largely by present money market considerations while those of actuaries are based on a study of the experience of centuries. About twenty years ago Mr. McClintock, Mr. Charlton Lewis, Mr. Nichols and the writer stated that they did not believe the rates of interest would fall further or continue low, but one of the large life insurance companies published a pamphlet giving the views of a score of eminent bankers to the effect that the market rate of interest would surely continue low and that no rate over 3 per cent. would be a safe basis for reserves.

As life insurance companies are the only corporations chartered for the issue of annuities it is as much their duty to meet the needs of the public in that respect as in connection with life insurance. If the annuity business is so conducted as not to furnish annuities at all ages at rates as low as can safely be done, the companies are falling short of their duty and the public suffer thereby. I know of at least one case where a lady was considering taking an annuity, but as the yield would be little more than could be obtained from mortgages she was persuaded to put all her money into an investment in which it was lost. Had she been offered an annuity with participation, which would probably have given her a more liberal return she doubtless would have taken the annuity and escaped loss.

If the annuity business is pushed and popularized, as it should be in this country, there can be no doubt of the safety of the McClintock as ultimate tables, indeed it seems probable that, as he believed, they will be found so conservative as to offset the selection against the companies in the earlier years. If these propositions are granted, it follows that the select and ultimate tables

above presented are as conservative as can be desired. The writer is inclined to think that they are ultraconservative, but if companies will adopt them on a participating plan, as suggested in a previous paper, it may be that not much injustice will be done.

The foregoing tables were prepared under the direction of Mr. William Breiby, one of our Associates, by using the formula:

$$a_{[x]} = vp_{[x]} + v_2^2 p_{[x]} + v_3^3 p_{[x]} + v_4^4 p_{[x]} + v_5^5 p_{[x]}(1 + a_{x+5})$$

in connection with the annuity values according to the ordinary tables, of which I have complete sets. The 3 and $3\frac{1}{2}$ per cent. annuity values published by our Society and by the New York Insurance Department, respectively, are stated to only three places of decimals and their use may have caused some slight errors of not over 4 cents in the select and ultimate values derived from them.

Requirements as to Health under Applications for Reinstatement of Policies. Mortality under Reinstated Policies.

BY

ARTHUR HUNTER.

For many years the custom of the life insurance companies has been to grant a month of grace in payment of the premium. During that time the sum insured would be payable in event of death, the past due premium, however, being deducted. If the premium is not paid at the end of the month of grace, the policy lapses and is not usually reinstated without some evidence of insurability satisfactory to the company. In this connection I shall refer to policies as "lapsed" when the days of grace have expired, even though the policy may have term extension benefits. The rules of the companies with regard to the requirements under lapsed policies have not been brought before this Society, and I have accordingly obtained the practice of twenty of the largest American companies and of five of the principal Canadian companies. In dealing with this subject, reference to the practice under policies which do not have a month of grace has been omitted.

The evidence of good health or of insurability of the insured generally takes the form of,

1. A statement signed by the insured, usually called a Personal Health Certificate.

2. A certificate signed by the insured in presence of a physician who completes a simple medical examination. For convenience this may be entitled a Medical Health Certificate.

3. A medical examination of the same nature as that required for new insurance.

Personal Health Certificate.

The personal health certificate usually contains declarations by the policyholder to the following effect:

1. That he is now in good health and has been in sound health since the date of issue of the policy in question. (In some

cases illnesses which have occurred in the last few years only are considered.)

2. That he has not been declined for insurance, has not had a policy issued to him different from that applied for, and has no application for insurance pending.
3. That there has been no change in his occupation or personal record since the policy was issued.
4. That he is of temperate habits. (He is sometimes asked to state his daily consumption of alcoholic beverages.)

Of the twenty American companies, sixteen use a form of personal health certificate, and of the five Canadian companies, four use it. Of these twenty companies, fifteen permit its use for a month after the days of grace have expired, three for more than a month and two for fifteen days only. Practically all companies reserve the right to require a medical examination if there is any unfavorable record of recent years against the policyholder. The rules of seven companies require an endorsement by an agent, bank official, or other person of standing in the community, that he has seen the applicant, who appears to be in good health. Under the practice of several other companies, the majority of the applicants for reinstatement are seen by agents or by branch office officials.

As the form of the personal health certificate may be of interest, the one used by a prominent company is given.

APPLICATION FOR REINSTATEMENT OF POLICY.

Policy No. ..... Amount $.....

Questions to be answered by the Applicant.

1. *a.* What is your present business or occupation? (Give full details.)	Answer, *a.*
b. If employed, give name of employer.	Answer, *b.*
c. What is your business address?	Answer, *c.*
2. *a.* Has any Life Insurance Company ever examined you, either on an application for insurance or for any other reason, without issuing a policy? (If so, state name of Company.)	Answer, *a.*
b. Has any Life Insurance Company ever issued or offered to issue a policy on your life differing from the one then applied for? (If so, state name of Company, and give particulars.)	Answer, *b.*
3. *a.* How frequently, if at all, and in what quantity do you use beer, wine, spirits or other intoxicants?	Answer, *a.*

b. How frequently, if at all, and in what quantity have you used any of them in the past? Answer, *b.*

c. Have you within the last five years used any of them to excess? Answer, *c.*

d. Do you now or have you ever used morphine, cocaine, or any other habit forming drug? Answer, *d.*

4. What illnesses, if any, have you had since the date of the above policy? Answer.

5. What was the nature of each illness, its date and duration? Answer.

6. What physicians have treated you or have you consulted since the date of the above policy? Answer.

7. Are you now in sound health? Answer.

I DECLARE on behalf of myself and of every person who has or shall claim any interest in or under the above numbered policy, that I made each and all of the foregoing answers; that I have carefully read them over, and find they are written exactly as I made them. Said answers each and all are, and I warrant them to be, full, complete and true. I have made said answers for the purpose of inducing the .... Company to reinstate my said policy, and I understand that they are each material to the risk and that said Company will, and I agree that it shall, rely and act solely upon my said answers in passing upon my application for the reinstatement of said policy which lapsed for non-payment of premium due on the ..... day of ..... 191.., and is not now in force except as may be provided by its non-forfeiture provisions.

I FURTHER AGREE that said policy shall not be deemed reinstated by reason of any cash paid or settlement made in connection with this application, or otherwise, unless and until said Company at its Home Office in acting upon this application shall have duly reinstated said policy during my lifetime and good health, notice of such reinstatement to be promptly mailed to me. If the Company does not mail to me from its Home Office within thirty days from this date notice that said policy has been reinstated, then this application is to be considered declined and the Company is, upon demand with surrender of the receipt given therefor, to return any payment made in connection with this application.

Witnessed by ..... Signature of Insured .....

P. O. Address ..... P. O. Address .....

.....

Dated at ..... Day ..... Month .....191..

It will be noticed that this certificate contains an agreement that the policy is not to be in force until the company has, during the life-time and good health of the applicant, formally approved the request for reinstatement. A number of companies have a clause of this nature in their certificates.

Medical Health Certificate.

A short form of medical examination, which we shall call a medical health certificate, is in use by sixteen of the twenty American companies, and by the five Canadian companies. It ranges from the type which requires the physician to state that the heart, lungs, respiratory organs, etc., are normal, to the one which is as nearly complete as the regular form of medical examination for insurance. In four out of the twenty American companies the applicant for reinstatement is required to furnish a medical health certificate after the month of grace has expired, while the other companies generally require it thirty or more days after that period. As the nature of the medical health certificate varies very greatly, there is naturally a wide difference in the time during which it may be accepted for reinstatement. Three companies accept it for six months after default in payment of premium; four companies for one year; two companies for two years; and the other twelve at any time during which the policy may be reinstated under its terms.

Complete Medical Re-examination.

In the case of the companies which employ a medical health certificate ten of the American and three of the Canadian companies require a full medical re-examination at varying times after the due date of the premium, ranging from six months to two years.

While the practice differs in some particulars, the companies may be divided into four well-defined groups:

(*a*) Those which use a personal health certificate requiring in some instances the endorsement of another person than the applicant, a medical health certificate after the expiration usually of sixty days from the date the premium was due, and a full medical re-examination from six months to two years after the premium was due and unpaid.

(*b*) Those which use a personal health certificate, and a medical health certificate but do not require a full re-examination at any time.

(*c*) Those, few in number, which do not use a personal health certificate but a medical health certificate, and which in some cases call for a full medical re-examination after a long period of lapse.

(*d*) Those, very few in number, which do not use a medical

health certificate, but require either a personal health certificate or a complete medical examination.

Payment of Medical Fee.

With regard to the fee for medical examination, I find that twelve of the twenty American companies, and three of the five Canadian companies require the applicant for reinstatement to pay it. Of the remaining ten companies, five pay the medical fee in all cases, while the other five do so with certain exceptions. For example, the fee is paid by Company A if the policy has been in force for at least one year, is for more than $1,000, and no previous lapse has occurred on the part of the life involved under any policy with the company; by Company B if the policy has not been in force for more than two years; and by Companies C and D if it has not lapsed more than once.

Policies Running under Term Extension.

All American companies are now facing the problem of the treatment of policies with overdue premium which have a period of term extension. Few companies have yet adopted rules for such cases, each applicant being considered on his merits. In the companies with fixed rules, the most liberal permits reinstatement without any action by the insured other than payment of arrears, provided there is at least three years of term extension after date of application for reinstatement. Five other companies reinstate the policies irrespective of the health of the insured, provided there is at least five years of term extension to run, while several others adopt the same practice where there is a "long" period of extended insurance. Under endowment insurance, policies are usually reinstated without evidence of insurability where the term extension covers the period of the endowment. In a paper to the Actuarial Society (Volume X) I brought this problem to the attention of our members, and in Vol. XI Mr. Moir submitted a solution. It is not practical to make an extensive calculation under each case to determine whether it is to the interest of the company to reinstate the policy; but a sufficient number of calculations may be made to indicate the line on which to proceed. In my judgment it is safe to reinstate policies with more than five years of term extension, irrespective of the condition of health of the insured. There are

many instances where, on account of the insured's being very seriously impaired, it would be advisable to reinstate under policies with much shorter periods of term extension than five years. It is a good rule to insist on the usual requirements for reinstatement in order to deter the policyholder from allowing his premiums to remain unpaid in the future, and also so that if he applies for new insurance there will be record of his history. Exceptions should however be made with judgment.

Evidence of Insurability.

At one time it was customary to call for evidence of *good health* satisfactory to the company, before the policy was reinstated, but the great majority of the companies now use the phrase "*evidence of insurability.*" I have obtained the opinion of several actuaries and lawyers regarding the degree to which companies are protected by both of these phrases. It seems to be practically the unanimous opinion that the phrase "evidence of good health" may be construed as referring only to the personal condition of the policyholder at the time the application for reinstatement was made, and that the company would be legally debarred from refusing reinstatement because of any change in occupation, family history, moral hazard, etc. On the other hand, it is generally believed that the phrase "evidence of insurability satisfactory to the company" would put the company in a position where it could judge an application for reinstatement precisely as it would judge an application for new insurance. In this connection the law case reported on page 158 of Vol. XVI of our *Transactions* is interesting as showing that the court apparently applied the same rules to the statement in a reinstatement application as to that in an application for new insurance. A company may therefore apply the same strict requirements in the case of reinstatement as it would in acceptance of a risk submitted for the first time, which would of course include changes in family history, personal history, occupation, financial condition, habitat, etc. While the companies have undoubtedly this right, the practice is to be more lenient in considering applications for reinstatement than in passing upon the merits of risks for new insurance, the older the policy the more lenient being the treatment with regard to reinstatement. The present warfare has resulted in the companies receiving applications for reinstatement from men who had purposely dropped their

policies and who desired to reinstate them because they had volunteered or intended to volunteer for military or naval service in Europe. It is the custom of several companies requiring "evidence of insurability satisfactory to the company," to insert a war clause before reinstatement although the original policy did not have any war clause in it. The Insurance Department of Canada has approved such a clause. The following is an extract from a letter from the Superintendent, dated December 11, 1914: "It would appear reasonable that this evidence of insurability should not be more exacting to the policyholder than the requirements as to insurability in the case of a *new* proposal, and it would perhaps be permissible to modify the conditions of the policy after restoration in accordance with any special conditions being included in new policies issued." As the company in question was including a war clause in new policies, it was entitled under the foregoing opinion to include a similar clause before restoring policies to full force.

Two companies which have had an unfortunate experience in connection with suicides in the first year after policies were reinstated, require the applicant to agree to a suicide clause applicable for one year after the policy is restored to force. It is held by these companies that as there is a suicide clause during the first year under new policies, they are entitled to exact the same conditions under reinstated policies. In one of these companies a lawsuit is pending at the present time because the insured had withheld the information that he was an embezzler at the time of reinstatement; he committed suicide after the policy was restored. An interesting case on these lines is reported in *T. A. S. A.*, XVI, page 154, the court holding that the period of contestability was revived under a reinstated policy.

Mortality under Reinstated Policies.

Some of the companies have not used the personal health certificate because of apprehension that advantage of it might be taken by policyholders who had learned after the days of grace had expired that they were poor risks. One or two cases of apparently that type made the New York Life Insurance Company decide to ascertain its relative mortality among reinstated policyholders. If we were to rely on our preconceived ideas, our opinion would probably be that the mortality among policyholders who were rein-

stated within two months after the premiums were due and unpaid would be no worse than among other policyholders whose insurance was originally taken at the same time and at the same age. This opinion would be based on a consideration of the reasons for failure to pay the premiums when due, the principal of which would be—

(*a*) Carelessness,

(*b*) Lack of ready money,

(*c*) Belief that the policyholder did not need insurance,

(*d*) Intention to drop the insurance because of having taken a policy in another company.

Keeping in mind that persons in poor health would not voluntarily let their policies terminate, it may be said that, with the exception of class (*b*), the foregoing would be "average" risks, and that this class would probably not contain a large proportion of under-average lives. It is reasoning of this nature that has apparently convinced the majority of the companies that a simple declaration by the policyholder regarding his health is sufficient protection under policies which have been recently lapsed. No experience has been published whereby it could be determined whether this line of reasoning is correct or not. I take pleasure, therefore, in presenting to the members of the Actuarial Society the experience of the New York Life under policies reinstated, with the exceptions hereafter mentioned, during the years 1908 to 1911, observed to the anniversaries falling between July 1, 1914, and June 30, 1915. As any selection against the company would make its appearance in the early years after reinstatement, an experience covering a longer period of time is not necessary.

The expected deaths in the various exhibits have been calculated by the Compound Progressive and by the American Tables, the former being a select table. If a policy had been taken at age thirty and had lapsed for non-payment of premium at the end of the third policy year, the expected deaths in the first year after reinstatement would be determined from the rate of mortality at age thirty at issue in the fourth policy year. It may be well to state that the C. P. Table represents the experience of the New York Life Insurance Company under policies issued during the thirty-one years prior to 1902, that it is based on amounts insured, and that the experience in recent years is distinctly lower than shown by this table. In calculating the expected deaths a suit-

able deduction was made for the period between the date the premium was due and the date of approval of reinstatement as the case would not have appeared in this investigation if the death had occurred during that period. In the case of the personal health certificate this was found to be sixty-seven days. No distinction was made between men and women, but all cases treated as under-average lives were excluded. The policies with few exceptions were issued in the United States and Canada. Where a policy was originally issued at the regular rate of premium, and on reinstatement was charged an advance in age, the expected mortality was taken at the rated-up age.

The practice of the company is to reinstate policies on receipt of a satisfactory personal health certificate within one month after the days of grace have expired, provided no unfavorable record appears against the policyholder. The officials at the branch offices had the right to approve the reinstatement of cases of small amount which had been lapsed for a short period, provided nothing unfavorable was recorded against the case. The average amount of insurance on these cases was about $1,200, while the average time elapsed between the end of the month of grace and the signing of the personal health certificate was less than thirteen days. It was not deemed worth while to investigate the mortality under such cases because they were expected to show a better mortality than the cases reinstated by the home office on a personal health certificate.

Personal Health Certificate.

In preparing these cases for investigation it was noted that in a goodly percentage the policyholder had been remiss in paying his premium on more than one occasion. The experience was therefore divided into (1) cases lapsed once only and restored under a personal health certificate; (2) cases which had been reinstated two or more times, a personal health certificate being accepted in each case. As the mortality under (2) was as good as under (1) the results have been combined. When a case was restored under a personal health certificate, and later was reinstated on receipt of a medical health certificate, or of a complete medical examination, it was considered to terminate at the date the premium was unpaid on the second occasion so far as concerns the investigation into the mortality under personal health certificates. It then entered into

another classification at the date prior to the time it was reinstated under a medical health certificate or a full medical examination. The following shows the experience under one or more health certificates:

REINSTATED UNDER ONE OR MORE PERSONAL HEALTH CERTIFICATES.

Years Since Reinstatement.	Actual Deaths.	Expected Deaths by Compound Progressive Table.	Ratio to C. P. Table.	Expected Deaths by American Table	Ratio to American Table.
By Policies.					
1	67	92.29	73%	126.07	53%
2	73	98.53	74%	129.67	56%
3	68	90.36	75%	114.96	59%
Thereafter	156	181.81	86%	221.76	70%
Total	364	462.99	79%	592.46	61%
By Amounts Insured.					
1	$142,000	$218,104	65%	$294,089	48%
2	193,500	238,794	81%	309,987	62%
3	192,000	215,041	89%	270,768	71%
Thereafter	359,800	435,639	83%	526,157	68%
Total	$887,300	$1,107,578	80%	$1,401,001	63%

It is evident from the foregoing table that the mortality experienced under policies reinstated on receipt of a personal health certificate was satisfactory, and justifies the opinion of the companies that this system is safe. In considering these results it should be remembered that the personal health certificate was not used if the applicant had been declined for insurance within recent years, was known to be an impaired risk, or was suspected of concealing any matter of importance. Furthermore, if the personal health certificate indicated a recent illness, or a change to a hazardous occupation, or in fact any change of importance, the applicant was required to furnish either a medical health certificate or a full medical examination and such cases were not included in the above synopsis.

A review of the death losses shows a very small proportion in which all the facts regarding the state of health of the insured had not been brought to the attention of the company through the medium of the personal health certificate, through an inspection report or through some other channel. These cases were too few

to justify a change in a system which is so convenient and which throws so few obstacles in the way of policyholders many of whom would seize slight excuses to defer reinstating their policies. The causes of death were not materially different from those among the policyholders as a body. One company found that there was an undue proportion of deaths from suicide, but our experience was not materially different from that under our total business. Another company stated that the proportion of death losses from tuberculosis to total deaths among reinstated policyholders was high, but we had only 9.4 per cent. from that cause, against 10.6 per cent. among all policyholders during the years 1913 and 1914.

As already stated an investigation of the experience under reinstatements with two or more personal health certificates indicates that the mortality is as good as under policies where there has been only one reinstatement.

Medical Health Certificate.

If application is made for reinstatement after sixty days and before six months have expired, our rule is to require a medical health certificate, but it is also requested in certain types, under which the premium has been overdue for less than sixty days. These generally consist of cases on which the company has unfavorable information on its records. The form of medical health certificate calls for a simple but fairly complete physical examination.

Where an insured had been previously reinstated on the basis of a personal health certificate he was considered to enter the present section of the investigation at the due date of the premium immediately preceding the reinstatement of the policy under a medical health certificate. Throughout the investigations the rule in brief form was as follows: If the later requirements are more stringent than the first, then the observation should be terminated under the less stringent requirement, and entered for further observation on the more severe conditions for reinstatement. If, however, a medical health certificate or a full medical examination were asked for, then a future restoration on easier terms (for example, under personal health certificate) would be ignored.

The experience under policies reinstated on medical health certificates was not sufficiently extensive to justify a division by policy years since reinstatement. It is as follows after making an

adjustment in the expected deaths for the time elapsed from date of lapse to date of reinstatement:

UNDER MEDICAL HEALTH CERTIFICATE.

	Actual Deaths.	Expected Deaths by Compound Progressive Table.	Ratio to C. P. Table.	Expected Deaths by American Table.	Ratio to American Table.
By policies	76	86.85	88%	110.85	69%
By amounts insured	$183,500	$171,295	107%	$216,915	85%

Evidently the form of medical health certificate was fitted to the type of cases to which it was applied by the company.

COMPLETE MEDICAL RE-EXAMINATION.

The last investigation deals with cases reinstated on furnishing a complete medical re-examination. These were all cases under which the premium was six months or more overdue, or under which the company decided, for special reasons, that a full medical examination was desirable, the latter being few compared with the former. In the following exhibit, as well as in the others, allowance has been made in calculating the expected deaths for the average time elapsed between the date the premium was payable and the date the policy was reinstated:

REINSTATED ON COMPLETE MEDICAL RE-EXAMINATION.

	Actual Deaths.	Expected Deaths by Compound Progressive Table.	Ratio to C. P. Table.	Expected Deaths by American Table.	Ratio to American Table.
By policies	50	80.76	62%	104.43	48%
By amounts insured	$196,300	$195,364	100%	$248,530	79%

As was to be expected, the mortality under the above conditions was light so far as the experience by policies is concerned. There was one claim in the second year after reinstatement for $45,000, the death of the insured being due to an automobile accident. This accounts for the large difference between the experience by amounts and by policies.

It has been the custom of this company to call for the usual re-

quirements regarding reinstatement, although there was a long period of term extension. Where that period was five or more years the policy was reinstated irrespective of the condition of the insured's health. Unfortunately, there was no way of segregating these cases except with a very large amount of labor. As there was not a large proportion of such cases, and as they are probably divided proportionately among the four groups under investigation, it was not thought necessary to go to the expense of classifying them separately.

I trust that the publication of the foregoing experience will result in the publication of the statistics of other companies, and in a full discussion of the treatment of policyholders who apply for re-instatement while their contracts are running on term extension.

Military Service and its Bearing on the Policy Contract.

The Recent European War Experience of the Mutual Life Insurance Company of New York.

BY

JOHN S. THOMPSON.

A few remarks on the risks incidental to military and naval service may be of interest, especially in connection with the problems which all European and several Canadian and American companies now face in reference to their European business.

Military and Naval Service Clauses.

America.

The development of the present American attitude towards the extra risk due to service in the army and navy, is a matter of history so far as the policy contracts are concerned and as such is probably familiar to a majority of the older members of the Society, but, for purposes of comparison, some details are reproduced here pertaining to the changing character of the military and naval service clauses during the past quarter of a century. In this investigation, reference was made to the policy forms of about a dozen of the largest companies which were in 1890 and are still highly prosperous, writing now about 60 per cent. of the total new standard business written each year in New York State.

Twenty-five years ago we find in the policy-forms of these companies, clauses such as the following, which are typical and are selected at random:

"This policy does not insure against death occurring while engaged in military or naval service in time of war or in consequence of such service. . . . In any such case the policy shall be null and void but on its surrender within six months thereafter, with satisfactory proofs of death, the net reserve (computed by the American Table of Mortality and interest at 4½ per cent.) after deducting all indebtedness will be paid therefor."

"Without the written permission of the President or Secretary of the Company, the insured shall not during the continuance of the policy . . . engage in military or naval service of any kind in time of war."

"If within three years from date, the insured shall . . . enter or be engaged in any military or naval service whatever (except in time of peace) . . . this policy shall be null and void."

"The insured may without previous notice to the Company . . . engage in any military or naval service . . . but in such case he shall pay . . . for the risk of military or naval service in time of war . . . an extra premium equal to that charged by the Company in similar cases, which, if not paid at the time of assumption of the risk, shall not invalidate the contract but shall be a lien upon the policy and deducted therefrom with interest upon its payment."

A single company made the liberal provision that the "policy after two years will be incontestable except for fraud or non-payment of premium" but with that exception each company was protected against the extra risk of military and naval service in one of two ways: (1) the insured must pay or allow to be charged as a lien against the policy, such an extra premium (in one or two cases a maximum was named) as the company found it necessary to levy on all policy-holders incurring the extra hazard, or (2) the company would pay to the insured's representatives in case of the insured's death in active military or naval service the net reserve on a prescribed basis; in one case the policy is declared to become null and void as a consequence of such service and the net reserve must be accepted in settlement of the policy when it becomes a claim whether the insured has died at the time of service or not. In one case the protection derived from one or other of these methods is made applicable only to the first three policy years but in the remaining ten cases extends "during the continuance of the policy."

Five years later it is found that the company whose policy was "incontestable after two years from date except for fraud or non-payment of premiums" is now asking applicants to sign an application wherein it is agreed that "if within two years from the date of the policy, the insured shall be engaged in military or naval service in time of war, this policy shall become null and void." The company which in 1890 was using the last clause quoted in the preceding paragraph, has introduced an agreement in its application blank to the effect that "if during the continuance of the policy the insured without the written consent of the Company shall

engage in military or naval service in time of war, the policy shall become null and void." About half the companies in question have made no substantial change in the provisions relating to military and naval service and the remainder have modified their policy forms so that service in the army or navy affects the settlement of the policy only during the first two or three years of the policy's existence instead of throughout its entire duration. In other words, at this time only seven companies make their policies subject to the conditions of their respective military and naval service clauses for the duration of the contracts, one retains a three-year limitation and the remaining companies provide for protection against extra hazard only for two years from date of issue.

In 1900, the experience of the Spanish-American war has had no noticeable effect upon the procedure of life companies in respect to the extra risk due to military and naval service. Excluding a company which utilizes a clause containing the following provision:

"If the insured shall at any time be engaged in army or naval serivce without a permit therefor signed by an officer of said Company, then this policy shall be null and void, but after one year from the date hereof and before such service is entered upon, a permit therefore will be granted, if requested, in consideration of an extra premium of not exceeding two and one-half per cent. of the sum insured, payable annually and in advance,"

there are now only five companies of those under observation which control the army and navy hazard during the continuance of the policies. Two companies since 1895 have made their contracts absolutely unrestricted by means of a comprehensive clause somewhat like the following:

"This policy is absolutely free of conditions as to residence, occupation, travel, habits of life and manner, time and place of death. No permit nor extra premium will be required for military or naval service in time of war or peace. The policy is incontestable from issue."

The remainder of the companies in question introduce a provision restricting military and naval service in time of war only during the first one or two policy years. One of the latter which formerly paid the Combined 4 per cent. reserve on the voiding of the contract through the violation of certain conditions, now pays, under similar conditions, the slightly larger American 3 per cent. reserve, and another, which voided the policy absolutely during the

first three years or paid the reserve in case of death if at least three premiums had been paid, now provides for the continuance of the policy in full force in case of active service after the first policy year or payment, in advance, of an extra annual premium of 2½ per cent. of the face of the policy.

These conditions continue until 1905 with respect to eight companies. Two companies have joined the ranks of those which make their contracts absolutely free from restrictions as to military or naval service although in these two cases policies are incontestable not from date of issue but after two years from date of issue. The company which in 1895 agreed to pay the Combined 4 per cent. reserve in case of death in service without a permit and the payment of the required extra premium and in 1900 under similar conditions agreed to pay the net American 3 per cent. reserve, now contracts to pay under the same circumstances the amount of premiums paid. There are therefore four companies writing unrestricted policies, five issuing policies which, throughout their continuance, do not permit military service without certain restrictions, and the balance of the companies impose intermediate limitations in their new contracts, usually restricting military service in the first two policy years. On the whole, there is a decided trend towards the liberalization of this feature of the policy form.

Between 1905 and 1910, four more of the companies under observation have adopted the most liberal clause providing that "the policy is free from restrictions as to residence, travel and occupation" discarding therefor more stringent clauses which have been in use, though modified from time to time, for a great number of years so that now, in 1910, new insurance contracts of eight of the companies are unrestricted as to military and naval service, the clauses relating to which are combined with incontestability clauses, variously providing for incontestability, except for fraud or for non-payment of premium, after one or two years. There remains now only a single company issuing policies containing an occupation provision controlling the policyholder's engagement in active service in the army and navy throughout the duration of the contract. At the beginning of 1914 we find that this company has changed its practice and adopted for its new policies the following more liberal provision:

"If within one year from the date of the policy, the insured shall without the written consent of the company, engage in military

and naval service in time of war . . . this policy shall be void and no claim shall exist hereunder."

Of the companies under observation in 1914, therefore, two employ "one year" military and naval service clauses and the remainder the unrestricted provision. At the time when it appeared that active intervention by the United States in Mexican affairs was a distinct possibility, one of the companies, until that time offering contracts unrestricted as to service in the army and navy, adopted the following provision which is still in use in 1915:

"If the insured engages in military or naval service or in work as a civilian in any capacity in connection with actual warfare during the first policy year, there shall immediately become due and payable a single extra payment of three per centum of the face of the policy. If the insured shall so engage and shall die within one year from commencement of such service or work, without having paid said additional charge prior to such commencement, the Company's liability shall be limited to one fifth of the face of the policy."

The great majority of the policies now being issued, however, are absolutely unrestricted as to military and naval service, and such as are not, contain a restriction which in none of the cases under observation extends beyond the first policy year.

This brief analysis exhibits in some detail in connection with one small feature of the policy contract a tendency which has been dealt with more generally by Mr. Moir in his paper entitled "Liberality of Modern Policies" (*T. A. S. A.*, XII, 175) and much that he says there may be specifically applied here. There is no doubt as to the propriety of having modified, for example, the "travel" clause of fifty years ago on the ground that the development of interstate and international communication has subtracted from travelling many of the hazards against which the old practice purported to guard the company, but recent history disproves the theory that the chance of war is remote and there is no doubt as to the deadly nature of modern conflict.

Great Britain.

It is probable that prior to August, 1914, the British practice in regard to military and naval service was in general only slightly less liberal than that of American companies and was also, like that of our companies, the outcome of an evolutionary process extend-

ing over a considerable period. "World-wide" policies would be issued to those applicants who had reached at least a specified age (say twenty-five or thirty years) and who satisfied the company that they were of such settled pursuits as to be unlikely to proceed beyond the usual "free limits" of travel or to engage in any hazardous occupations, of which, of course, active military service would be one. But at present, in contrast with their former position and with the prevailing practice of American companies, a common war clause in British life policies is one substantially as follows:

"The policy will be void if the assured shall, without the previous permission of the directors (which may be obtained on payment of such additional premium, either single or recurrent or on such reduction of the sum assured as the directors may think proper) . . . take part in any naval or military operations during actual warfare or in any insurrection or any expedition or operation of a warlike character either as a combatant or a non-combatant . . . or engage in any trade, business or occupation involving special or unusual risk to life or health unless the fact of such engagement or employment be mentioned in the policy."

It is interesting to observe, that, as an act of patriotism, the majority of British companies are not charging extra premiums on account of the war service of old policyholders, even when authorized by the policy contract to do so, except as they may be in some cases professional soldiers in the regular army, although the feeling has been frankly expressed by some managers that the government should pay reasonable extra premiums levied for active service, in order that the extra loss due to the service of policyholders might be as widely distributed as possible and not fall, perhaps excessively, on the members of individual companies.

France.

The late E. B. Phelps is authority for the statement that the military and naval service clause found in French policies usually stipulates that

"the policy shall not be forfeited if the extra premium is not paid but if the assured die within eight months of the cessation of hostilities, the Company will pay to the beneficiaries only the reserve instead of the full face value of the policy."

On the same authority, the French companies since the beginning

of the war have, at the suggestion of the government, expressed their willingness to reduce the eight months' limitation to one of three months and have agreed to divide among the insured any excess of war premiums over the actual excess mortality due to the war.

Austria and Hungary.

A manual of Austrian policy conditions discloses the following facts concerning the Austrian companies' attitude in 1912 towards military service of civilians in the "Landwehr" and "Landsturm" in time of war:

1. The war risk was assumed for the full amount of the policy without extra premium by two companies with respect to both the 'Landwehr' and the 'Landsturm' and by two others with respect to service in the 'Landsturm' only.

2. Eighteen companies assumed the extra risk for the full amount without extra premium up to a certain maximum which varies with the company and may in the same company be different for the 'Landwehr' and 'Landsturm' and in no case cited exceeds 30,000 crowns or about $6,000. Full risk would in some cases be assumed if extra premium were paid on the portion not assumed free by the company.

3. In eight companies the risk, with or without a maximum, is assumed without extra premium to the extent of only one half of the face amount, the maximum, where operative, in no case exceeding 15,000 crowns or about $3,000. Full risk would in some cases be assumed, if extra premium were paid on the portion not assumed free by the company.

4. Two companies with respect to service in the 'Landwehr' and two others with respect to both the 'Landwehr' and the 'Landsturm' declined to assume an extra risk without an extra premium. Three of these companies allow the reserve or surrender value as a claim where the insured dies in service and has not paid the requisite extra.

All the companies other than the three mentioned in (4) also allow in case of death in active service either the net reserve, the surrender-value or full or partial premium return in case the company "declines to pay a war claim in full or in part." One company undertakes under such circumstances to pay the net reserve if it exceeds the maximum on which the company assumes the

extra risk without the extra premium. Several companies that assume a war risk do so only provided the policy shall have been in force one, three or six months prior to the declaration of war.

Many of the companies mentioned in the preceding paragraph do business also in Germany and it is probable that the practice in general resembles that in Austria but, in the absence of data, it was impossible to make a similar analysis. Neither is there information at hand as to what modifications, if any, since the beginning of the war, have been made in either country in the insurance companies' methods of dealing with civilian applicants liable to military duty.

War Claims in the Mutual Life.

The war losses of which intimation has been received by the Mutual Life to date amount to about $400,000. This sum includes not only those killed in active service but also those dying from disease contracted in service or in preliminary training and a few even more remotely related to active hostilities. In view of the necessary delay in the reporting of deaths by the company's foreign representatives this may be said to approximate to the extra mortality due to the first year of war or perhaps to even a shorter period than the first full year. The following table exhibits the amount of these losses incurred in each of the countries or divisions thereof at war and the amount of business in force at the middle of 1914 in each country:

Country.	Business in Force July 1, 1914 (Approximated).	War Losses.		Percentage Loss.
		Lives.	Amounts.	
Great Britain	$57,950,000	27	$156,213	.270
South Africa	12,310,000	5	22,172	.180
Australia	7,810,000	1	493	.007
Canada	34,340,000	11	36,701	.107
Subtotal	$112,410,000	44	$215,579	.192
France	40,450,000	40	87,117	.215
Belgium	9,000,000	4	7,496	.083
Italy	9,380,000	1	1,930	.021
Germany	9,720,000	10	18,517	.190
Austria Hungary	22,030,000	14	39,267	.178
United States	.....	5	30,250	.....
Total	$202,990,000	118	$400,156	.196

Included in the above mentioned losses are eight lives insured for $57,907 whose deaths were caused by the destruction of the steamship Lusitania. Of this latter amount of insurance almost half was on the lives of American citizens, and of the total war losses, the total United States issues amount to $54,025 or about 13½ per cent. This indicates that even those companies which have always underwritten only domestic business may not be entirely exempt from "war mortality."

The distribution of these losses according to ages at death is as follows:

Ages at Death.	Number of Lives.	Amount of Claims.
25 and under	11	$ 31,202
26–35	28	102,071
36–45	45	105,586
Over 45	34	161,297
Total	118	$400,156

A classification based on causes of death has been roughly established with the following results:

Cause of Death.	Amount of Insurance.	Percentage.
(1) Killed in service	$101,212	33.0
(2) Died from wounds received in service	36,525	11.9
(3) Illness and disease due to active service	105,105	34.3
(4) Accidents to non-combatants (includes "Lusitania" losses)	63,828	20.8
	$306,670	100.%

There are claims amounting to $93,486 about which we have as yet no information other than the fact of "death from war."

Hence, excluding "accidents to non-combatants" and throwing classes (1) and (2) together, it may be seen that of those losses concerning which we have information at the present time 56.7 per cent. are incurred in battle and 43.3 per cent. through the operation of illness and disease. For the sake of simplicity and in consideration of the smallness of the figures involved, these classes have been given considerable latitude. For instance, class (1) includes not only those shot in action but also those shot by "snipers" at times of comparative inaction on the firing line and those killed while reconnoitring; one among them was court-martialed and shot for treason and another was an engineer drowned in the sinking of an armed merchantman. One in class (2) was a chaplain

wounded while in an ambulance at the front. Class (3) is decidedly heterogeneous and, if the data were extensive, offering an opportunity for instructive deduction, would present the most difficulties. Some in the class died from pneumonia contracted in preliminary training; many others were not, strictly speaking, in active service such as "quartermaster, injured in accident," "railroad guard, cause of death, gastritis," "clerk in war office—cause of death, tumor of brain," "barrister, who assisted in the recruiting and training of troops," etc. Two doctors were included who died of variola and typhus fever respectively, these diseases being contracted in military hospitals.

The following table computed by Mr. A. G. Mackenzie and reproduced from Messrs. Schooling and Rusher's paper (*J. I. A.*, XXXVII, 545) on the mortality due to the South African war, gives the comparative losses from battle and disease in several important campaigns:

War.	Losses Per Cent. from Battle.	Losses Per Cent. from Disease.	Total Losses, Per Cent.	Proportion of Losses.	
				In Battle, Per Cent.	By Disease, Per Cent.
Crimean.					
English, per annum..	3.3	9.3	12.6	26.2	73.8
French, per annum ..	3.4	12.1	15.5	21.9	78.1
American Civil.					
North, first year.....	1.7	5.0	6.7	25.4	74.6
North, four years....	3.9	7.4	11.3	34.5	65.5
Austro-Prussian, 1866.					
Prussian losses, seven weeks.....	1.4	1.8	3.2	43.8	56.2

Hence from the scanty facts now to hand in connection with the present war, the indications are that the machinery of battle is becoming more deadly and that, through increase in medical skill and increased attention to sanitation, disease and infection claim a proportionately smaller number of victims.

The distribution of losses according to occupation at death is as follows:

Class.	Lives.	Amount of Insurance.	Average Insurance.
(i) Officers.....	35	$152,439	$4,355
(ii) Non-commissioned officers and privates.....	33	70,559	2,138
(iii) Others.....	19	83,672	
Total.....	87	$306,670	

No distinction is made in class (i) between those who are volunteer officers and those who are officers of the regular army.

Class (iii) is so miscellaneous that further detailing would not be instructive.

It is, however, the ratio of the war losses to the amount of insurance which has been taken into the war through the enlistment or conscription of policyholders that is of interest rather than the ratio, exhibited in a preceding paragraph, of the war losses to the total business in force, much of which, for various reasons, is definitely removed from the risk of mortality due to the war.

Unfortunately, because of the large proportion of outstanding insurance in connection with which permits for service are not required, we cannot say what amount of insurance is carried on policyholders who have entered the war, but an attempt may be made to estimate it from the general considerations outlined herewith.

The business in force on male lives in the Mutual Life as of December 31, 1913, in the various foreign countries at war was distributed approximately as follows, according to the indicated age groups:

Country.	Ages 30 and under.	Ages 31–45.	Ages over 45.	Total.
Great Britain....	Amt. $1,210,000	$16,050,000	$30,060,000	$47,320,000
	% 2.6	33.9	63.5	100
Australia.....	Amt. 150,000	1,760,000	4,480,000	6,390,000
	% 2.3	27.6	70.1	100
South Africa.....	Amt. 690,000	5,820,000	4,120,000	10,630,000
	% 6.5	54.8	38.7	100
Sub-total.....	Amt. $2,050,000	$23,630,000	$38,660,000	$64,340,000
	% 3.2	36.7	60.1	100
France.....	Amt. 40,000	13,700,000	25,110,000	38,850,000
	% 0.1	35.3	64.6	100
Belgium.....	Amt. 240,000	2,900,000	5,180,000	8,320,000
	% 2.8	35.0	62.2	100
Italy.....	Amt. 80,000	2,660,000	5,680,000	8,420,000
	% 1.0	31.6	67.4	100
Germany.....	Amt.	1,200,000	7,760,000	8,960,000
	%	13.4	86.6	100
Austria-Hungary .	Amt. 610,000	7,330,000	10,570,000	18,510,000
	% 3.3	39.6	57.1	100
Total amount..	$3,020,000	$51,420,000	$92,960,000	$147,400,000
Total per cent..	2.1	34.8	63.1	100

The reason for the abnormally low percentages of business at the lower ages in some countries is due to the fact that in those

countries the company has not written new business for a number of years. The writing of new business was discontinued in Prussia in 1895, in the remainder of Germany in 1904, in France in 1906, in Italy in 1907, and in Austria Hungary in 1913.

Consider in connection with this the following table showing the distribution of the male population of the chief countries according to approximately the same age groups:

Country.	Ages 15–24.	Ages 25–34.	Ages 35–44.	Total ages 15–44.	Ages 45 and over.	Grand Total.
Great Britain.	4,012,000	3,506,000	2,909,000	10,427,000	4,606,000	15,033,000
France.....	3,198,000	2,879,000	2,563,000	8,640,000	5,256,000	13,896,000
Italy.....	2,799,000	2,066,000	1,846,000	6,711,000	3,772,000	10,483,000
Belgium.....	629,000	518,000	397,000	1,544,000	716,000	2,260,000
Germany.....	5,206,000	4,187,000	3,218,000	12,611,000	5,296,000	17,907,000
Austria-Hungary.....	4,063,000	3,124,000	2,718,000	9,905,000	4,598,000	14,503,000
Total.....	19,907,000	16,280,000	13,651,000	49,838,000	24,244,000	74,082,000

With the exception of the figures relating to Great Britain, which are drawn from the census of 1911, the populations are based on censuses taken in or about 1901 (brought together in Webb's Dictionary of Statistics), more recent data not being immediately available.

Let it be supposed that in the active service of the six countries mentioned above, there are armies totalling fifteen million men: a great proportion of these are below age 30 but as our experience shows, a substantial number must be above age 45. It was found that such an army could be recruited by enlisting 40 per cent. of the male population at ages 15 to 24, thirty per cent. of those aged 25–34, fifteen per cent. of those aged 35 to 44, and, say, five per cent. of those aged 45 and over. If the insurance in force in the three age-groups cited above is drawn upon in similar proportions, say, to the extent of thirty-five per cent. for that in force under age 30, fifteen per cent. for that at ages 31 to 45, and five per cent. for that at ages over 45, we have the following table giving the rates of loss based on these assumptions, the smallness of the Italian rate being due, of course, to the fact that Italian business has been at risk for considerably less than one year:

Country.	Estimated Exposure to Risk.	War Loss.	Loss, Per Cent.
Great Britain.....	$4,330,000	$156,213	3.6
France.....	3,330,000	87,117	2.6
Belgium.....	780,000	7,496	1.0
Italy.....	710,000	1,930	0.3
Germany.....	570,000	18,517	3.2
Austria-Hungary.....	1,840,000	39,267	2.2
Total.....	$11,560,000	$310,540	2.7

On this basis, therefore, a war extra of 5 per cent. per annum would probably be sufficient protection in any of the warring countries.

Officers and men are subject to such different rates of mortality that some attempt should be made to further subdivide the data. Let the following assumptions be made:

1. That for every male individual in the general population who is likely to be enlisted as an officer in the army there are ten who will enlist as privates. That is to say, if the male population from which the armies of the nations named above are to be drawn numbers 50,000,000, this is divided on this supposition into, say, 5,000,000 prospective officers and 45,000,000 available for the rank and file.

The basis for this rather broad statement is derived from the occupational classification of male lives of England and Wales as is shown in the 1911 Census. According to that enumeration the total number of occupied and unoccupied males in England and Wales between the ages of twenty and forty-five was about 6,670,000. Of these it was found on examination, that the learned professions, students, merchants and high-class retail tradesmen, municipal officers, those of private means, etc., constituted a group of about 500,000. This left in the remaining class domestic and commercial clerical employees, general laborers and the great mass of industrial employees of which the agricultural, shipping and textile manufacturing interests claim such enormous individual groups. If ages above 45 had been included, the result would doubtless have been to lower the ratio of the secondary to the preferred class. It was believed that on the continent, the relative extent of the "officer" class and of the "private" class would be much the same. It is realized that such a classification is not to be undertaken lightly and involves a breadth of knowledge which

is not claimed, as to European social and industrial conditions; the conclusion is highly speculative but appears at least reasonable and is adopted for purposes of discussion.

2. That in the armies on a war basis there is one officer to each forty non-commissioned officers and men.

The relative numbers of officers and men in the principal European armies (excluding Russia) on a peace footing are, as given in the 1915 edition of the Statesmen's Year Book and other up-to-date sources:

Country.	Officers.	N. C. O's and Men.	Ratios.
Great Britain.....	12,800	258,500	1 : 20
France.....	31,600	613,700	1 : 19
Italy.....	15,200	289,900	1 : 19
Germany.....	36,600	799,000	1 : 22
Austria.....	34,000	390,200	1 : 11
Total.....	130,200	2,351,300	1 : 18

On a war footing, the regiments are expanded; in connection with the actual details of this process, little official information is available but we have the following furnished by the Current Literature Publishing Company in their 1911 Encyclopedia:

	Ratios. Officers.	Men.
GREAT BRITAIN.		
"The war strength of the battalion is 29 officers and 955 men with an additional officer and 90 men left at the immediate base to fill the first vacancies."	1	35
FRANCE.		
"The war strength of battalions is given by Van Loebell as 19 officers and 1009 men."	1	53
"The war strength of cavalry squadrons is five officers and 160 men."	1	32
"The war strength of batteries is 4 officers, possibly 5, and 170 men except that mountain batteries have 200 men."	1	43
ITALY.		
"The war strength of batteries of field artillery probably follows the German organization, at least for the new matériel, viz., five officers and 150 men."	1	30

No information is given as to infantry unit.

GERMANY.

"It is probable that the war strength of an infantry battalion is able to count 25 officers and 1,000 rifles."	1	40
"The cavalry squadron on a war footing numbers 6 officers and probably 172 men."	1	29

AUSTRIA-HUNGARY.

"The war strength of batteries is six officers and about 250 men."	1	42

No details are given as to infantry formations.

Hence, on this supposition in armies numbering 15,000,000 non-commissioned officers and men there would be 375,000 officers.

3. That the insurance in the Mutual exposed to war risk respectively in the class of officers and in the class of privates is approximately in the ratio of two to one.

A classification along the same lines as that made above in connection with the English Census was instituted in connection with 172,057 lives resident in the United States and Canada to which insurance was issued in the years 1907–1912 inclusive. As far as possible the same occupation groups were thrown into the respective classes as in the former case. The total number of lives belonging to the issues in question was about 191,000, so that over 90 per cent. of them entered into the observations.

It was found that 86,901 belonged to the "preferred" class and 85,156 to the secondary division. As it was noted from those policies which have already become claims that the average policy in the former class was about double that in the latter it was determined to assume that the amounts of insurance outstanding in the two divisions were in the ratio mentioned above.

The insurance in force on male lives up to age 50 inclusive in the warring countries where the Mutual has business in force amounts to $75,000,000. On the above basis, therefore, $50,000,000 is assumed to be carried by men eligible for commissions if entering the army and $25,000,000 by men who on enlistment would likely be called upon to fill the ranks.

4. That the total war claims intimated to date would be charged to the extra mortality of the respective classes in the same ratio as are those about which information sufficient to make a classification possible is available. If this is the case the $400,156 claims

are divided into $273,307 on lives of officers and $126,849 on lives of privates.

These suppositions mean that on the average about seven and one half per cent. of those eligible for commissions as officers have enlisted and about one third of those likely to enter the ranks as privates. If these ratios are applied to the insurance in force at age fifty and under, the result is that officers in service carry about $3,750,000 in the Mutual and the privates about $8,000,000. It is to be observed that the total of these two items of insurance corresponds fairly closely with the amount arrived at previously from other considerations. Hence, a minimum extra premium for officers would be about seven and one half per cent. of the sum assured and for the privates about one and one half per cent. It would have been interesting to introduce an age classification of the two groups. This would be of importance in consideration of the probable difference in the average ages of their members but the scantiness of the data rendered it impossible.

Two of the assumptions leading to these conclusions are highly arbitrary, but if it is determined and proves practicable to assess officers and privates at different rates on enlistment or to increase the rate of extra premium when a non-commissioned officer or private is advanced to the rank of officer, some such suppositions must be made. The different rate of mortality among officers and men was commented upon by Messrs. Schooling and Rusher who estimated that the death rate of British officers in the South African War was 7.8 per cent. per annum in the first year and 5.3 per cent. per annum for the whole war, whereas the death rate among the non-commissioned officers and men was estimated at 3.8 per cent. per annum for the war.

British Companies and the War.

The recent experience of British companies cannot, of course, be examined in detail but a generous budget of 1915 annual reports to which I had access threw some light on the subject. Ten companies gave approximately or exactly their war losses and the following table exhibits these as well as the total amount of death claims paid by the company during the fiscal year:

TEN BRITISH COMPANIES.

Total Year's Claims.	War Claims from Beginning of War to Date of Annual Report.	Ratio Percent of War Claims to Total Claims.
£ 809,986	£ 42,707	5.3
92,041	5,746	6.2
306,502	27,463	9.0
1,231,646	50,000	4.1
73,000	10,000	13.7
371,837	27,335	7.3
333,210	70,000	21.0
543,917	25,000	4.6
570,772	54,000	9.5
572,010	114,400	20.0
Totals £4,904,921	£426,651	8.7

Although the total claims is in practically all, if not all, cases within the "expected" claims according to generally accepted standard tables, the feeling seems to be that the rate of loss already experienced is but little guide to the future and in some cases substantial reserves have been appropriated to take care of adverse mortality due to the war in anticipation of greater losses when the "new army" has done more work on the firing line. As stated previously, policies issued prior to August, 1914, on the lives of civilians are usually free of extra premium for military service, but members of the regular army are charged extras varying from three to seven guineas per cent. according to the branch of the service and the practice of the individual company.

The complaints as to the depreciated and uncertain security values, the heavy income-tax and the curtailment of new business are universal. Because of the strong reserves accumulated under modern conditions the disturbance to companies' finances though great has not exceeded manageable proportions. An increase in the already heavy rate of income tax is imminent and is viewed with apprehension in some quarters. The writing of a reduced amount of new business was inevitable and was due not only to a lessened demand for insurance and to scarcity of money but largely also to the absence of office and branch representatives on active service. From 30 per cent. to 60 per cent. of various departments of various companies have joined the colors; from one company as many as 175 men have enlisted. Those who have not gone into active service are bearing the increased burdens of the work at

home, the positions of those enlisting being in most cases kept open and the full salaries being paid for the duration of the war. To read these reports impresses one with the conviction that there are many adverse contingencies from which a company is not protected by the most stringent of military and naval service clauses.

Legal Notes.

BY

WENDELL M. STRONG.

Power of Superintendent of Insurance:—(Stern *vs.* Metropolitan Life Ins. Co., New York Supreme Court, Appellate Division, 154 N. Y. Supp. 472.) Stern sued the company for commission for writing new business. The company demurred on the ground that the plaintiff had no license to act as agent. The question raised was that of the constitutionality of the section providing for the licensing of agents. The statute states: "The Superintendent of Insurance shall have the right to refuse to issue or renew any such certificate in his discretion." It had been held in the court below that this vested in the Superintendent an unrestricted discretion, or, in other words, arbitrary power to prevent any person from pursuing a lawful calling. The Appellate Division held that, as qualified by the other clauses not here quoted referring to forms of application intended to give the Superintendent such information as he might require, this clause did not indicate arbitrary action but contemplated the adoption of a common standard for all applicants, retaining, however, "in the Superintendent . . . authority to determine whether the applicant conforms thereto." The court, consequently, held that the statute was constitutional.

Tax upon Premiums Paid by Resident of State to Foreign Life Insurance Co.:—(Equitable Life Assurance Society *vs.* Pennsylvania, U. S. Supreme Court, 238 U. S. 143.) The question involved was whether in determining the amount of a privilege tax the State of Pennsylvania could consider premiums paid outside of the state by residents of the state. The supreme court of Pennsylvania had held that the state could do so, holding the tax in question to be one for the privilege of doing business in the state. The ground upon which the case went to the Supreme Court of the United States was the question whether the Pennsylvania statute as construed deprived the company of property without due process of law. The court held that it did not.

TAXES:—"DOING BUSINESS" WITHIN A STATE. (Kentucky *vs.* Provident Savings Life Assurance Society. Supreme Court U. S., not yet reported.) The question was that of taxation of life insurance premiums. The statute provided for taxation upon the premiums of all companies "doing business" in the state. The Society had withdrawn from the state on January 1, 1907, "had closed all its offices and had ceased to solicit or write insurance, or maintain any agent, or to collect any premiums under that jurisdiction." In the Kentucky Court of Appeals it was held, nevertheless, that, while its old policies continue in force and the premiums are paid to an agent in another state, a company is "doing business" within the meaning of the statute (*T. A. S. A.*, XIII, 333). The Supreme Court of the United States held, however, contrary to this. The following are extracts from the opinion:—

"As the policies remained in force, it is said that the company continued to furnish protection to citizens in Kentucky. The renewal premiums, as already stated, were paid in New York. There is, however, a manifest difficulty in holding that the mere continuance of the obligation of the policies constituted the transaction of a local business for which a privilege tax could be exacted. As a privilege tax, the tax rests upon the assumption that what is done depends upon the state's consent. But the continuance of the contracts of insurance already written by the company was not dependent on the consent of the state. It is true that acts might be done within the state in connection with such policies, as for example in maintaining an office or agents although new insurance was not written or solicited, which could be considered to amount to the continuance of a local business. In such case it would be the actual transaction of business that would furnish the ground of the license exaction, and not the mere existence of the obligation under policies previously written. These policies are contracts already made; the state cannot destroy them or make their mere continuance, independent of acts within its limits, a privilege to be granted or withheld. Neither the continuance of the obligation in itself, nor acts done elsewhere on account of it, can be regarded as being within the state's control."

"Assuming this to be the point in dispute, the question at once arises whether the matter is reviewable in this court. . . . Taxation without jurisdiction has been held to be a violation of the Fourteenth Amendment . . . and the principle involved applies to the assertion of authority on the part of the state to exact a license tax for the privilege of doing acts which lie beyond the sphere of local control."

This case was distinguished in the words of the court from the previous one of The Equitable Life Assurance Society *vs.* Pennsylvania as follows:—

"The present case thus differs from that of Equitable Life Assurance Society *vs.* Pennsylvania. It was not disputed that the Equitable Company was actually doing business in Pennsylvania. The question was as to the permissible measure of a tax exacted for a privilege admittedly exercised. As this court said: "The tax is a tax upon a privilege actually used. The only question concerns the mode of measuring the tax." In the present case it is not the measure of the tax for doing business, but the very basis of the tax—that is, whether the company was doing business within the state—that is in controversy."

Failure to Attach Application to Policy:—(Archer *vs.* Equitable Life Assur. Soc. of U. S. Supreme Court of N. Y., Appellate Division, 154 N. Y. Supp. 519.) The question involved in this case was whether fraud and misrepresentation in procuring the issuance of the policy could be proved, although the statements involved were not contained in any paper attached to the policy. The statutes in a number of states provide that no statement of the insured

"shall avoid the policy unless it is contained in a written application and a copy of such application shall be endorsed upon or attached to the policy when issued."

In New York, however, the provisions of the statute are very different, being simply that every policy of life insurance

"shall contain the entire contract between the parties, and nothing shall be incorporated therein by reference to any constitution, by-laws, rules, application or other writings, unless the same are endorsed upon or attached to the policy when issued."

In the states whose statutes contain the provision first quoted above, it is evident that the intention was to exclude any proof of fraudulent statements or misrepresentations in the procuring of the policy unless such were contained in the application and the application were attached to the policy. The provision in the New York statute, quoted above, is very different in its statement: it is that the policy shall contain the entire contract, following which there is a specific prohibition of incorporation by reference to "any constitution, by-laws, rules, application or other writings." It is

only necessary to go back to the time of the Armstrong investigation to find that it was customary to incorporate by reference, as part of the policy contracts, conditions contained in the application, or, in some cases, in the charter or by-laws of the company, and which were not stated in the policy. Thus the insured had no way to determine in full from his policy what his contract with the company was, and this was evidently a real evil to be remedied; the clause quoted did remedy this very specifically, and would appear to have been directed solely to such purposes.

Nevertheless, the court, following the case of Murphy *vs.* The Colonial Life Insurance Company, held that the New York statute excluded the introduction of proof of misstatement in the answers of the insured when a copy of the application was not attached to the policy; in other words, the New York statute was interpreted as if it had read like the clause first quoted above contained in the statutes of certain other states. Besides the reason quoted above for thinking that the New York statute did not really have the meaning attributed to it by the Appellate Division, there is the further reason that a state like Minnesota, where the first provision quoted above is in force, has a separately numbered provision of the Insurance Law, requiring that there be included in every policy the statement that the policy shall constitute the entire contract between the parties; evidently, then, the legislators of Minnesota and of several other states having similar provisions did not regard the provision that the policy shall constitute the entire contract between the parties as ruling out proof of fraud in statements in the application when such application was not attached to the policy, since it took particular pains to cover this latter by a separate special provision.

Suicide:—(State Mutual Life Ins. Co. *vs.* Long, Court of Civil Appeals of Texas 178 S. W. Rep. 778.) The interest of this case lies in the fact that the court held that the verdict of the jury against the company was contrary to the evidence and remanded the case for a new trial with the suggestion to the lower court that if the testimony were the same as on the first trial the jury should be instructed to find in favor of the company.

The policy contained the usual one year clause against suicide. The insured was in a store in the evening with two other men. One of these he shot and killed and then shot himself. The only

witness was the third man. His evidence, however, was uncontradicted and there was also evidence to show that the insured had been drinking and that he acted queerly when he had been drinking. No suspicion attached to the third man present who was the witness and the only evidence tending to show that the insured had not shot himself was the absence of powder marks.

It is well known that no matter how strong the evidence it is difficult for an insurance company to defend on the ground of suicide and it is encouraging to find a court taking such ground to prevent an unjust verdict from standing.

Effect of Wrongful Statement that Policy Was Lapsed: —(The Capital Life Ins. Co. of Canada *vs.* Parker, Supreme Court of Canada, 51 Canada Supreme Court Reports, 462.) A note was taken by an agent of the company for a quarterly premium which was overdue, and there was included in the note a small instalment due on the previous quarterly premium. The note was sent to the head office and nothing more was heard from it until the company's superintendent of agents, nearly a month later, and but a few days before the note matured, stated to the insured that the policy had lapsed because of the inclusion in the note of the instalment of the previous premium, contrary to the company's rule and beyond the authority of the agent. The insured, because of the statement that the policy had lapsed, did not tender payment of the note or the premium due very near the same time, or the following quarterly premium, and died within the days of grace after the due date of the latter premium.

The court held:—

First: That the receipt and retention of the note at the home office was an acceptance, even though the agent exceeded his authority in including a portion of the previous premium.

Second: That the statement to the insured by a high official of the company that his policy was lapsed and could be restored only on satisfactory evidence of insurability, made the tender of the premium and of the amount of the note coming due in a few days unnecessary in order to keep the policy in force, as it was a distinct notification that they would not be accepted.

Bankruptcy; Ownership of Policy:—(In *re* Dreuil & Co., U. S. District Court, Eastern District of Louisiana, 34 American

Bankruptcy Rep. 373.) The policies in question were endowment policies payable to the insured at maturity and to his wife in case of prior death. The court held that the respective interests of the insured and the beneficiary should be determined actuarially and the insured given the right to redeem the policy (presumably by securing payment to the trustee of a share of the surrender value proportional to the value of his interest in the policy).

This decision should be considered in connection with In *re* Churchill (Fed. Rep. 766, see *T. A. S. A.*, XV, 122), in which case the policy provided for a surrender value to the insured of not less than the face of the policy at the end of the twenty-year period, but in case of death the policy was payable to the insured's wife. The bankruptcy occurred only about two years before the end of the period. Thus the policy was very similar to an endowment policy which would mature in two years and which was payable to the insured at maturity. The court, however (Circuit Court of Appeals), held that, the insured having no present right to surrender the policy, the trustee took nothing.

Presumption of Death from Absence:—(N. Y. Life Ins. Co. *vs.* Holck, Supreme Court of Colorado, 151 Pac. Rep. 916.) The insured had been absent and unheard of for more than eight years and the attempt was made to obtain payment of the policy on the theory of the presumption of death in case of more than seven years' unexplained absence.

The circumstances were somewhat unusual. The insured quarrelled with his wife and, though living in the same town with her for more than a month afterward, did not speak with her or communicate with her. After leaving this town he was heard of occasionally for several years and the Christmas following his leaving, a package was received for his little daughter with a note in his handwriting, but in this was no communication for his wife. The insured had four brothers and sisters but each of them testified to not having seen him or heard of him directly for some time before his disappearance. In the lower court the case was decided against the company. There were two important exceptions taken. One was to the refusal to allow the company to introduce mortality tables showing the expectancy of persons the age of the insured at the time of his disappearance. A section of the Colorado statutes of 1908 provided for the use of a certain table in civil cases when

it should be "necessary to establish the expectancy of continued life of any person from any period in such person's life." The Supreme Court held that such table was competent as evidence and should have been admitted for the consideration of the jury in connection with the other testimony as of weight in helping to determine whether or not the insured was dead.

The other exception was to the charge of the lower court as to the presumption existing in case of absence unheard of for seven years. The Supreme Court held regarding this as follows:—

"Without approving or disapproving the instruction given, it failed to clearly advise the jury and distinctly call their attention to the proposition that, if the circumstances of the case were such as to account for the absence of the insured without tidings, without assuming his death, the presumption of death would not attach. That was the vital question, and the law as laid down in one or the other of the instructions requested should have been given, so that the jury would have been required to consider any testimony which would account for the absence and silence of the insured upon any reasonable hypothesis consistent with the evidence, other than that he was dead."

Because of these errors the judgment against the company was reversed and the cause remanded. The conclusion reached by the Supreme Court in this case is the logical one and is more worthy of notice because of the tendency in suits on life insurance policies to interpret the seven years disappearance of the insured as showing death, whether there is real reason to infer from such disappearance that death had occurred or not. The allowing of the introduction of a proper mortality table as evidence to show the probability of the insured still being alive is a logical method of arriving at real justice and such table should have great weight when the circumstances are such that no presumption exists, from the mere fact of absence unheard of, that the insured is dead, or when any presumption therefrom would be weak.

Right to Levy Assessment:—(Hartford Life Ins. Co. *vs.* Ibs, U. S. Supreme Court, 237 U. S. 662.) The question on which this case went to the Supreme Court of the United States was that of giving full faith and credit to the judicial proceedings of another state. The action in this case was brought in Minnesota and the court refused evidence offered by the insurance company of the

judicial proceeding involving the same question in Connecticut, the home state of the company.

The company maintained a department which was on the assessment plan and had in this accumulated a "mortuary fund" to enable it to pay claims promptly. The insured was assessed on May 2 to meet claims which matured during the quarter ending March 31. He failed to pay and the policy was cancelled shortly before his death. The plaintiff in the action against the company claimed that most of these losses had been paid from the mortuary fund and that sufficient remained in that fund to pay the rest so that the assessment was unnecessary and void.

An action had been previously brought in Connecticut by a number of members of this assessment branch asking, among other things, the distribution of the mortuary fund among the members. The Supreme Court of Connecticut had held that the creation and holding of such a fund by the company as a source from which to make prompt settlement, was legal.

The Supreme Court of the United States held that this should have been admitted in the present case, stating as follows in regard to the point that the plaintiff was not a party to the Connecticut action:—

"But it was impossible for the company to bring a suit against 12,000 members living in different parts of the United States. It was equally impossible for the 12,000 members to bring a suit against the company to determine the questions involved. Under these circumstances Dresser and thirty other members, holding certificates, brought suit "in their own behalf and in behalf of all others similarly situated."

The court elsewhere in the course of the opinion said:—

"The fund was single, but having been made up of contributions from thousands of members their interest was common. It would have been destructive of their mutual rights in the plan of mutual insurance to use the mortuary fund in one way for claims of members residing in one state, and to use it in another way as to claims of members residing in a different state."

The plaintiff also made claim that the cause of action in the two cases was different. In regard to this the court said:—

"Even if the second suit is for a different cause of action, the right, question, or fact once so determined must, as between the same parties or their privies, be taken as conclusively established so long as the judgment in the first suit remain unmodified."

MATERIALITY UNDER MISSOURI STATUTE:—(Stephens *vs.* Metropolitan Life Ins. Co., St. Louis Court of Appeals, 176 S. W. Rep. 253.) The Missouri statute provides that no misrepresentation of the insured shall be deemed material to defeat the policy unless the matter misrepresented shall have contributed to his death, and that the question whether it did contribute shall be a question for the jury. In the present case the insured was admitted to the hospital for cirrhosis of the liver and myocarditis shortly after making the application and some two weeks later died of these causes. The policy was issued and was delivered to the plaintiff, who was beneficiary, the day before the insured's death, without the company's knowledge of his illness. The policy contained the usual clause that no liability was assumed unless the insured was in good health at time of delivery.

The jury gave a verdict against the company. On appeal the court held that the case was concluded by the statement made in the proofs of death which was signed by the attending physician that the insured died of these two diseases, no evidence contra having been introduced, saying of the certificate of the physician:—

"The admission it contains is conclusive of the fact that the insured was confined in the hospital suffering from cirrhosis of the liver and myocarditis at the date of the policy and when it was delivered, and that he was therefore not then in sound health; also that he died on the following day from these diseases. Obviously no question of fact concerning this appeared for the jury to determine as it is conclusively established in such admissions that the contingency on which the policy became payable, that is, the death of the insured, resulted from the identical disease from which he was suffering at the date of the policy and when it was delivered."

CHANGE OF BENEFICIARY CLAUSE WHEN BENEFICIARY IS WIFE: —(National Life Insurance Co. of United States *vs.* Brautigam, Supreme Court of Wisconsin, not yet reported.) The policy had the usual clause providing for change of beneficiary and the insured, in accordance with the provisions of this clause, changed the beneficiary from his wife to his children. After his death the wife claimed the proceeds.

The Wisconsin statutes contain provisions regarding policies in favor of a married woman of which the essential part is as follows:—

"And every such policy, when expressed to be for the benefit of or assigned or made payable to any married woman or any such trustee shall be the sole and separate property of such married woman and shall inure to her separate use and benefit and that of her children, and in case of her surviving the period or term of such policy the amount of the insurance shall be payable to her or her trustee for her own use and benefit, free from the control, disposition, or claims of her husband and of the person effecting or assigning such insurance and from the claims of their respective representatives and creditors."

The court held that when a wife had been legally made beneficiary the statute prevented her rights from being divested without her consent. This decision, apparently, makes the change of beneficiary clause of no effect in the case of a Wisconsin policy when a married woman is beneficiary.

No such rule obtains outside of Wisconsin.

What Constitutes "Removal":—(Laue *vs.* Grand Fraternity, Supreme Court of Tennessee, 177 S. W. Rep. 941.) The contract provided that the insured should "not remove himself from that part of the North American continent lying between the northern boundary of Mexico, or the twenty-fifth parallel of North Latitude and the fifty-fifth parallel of north latitude." There was also a provision in the laws of the order that no benefit certificate should be granted to any one "residing" outside of the above-mentioned limits. The insured went to Panama and worked there for several months, but did not change his legal domicile. The order attempted to expel him for this. One of the questions before the court was whether such attempted expulsion was valid. The court held that he had not violated the terms of the contract, since the agreement not to "remove himself" must be construed in consideration of the rule of the order not to issue certificates to those residing outside of the same limits as meaning to remove his permanent residence.

If Change of Beneficiary is Wrongfully made can the Old and the New Beneficiary each Recover the Full Amount?:—(Hagar *vs.* Grand Lodge, A. O. U. W. of Kansas, Supreme Court of Kansas, 150 Pac. Rep. 528.) The insured had a certificate in favor of his wife. Upon the death of the wife he

changed the beneficiary to his nephew, under an agreement that the nephew should pay assessments as they came due, and put the certificate in the possession of the nephew. Afterwards the insured married again and attempted to change the beneficiary. The nephew refused to surrender the certificate. The insured took up the question with the order and they, with apparently full knowledge of the facts, issued a new certificate with another beneficiary. After the death of the insured both the nephew and the second wife claimed the amount payable. The lower court gave judgment for the full amount against the company in favor of each. The superior court, without attempting to determine which beneficiary was entitled to the proceeds, held that judgment should not have been given to each, thus making four thousand dollars payable when the certificate called for only two thousand dollars.

Finding of Jury Overruled:—(McKnelly *vs.* Brotherhood of American Yeomen, Supreme Court of Wisconsin, 152 N. W. Rep. 169.) The jury returned answers to several specific questions, one of which was, did the insured have tuberculosis at the time of obtaining the certificate. The answer was, "No." This the court changed to "Yes," and appeal was taken to the Supreme Court, which affirmed the action of the lower court. About the only evidence that the insured did not have tuberculosis was the general statement of the examining physician that he found no evidence of it. On the other side there was positive evidence that a little over a year before the insurance was obtained the insured visited a physician who examined his sputum and found tubercular bacilli. This the physician testified to positively. Another physician testified that a few months after the obtaining of the insurance the insured came to him and this physician concluded from his symptoms that he was suffering from tuberculosis and had been so suffering for several years. Another physician, testifying as an expert, stated that the finding of the examiner for the order was not at all inconsistent with the insured's having tuberculosis at that time, and he also testified that "if a microscopical examination of the sputum discloses tubercular germs their continuance in the system is certain."

The court held that there was no evidence to cast any doubt upon the credibility of any of the physicians testifying and that two of them testified to facts concerning which they had certain

knowledge and that there was no testimony which necessarily contradicted this affirmative evidence, hence there was nothing to sustain a finding that the insured did not have tuberculosis.

Limitation of Time in Which Notice of Death Must be Given:—(Order of United Travelers *vs.* Boaz, Court of Appeals of Colorado, 150 Pac. Rep. 823.) The membership certificate did not specify rights and conditions but referred to the constitution of the order. One of the provisions of the constitution was that, in event of death by accident, "notice of the death must be given in writing to the Supreme Secretary within ten days after the death" and it was provided that failure to give such notice invalidated all claims against the order. In the present case a letter giving notice was mailed in time to reach the Supreme Secretary within the ten days, but by error was directed to Chicago instead of Columbus, Ohio. It was forwarded from the Chicago postoffice to Columbus, but, owing to the delay, did not reach the Supreme Secretary until nineteen days after the death of the insured. The court held that the claim was invalidated by the failure to give notice as provided for in the constitution. This seems a very strict construction, especially in view of the fact that the time limit was extremely short and that attempt was made to give notice within the required time which failed only because of an error in the direction of the envelope.

Surrender Value:—(Devitt *vs.* Mutual Life Ins. Co. of Canada, Supreme Court of Ontario, Appellate Division, 33 Ontario Law Reports 473.) The question involved was whether the policy was in force at the death of the insured. This decision depended, under the provisions of the policy, upon the amount of surrender value on Sept. 30, 1913. The only provision in the policy as to cash surrender value was a table giving the amount of the cash surrender value at the end of each policy year, and the year in this case ended on March 28. If the cash surrender value on Sept. 30 was that of the previous anniversary then the policy was not in force at the death of the insured. If it increased pro rata during the year then the policy was in force. The lower court held that the surrender value did so increase. The Appellate Division, however, after examining the policy held that the policy made no provision for any such increase, but rather provided for a surrender

value at the end of each year, so that the increases came at the end of each policy year rather than during the year, and that, consequently, the surrender value on Sept. 30 was the same as on the previous anniversary, so that the policy was not in force at the death of the insured.

In the course of the opinion the court pointed out that the surrender value was not something that existed independently of the terms of the policy but was created by the contract and the amount was only such as was provided by the contract.

Effect of Later Statute on Right under a Certificate in a Fraternal Order:—(Bush *vs.* Modern Woodmen of America, Supreme Court of Iowa, 152 N. W. Rep. 31.) The certificate contained the right to change the beneficiary and did not make restriction as to who might be designated as beneficiary. The laws of Illinois, the state in which the order was organized, contained no restriction as to who might be beneficiary at the time the certificate was issued, but later were changed to make such a restriction. After the change in the law the insured designated as beneficiary one not of the restricted class and on his death her right to the proceeds was contested by the heirs. It was held that the insured had no vested right to designate whomever he pleased as beneficiary and that the restriction imposed by the later change in the law was binding, so that the beneficiary designated in disregard of such law could not take the proceeds.

Delivery of Policy:—(Donovan *vs.* Excelsior Life Ins. Co., Supreme Court of New Brunswick, Chancery Division, 43 New Brunswick Reports 325.) Shortly after the application for the policy the premium was paid to the agent of the company and he issued a receipt for the amount of the same, which, however, was not the official receipt of the company. At the home office of the company a discrepancy was noted between the age given in the application and the one given in the medical examiner's statement. The application was approved and the policy sent out to the provincial manager with, however, the qualification that before delivery he was to ascertain if the medical examiner's confidential report had been sent in and whether it was satisfactory, and was also to reconcile the statement as to age. When the matter of age was taken up it was found that the applicant was one year older than

stated in the application. In consequence the policy was not actually delivered but was sent back for the change of age and the difference of premium was collected. After this happened the applicant fell sick and soon after died. The policy provided that

"*When policy is in force.*—This policy shall not take effect until the same has been delivered, the first premium thereon paid and the official receipt surrendered by the company during the life and continued good health of the assured."

The court held that there had not been

"any such delivery of the policy or surrender of the official receipt as is contemplated and required by the policy as a condition precedent to its taking effect."

Effect of Agreement in Restoration:—(Knights of Maccabees of the World *vs.* Brown, Supreme Court of Michigan, 152 N. W. Rep. 1085.) The beneficial certificate was made payable to the wife of the insured and the proceeds were claimed both by the divorced wife, who was wife at the time the certificate was taken out, and a later wife, who was wife of the insured at his death. A by-law of the order provided that in case of divorce of a beneficiary the payment should be made the same as if the designated beneficiary had died. Hence, if this by-law were to govern, the divorced wife would have no claim.

When the certificate was taken out no agreement was made that it was to be subject to the by-laws of the order. The insured was afterwards suspended from the order for non-payment of dues and at the time of reinstatement, besides furnishing a certificate of good health, agreed that the constitution and the by-laws then in force or thereafter adopted should be the basis of the contract. The question was, whether such agreement, signed by the insured alone, could prevent the continuance of the interest of the then wife. The court held that such wife "having no vested interest in the certificate, indeed she could not herself have removed the default, could not be heard to say that the new arrangement, assuming that in this respect it was a new arrangement, did not bind her."

Verdict Contrary to Evidence:—(Timlin *vs.* American Patriots, Supreme Court of Pennsylvania, 95 Atl. Rep. 104.) The company resisted claim after the death of the insured on the ground of false statements in the application. It was stated in the appli-

cation that the insured had not consulted a physician for eight years and that the last medical attendant had been Dr. McLean. Several physicians testified that they had attended the insured after the date stated and, furthermore, there was undisputed testimony that Dr. McLean had been dead for some years at the time it was stated he had attended the insured. The jury, nevertheless, gave a verdict for the plaintiff. The Supreme Court reversed this and entered judgment for the defendant.

The stand taken in this case by the Supreme Court of Pennsylvania in recognizing the materiality of the answer to the question as to what physicians had been consulted, and the refusing to allow the verdict of the jury against the company to stand because not supported by the evidence, is significant.

Conditional Receipt for Premium:—(Jaggi *vs.* Prudential Insurance Co., St. Louis Court of Appeals, 177 S. W. Rep. 1064.) The chief question involved was whether the company waived a forfeiture by accepting premiums under a conditional receipt. Premiums were paid on the policy, which was an industrial one, up to Dec. 4, 1911, after which no premiums were paid until Feb. 16, 1912, when $1.65, the amount of the premiums then due, was paid and a "temporary receipt" given. The wording of this receipt is not quoted, but in the opinion it is stated that the receipt "stated that the same was given upon the condition that the company would under no circumstances be liable under the policy unless it was in force in accordance with the terms when the said payment was made." The policy provided for forfeiture if premiums were in arrears more than four weeks and also provided for reinstatement within one year upon payment of arrears and evidence of insurability.

The court held that the acceptance of the arrears and the retention of the money, even though the receipt given was conditional, and the regular entry in the "receipt book" held by the insured was refused, operated as a waiver of the forfeiture. There was some evidence offered tending to show that the reason why the regular receipt was refused was not explained by the company's agent.

Character of Policy Loan:—(Rustin *vs.* Aetna Life Insurance Co., Supreme Court of Nebraska, 153 N. W. Rep. 548.) This

case, which involved a dispute as to the amount payable, and which was decided in favor of the company in accordance with the clear provision of the contract, is of interest because of the following statement made in the course of the opinion as to the character of a policy loan.

"Such a loan is not like an ordinary loan, secured by mortgage or personal security. It is in the nature of a withdrawal of the reserve, pro tanto, for the reason that, under the terms of the contract, the insured is entitled at any time to demand and receive a loan, and he alone has the option of deciding whether he will ever repay it or not. Of what avail would it be to the insurance company to collect the amount of the loan when the borrower could at once, by virtue of the contract, demand and receive a new loan for the same amount? The practical operation of the loan is to withdraw the reserve, at the option of the policyholder as to repayment."

A further statement in the opinion is also of interest:

"Courts should interpret life insurance contracts in such a way that they can in all instances be carried out, and fair dealing and good faith should be upheld both for and against the insurer. We must remember that, if we take an undue amount of the assets of the company to pay one policyholder, we are thereby lessening the protection afforded to all other policyholders. The amount tendered by the amended answer seems to be the amount due under the contract, and the judgment should be for that sum."

Incorporation of Statutory Provision into Policy Contract:—(Noem *vs.* Equitable Life of Iowa, Supreme Court of South Dakota, 153 N. W. Rep. 652.) The insured took a policy on which premiums were payable semiannually. The second semiannual premium was not paid and the insured died twenty-eight days after the due date of such premium. The policy provided for a month's grace in the payment of premiums after the first policy year. The statutes of South Dakota, however, contained a provision that no policy of life insurance should be issued in the state unless it provided for a grace of one month for the payment of every premium after the *first*. The court held that the month of grace attached to the second semiannual premium.

An extract from the opinion is:—

"In construing this policy we must substitute the words of the statute in regard to the allowance of grace for the words contained in the policy, because provisions of the statute are as

much a part of the policy as though actually incorporated therein, and the exact language of the policy must give way to the language of the statute. It is our opinion that the words of the statute 'a grace of one month . . . shall be granted for the payment of every premium after the first' contemplate the allowance of grace after the payment of the first premium, whether such premium be an annual, a semiannual, or a quarterly premium."

Change of Beneficiary:—(N. Y. Life Insurance Co. *vs.* Murtagh, Supreme Court of Louisiana, 69 So. Rep. 165.) The policy provided that the insured should have the right to designate a new beneficiary "by filing written notice thereof at the home office of the company, accompanied by the policy for suitable endorsement thereon" and that the change should "take effect upon the endorsement of the same upon the policy by the company and not before." The insured, just three weeks before his death, delivered to the branch office of the company the requisite notice, and the policy was already held by the company. The notice did not reach the home office, probably through loss in the mails, and so no change had been made at the death of the insured. The court held that, because of the provision of the policy that the change should not take effect until endorsed on the policy, no change had taken place.

This is in conflict with Mutual Life Insurance Company *vs.* Lowther (*T. A. S. A.*, XIV, 81), in which case the insured had fulfilled all the requirements in regard to notice, etc., but died before the notice and policy reached the home office. The provision of the policy was that the change should take effect "upon the endorsement of the same on the policy by the company." The court held that nothing remained to be done except for the company to make the formal endorsement, a mere ministerial act, which it had no option to refuse, and that, consequently, the change was effective.

In the present case the reason for making the attempted change effective was possibly stronger in that the insured had performed his part a sufficient length of time before his death so that in the natural course of events the endorsement would have been made before his death.

Change of Beneficiary:—(O'Donnell *vs.* Metropolitan Life Insurance Co., Court of Chancery of Delaware, 95 Atl. Rep. 289.) The insured attempted before his death to change the beneficiary under the policy in accordance with the policy provision,

which required written notice accompanied by the policy and provided that the change should take effect on the endorsement on the policy. The clause also contained the expression "subject to the approval of the company." The policies were in the hands of another person and the insured did not and was unable to deliver them to the company. The court held that no change of beneficiary had taken place.

Date on which Policy takes Effect when Premium is Deposited with Agent at Time of Application:—(Rushing *vs.* Manhattan Life Ins. Co., Circuit Court of Appeals, Eighth Circuit, 224 Fed. Rep. 74.) The insured made application for a policy of $10,000 on June first and deposited with the local agent the first annual premium, taking a receipt therefor to the effect that the deposit should be applied on account of the first premium if the application should be accepted by the company, but that otherwise the deposit should be returned. The application provided that there should be no contract of insurance until the policy should be "manually received and accepted . . . during the good health of the person whose life is to be insured. . . ." The policy when issued, on June 18, provided it was to date from June 1. The day after the policy was mailed to the agent the company learned that the applicant was not in good health and wired and wrote the agent to hold the policy until applicant recovered. Applicant died on June 29.

Suit was brought on the policy on the theory that the making it date from June 1 waived the condition of manual delivery during good health.

The court held that there was nothing inconsistent in making the contract date as of June 1, although not to go into effect until later and then only when the condition named was fulfilled. The position of the applicant was that of one who had made a proposition to the company which the company had accepted provisionally and conditionally, namely, on the condition that delivery be made while the applicant was in good health. The fact that, when consummated, the contract would date as of several weeks earlier was not unlike the very common practice in many kinds of contracts, such as promissory notes, mortgages, etc., to make them date as of an earlier date than that of their execution.

It is to be noted that this case dealt with a deposit of the pre-

mium with the agent and not with payment to the agent at the time of making the application under the provisions of a "binding receipt."

Rights of Assignee of Policy:—(Feliciana Bank and Trust Co. *vs.* Union Central Life Insurance Co., Supreme Court of Louisiana, 69 So. Rep. 91.) The bank took a policy as collateral security and, the notes given not being paid at maturity, sold the policy in accordance with the terms of the pledge and became the purchaser. It applied to the company to be recognized as the owner of the policy in regard to loan and surrender privileges and, on the company's refusal, brought suit. The company defended on the ground that it was not bound to recognize such ownership until the same had been decreed by a judgment in court to which the insured was one of the parties. It is evident that the position of the company was taken simply to protect itself as to any claim that might later be made by the insured. The court held that the company could not make such a demand, saying of such a suit that "it would be a suit to remove an imaginary cloud upon personalty."

Payment of Assessments:—(Knights of the Maccabees of the World *vs.* Parsons, Court of Civil Appeals of Texas, 179 S. W. Rep. 78.) The insured was record keeper of the local lodge and it was his duty to collect the monthly assessments and deposit them, for which he received a certain commission. Under the laws of the order a member who was more than a month in arrears in the payment of dues was automatically suspended. The insured was in default for the dues due Feb. 1 and was killed March 2. He had, however, made the collections during February and deposited them in the bank without deducting the commission to which he was entitled, which commission amounted to more than his own dues. The court held his dues of the first of February were paid from these commissions and that, consequently, he was in good standing.

Accounting; Production of Records:—(Equitable Life Ins. Co. *vs.* Hardin, Court of Appeals of Kentucky, 178 S. W. Rep. 1155.) A suit had been brought for an accounting on a participating policy, and the lower court referred the cause to commissioners to take evidence and to make and report a settlement of accounts between the parties. The order provided that they should hold sittings at Stanford, Kentucky, and that

"The defendant society and its officers and agents are ordered to appear before the aforesaid commissioners at their sittings at Stanford, Ky., at such reasonable times and occasions as the said commissioners may require, and shall bring with them for the examination and use of the said commissioners such books, papers, documents, and records of the said society as may enable the said commissioners to comply with this order of reference, and the said defendant society is ordered to leave with the said commissioners copies of such entries upon such books as the said commissioners may require, so as to enable them to make the settlement herein directed."

The company filed a petition in the Court of Appeals for a writ of prohibition. The court held that an accounting could be required but that the lower court proceeded erroneously in requiring the company to produce its books and records in Stanford, Kentucky, and in requiring the personal attendance there of the officers and agents, and that, instead, the court should have required the company to produce its records and documents bearing upon the issues at its office in New York and to permit the plaintiffs to examine them there, and that evidence should be taken by deposition in New York at a place convenient to the office of the company.

In this decision the principle is upheld that the investigation of facts incidental to an accounting should be conducted at a place and in a way where it would not cause unreasonable inconvenience or unreasonably interfere with the business of the company. It is readily seen that such a decision is imperative unless the accounting in itself was to cause the greatest injustice by being a much greater penalty than the loss of the case itself.

Premium Notice Required in New York State:—(Flint *vs.* Provident Life and Trust Co., Court of Appeals of New York, 215 N. Y. 254.) The notice is required by the New York statute to state that "unless such premium, interest, installment, or portion thereof, then due shall be paid . . . by or before the day it falls due" the policy will be forfeited. The notice in this case did not tell when the premium was due and simply stated that, if the premium was not paid, the policy would become forfeited. It did not state that this would be the consequence if the premium was not paid by or before the day it fell due. The court held that the notice as given did not negative the possibility that a forfeiture

might be avoided by paying the premium after the day it fell due, saying:

"'The notice described by the statute is designed to impress upon the insured the absolute necessity of paying by or before that day if he would avoid forfeiture. This feature is so essential that its absence is fatal.'"

Surrender of Policy:—(Roberts *vs.* Northwestern National Life Ins. Co., Supreme Court of Georgia, 85 S. E. Rep. 1043.) The policy provided for the right to the insured to change the beneficiary. It also provided for the right to the insured to surrender as follows:—"If after three full years' premiums have been paid any subsequent premium be not paid the insured may, within one month after the unpaid premium shall have become due, select one of the methods of surrender settlement, etc." The insured gave a note for the first year's premium, but did not pay the note and, after some eight months, surrendered the policy to the company in return for the cancellation of the note. The insured died before the end of the year and the beneficiary made claim on the ground that the insured had not the power to surrender the policy. The court held that the insured did not have such power under the contract, since the right to change the beneficiary did not include the right to surrender, and since the right to surrender that was given was strictly limited to a date after three full years' premiums had been paid.

Power of Fraternal Order to Increase Assessments:—(Thomas *vs.* Knights of Maccabees of the World, Supreme Court of Washington, 149 Pac. Rep. 6.) The bylaws of the order stated the amount of assessments. There was also the right given to change the bylaws. Furthermore the right was specifically given to levy more assessments than one a month. The court held that the order had the right to increase the assessments.

In the course of the opinion the contrary rule which prevailed in New York was considered and criticized. The theory underlying the organization of fraternal orders was examined and the court brought out clearly the fact that each member was both an insured and an insurer. The opinion is long and deals thoroughly with the fundamental questions involved in the question at issue and the law upon the subject in the different jurisdictions. It is one which is well worth reading.

Abstract of the Discussion of the Papers Read at the Previous Meeting.

PRACTICAL TREATMENT OF UNDERAVERAGE LIVES—ARTHUR HUNTER.

VOL. XVI, PAGE 64.

WRITTEN DISCUSSION.

MR. JOHN K. GORE:

Mr. Hunter's paper naturally divides itself into two sections: the determination of the proper extra premium for a given extra risk, and the modification, if any, of reserves and surrender options made necessary by the increased rate of mortality.

With regard to the lien system, Mr. Hunter states that, at age 40, a lien commencing at $300 per $1,000 of insurance and diminishing by about $26 per annum will cover an extra mortality of only 6 per cent. of the normal. This statement assumes, however, that the extra risk takes the form of a constant percentage of the normal mortality throughout life, an assumption not often warranted by the facts. When we consider that the lien system is supposed to be especially applicable to the case of decreasing extra risk, the idea naturally arises of treating the charge for each individual year as applicable to the extra risk of that year only. On this supposition, the percentages of extra mortality covered may be readily obtained as shown in the illustrations on the following page.

The last column gives the percentage of extra mortality covered. While the figures for the net amount at risk are based on American 3 per cent. reserves, the results would vary but little among the current standard bases of valuation. The extra mortality is, of course, a percentage of the normal, not of the American. This method seems to me to give a better idea of the sufficiency or insufficiency of a given lien to cover such an impairment as tuberculous family history.

On the basis assumed, no change in reserves or cash surrender values is required. Extended insurance should, theoretically, be adjusted, but if the lien still existed, no material injustice would result from leaving the extended insurance period unchanged. A paid-up policy would be sufficiently adjusted by charging against it a proportionate part of the lien.

The regular reserves in the case of a company making all its extra ratings on this plan would be rather too low, as Mr. Hunter sug-

gests, as the actual incidence of extra mortality does not bulk so largely in the early policy years as the plan would imply. The reserves suggested by Mr. Hunter, however, would be too high to a much greater degree, as they are based on the assumption that the greatest weight of extra risk is postponed until extreme old age. This assumption is not supported by the facts, which indicate that extra risk cases in the aggregate exhibit an extra mortality increasing for a few years after entry—this being due no doubt to the effect of medical selection—but attaining its maximum, regarded as an absolute addition to the mortality, within ten or fifteen years after entry. For practical purposes, a reserve based on the assumption that the extra mortality took the form of a constant absolute addition to the normal would probably be sufficiently accurate. Reserves calculated on this assumption would be too small at the early durations and too high at the later; in the aggregate they would probably be very nearly correct. Under this system, reserves would always be higher on liened than on regular policies, but not to nearly the extent that Mr. Hunter's method would show.

POLICY FOR $1,000—RESERVE AMERICAN 3 PER CENT.

Policy Year.	Net Amount at Risk.	Net Lien.	Net Claim.	Lien P. C. of Net Claim.
	Ordinary Life—Entry Age 25. Nominal Lien $300.			
1	$991	$284	$707	40
2	983	267	716	37
3	973	251	722	35
4	964	234	730	32
5	954	218	736	30
10	901	135	766	18
15	840	53	787	7
	Ordinary Life—Entry Age 40. Nominal Lien $700.			
1	$984	$674	$310	217
5	916	572	344	166
10	823	444	379	117
15	722	316	406	78
20	617	188	429	44
25	512	60	452	13
30	413	nil	413	0
	20-Year Endowment—Entry Age 25. Nominal Lien $700.			
1	$966	$659	$307	215
2	930	617	313	197
3	893	576	317	182
5	815	493	322	153
10	592	285	307	93
15	324	78	246	32

Personally, I have no enthusiasm for the policy lien plan. The liens are open to the suspicion of sometimes existing without the insured's knowledge. Moreover, the very fact of their imposition indicates that the policyholder is more than ordinarily in need of insurance, and that his is, therefore, a case where the full amount of insurance is specially desirable if the extra risk can be provided for in another manner—as by the imposition of an extra premium.

When the rating up is effected by charging an extra premium, the method of determining the extra is comparatively unimportant, provided the actual amount be neither inadequate nor excessive. In practice, however, when the age-rating method is used, an addition of so many years may be prescribed for a given occupation, irrespective of the age of the applicant. This means a very much greater charge at the older entry ages than at the younger, while the extra hazard can generally be fairly well met by an addition to the premium constant for all entry ages, or increasing slowly with age. In the case of medical impairments also, if the custom of adding the same number of years to age for a given impairment be followed—as has not infrequently happened—the extra premium will ordinarily be too small at the younger entry ages and too high at the older. Of course, the age-rating method may be applied with due allowance for its effect at varying entry ages, but in such case the extra premium is really arrived at independently, and the rated-up age is used rather for calculation of reserves and surrender values than for finding the extra premium. A further serious disadvantage of the plan is that by using the same number of years' rating for life and for endowment policies, extra premiums on the latter are seriously understated. A system of rating that provides for extra hazards according to some plan bearing a more or less definite relation to the expected incidence of the additional mortality has, therefore, certain important advantages.

In this connection I should be inclined to qualify Mr. Hunter's statement that "if the additional hazard is expected for many years to be a percentage of the regular mortality, the advance in age is the better plan," the comparison made by him being with the alternative plan of an addition to the premium assumed to meet yearly the current extra risk. The following figures show the extra mortality per thousand of entrants at age 40 provided for in various policy years on the assumption, (*a*) that the true mortality corresponds with the rating-up of five years, (*b*) that the extra mortality is a constant percentage of the normal commencing at the same amount as in (*a*), and (*c*) that the extra mortality is that covered by a uniform extra premium of an amount sufficient in the first year to cover the extra shown in (*a*).

It will be observed that by the thirtieth year the rated-age method is providing for four times the necessary extra mortality as shown in column 4, while the uniform annual charge covers about half the requirements. In practice, however, column 5 is likely to prove rather more typical of actual experience than column 4.

WHOLE LIFE, ENTRY AGE 40.

Extra Mortality per $1,000 provided for.				
Policy Year.	Net Amount at Risk.	Rating Age 5 Years.	Uniform Percentage.	Constant Extra Premium.
(1)	(2)	(3)	(4)	(5)
1	$984	$ 1.37	1.37	$1.37
5	916	2.28	1.51	1.47
10	823	4.29	1.83	1.64
20	617	12.15	3.46	2.18
30	413	30.27	7.94	3.26

So far as the extra premiums themselves are concerned, it has already been suggested that these can be found in fairly satisfactory fashion by more than one plan, but there are points in favor of a scale of extra premiums calculated to provide for extra mortality of a certain definite extent. Thus, if a scale of premium rates covers 30 per cent. extra mortality—the percentage being taken of the actual normal mortality and with due allowance for expenses—the medical director has a tangible basis in making use of the results of the Medico-Actuarial Investigation, as these were expressed in percentages of standard mortality. As against this, the age-rating method shows relatively small extra mortality in the early policy years and an enormously increased mortality in the later; rendering it very difficult to say how many years' addition to age will provide for an extra mortality as shown in the Medico-Actuarial Report.

When the age-rating method is used merely as the means of determining extra premiums, substantial justice may still be achieved, but when the rated-up age is used for calculating reserves and surrender values a very unsatisfactory position results. As pointed out by Mr. Hunter, a large part of the extra premium is generally returned upon surrender, while a further portion will be returned annually in the shape of extra dividends in the case of participating policies.

In order to determine the proper reserve to be held on rated-up policies, consideration must be given to the incidence of the extra mortality. Here we are met with the fact that in a large group of cases—such as those with family history of consumption—the extra risk, except as modified for the first two or three years by medical selection, decreases from the outset, while in other groups of cases the extra risk is an increasing one for a considerable period after entry. To make a series of reserve tables suited to varying phases of extra mortality would be extremely troublesome, but there is no necessity for so doing. It is entirely sufficient if our reserves are such as fit the aggregate of all varieties of extra risk for which a uniform extra premium is charged. This combination of groups with a uniform total cost for mortality but with dissimilar inci-

dence of such cost is exactly what takes place when, for instance, young underweights and overweights—but within the limits for standard acceptance—are treated alike as to reserves and surrender values, as the former will show the higher mortality in the earlier years, but will offset it by exhibiting superior vitality later on.

No exact statement of the average incidence of extra mortality is possible. We may, however, by considering a few general groups get a more or less definite impression as to where the truth lies. A careful study of the impairment and occupation groups of the Medico-Actuarial Investigation and of the earlier Specialized Investigation shows strong evidence that the extra mortality does not persist into old age in as great a degree as in middle life. As an instance, I have had calculated the expected and actual mortality, exclusive of the first five policy years, of all the impairment groups in Volume IV of the Medico-Actuarial Report which showed aggregate ratios of 125 per cent. or more, based on not more than one hundred deaths.

The figures are as follows:

DEATHS AFTER FIRST FIVE POLICY YEARS.

Entry Ages.	Expected.	Actual.	Ratio Actual to Expected.
15–29	292	380	130%
30–39	603	860	143
40–49	623	927	149
50–59	402	565	141
60 and over	117	113	97
Total	2,038	2,845	140

These results are strongly suggestive of the probability that after about age 60 there is a tendency for the mortality among the survivors of any group, formed by selection a number of years earlier, to approach the general rates, even though in the intervening period there had been a wide difference. It is probably fair to conclude that even for the most persistent causes of extra mortality the assumption of a constant percentage addition to the normal mortality decidedly overstates the mortality in later life. This assumption is known to produce generally reserves higher than the standard on ordinary life policies, more decidedly higher on limited-payment policies, with but little change on endowments, but the error in the assumption referred to will be reponsible for a part —probably the greater part—of the indicated extra reserves. On these classes of extra risks, therefore, it is probable that, in the aggregate, reserves somewhat in excess of the standard are called for.

On the other hand we have a large group of decreasing or approximately constant extra risk cases. The effect of constant ad-

dition to the mortality may be found for ordinary life and endowment policies by using an increased rate of interest, that is, reserves are uniformly and substantially decreased. Limited-payment policies must necessarily come to a larger reserve when fully paid-up, but for the earlier durations the reserves would generally be lower than the regular. In the aggregate, for business subject to a constant extra risk, the proper reserves would be lower than the standard, and this condition would be accentuated in the case of decreasing extra risk.

As some classes of extra risks call for higher and others for lower reserves than those of standard risks, there will apparently be but little error introduced into the aggregate by using the standard reserves in all cases.

For particular kinds, however, this condition will not hold. Ordinary life policies will average about the normal reserves, but limited-payment reserves would probably be somewhat understated and endowment reserves certainly overstated by using the standard figures in all cases. This would suggest that increased and decreased cash surrender values should be paid on limited-payment and endowment policies, respectively. A larger view, however, finds justification for retaining the standard values in the fact that they represent the result of averaging the mortality under all plans, being based on a single table. The actual normal mortality under the three kinds named varies considerably, and the use of an average for all kinds suggests that extra premiums in the bulk received in a single year may be regarded as providing for the cost of the extra risk in that year. The same broad view would justify retaining the normal tables of extended insurance periods and of paid-up policies, although, theoretically, these would generally be reduced. If adjustment were deemed imperative, it might be made in the dividends rather than in the surrender allowances, but it seems to me that the degree of precision attained as yet in the selection of risks is not such as to warrant minute adjustments, and that an entirely safe and convenient plan would be to apply to policies on rated up lives the standard dividends and surrender options.

Certain minor advantages inhere in this plan. When an extra rating is removed subsequent to the issue of the policy for any cause, such as change in occupation, no change in the subsidiary benefits or dividend scale is involved. It is also more readily understood by the insured who realizes that the policy issued to him is in all respects that which he applied for, except that an extra charge is made for it.

With regard to the method of changing the plan applied for to an endowment in cases where the extra risk is presumed to be almost or wholly deferred till after the endowment period, I have considerable doubt as to its value. The Specialized Experience showed a quite high mortality on such policies, and the experience within five years of issue was worse than that of the later years.

Possibly the method had been used with poor judgment by some companies, but it is at least possible that, as about half of such policies are not placed, the issued policies are accepted by the worse section of those to whom they are offered. It should be observed that the effect of limiting applicants to endowments, after they have already applied for other forms, is quite different from limiting such a class as saloonkeepers to endowments. In the latter case those desiring life policies will generally apply to companies granting them that form of insurance, while those applying to the company with the limitations will consist largely of those who prefer endowments, that is, of the better risks within their class. No such selection prior to application exists in the modified policy group, and such experience as is available does not indicate that any improved selection is obtained by this scheme. The actual loss from excess mortality in the group is, of course, less than if the same individuals carried life policies, but for a very definite extra risk no premium whatever is received. In passing, it may be noted that on the assumption of uniform extra risk the whole-life net extra premium at age 35 for about 5 per 1,000 extra mortality comes out at $3.52 per $1,000, while for a 20-year endowment the figure is $2.84. In such a case only a comparatively small part of the loss is avoided by insisting on the endowment plan.

Mr. E. E. Rhodes:

Mr. Hunter, by the submission of the present paper, has greatly increased the debt which the Society owes him on account of the knowledge which he has given of one of the uncharted seas of life insurance. Whether, however, he has given us sufficient information to enable us to avoid the dangers of insuring under-average lives, is a fair question.

Before we can define an under-average life we must, I think, have a clear conception of what constitutes, for insurance purposes, an average life. "Average" and "standard" are assumed to be synonymous.

My conception of an average male life is as follows: the height should not be under five feet seven, or over six feet. The weight should not vary more than ten per cent. from the following figures: ages 20–29, 160 pounds; ages 30–39, 150 pounds; ages 40–56, 140 pounds; ages 57–65, 135 pounds. The measurement around the abdomen should not exceed that around the chest at rest. The pulse should not intermit or become unequal or irregular, and the rate should be between 60 and 85 per minute. The systolic blood pressure should be between 115 and 140 mm. The urine should be normal. The chest should be well formed, and there should be no marked physical deformity. There should be no record of past excesses in the use of alcohol, and the present use should be moderate. I presume that men whose steps tend toward convivial sur-

roundings, but who do not take more than two or three social drinks a week, would be classed as average. The family and personal record should not indicate any weakness. The occupation should not involve any special hazard.

If a company is satisfied with the death rate which would be experienced on such a class of average, or standard risks, it can reasonably and properly accept risks which are within certain limits sub-standard, for the reason that it will write other risks which are super-standard. We may assume, for example, that if it accepts risks upon which a mortality of 110 per cent. of the standard may be expected, its mortality on its total business will still be 100 per cent. of the standard. This assumption, of course, is based upon the further assumption that there will be a good primary selection by the agency force, and that the standing of the company is such that the best class of risks will be attracted to it. When I speak of a mortality of 100 per cent. it will of course be understood that I refer to the experience upon average lives.

Our problem, then, is to determine whether in a particular case we may expect a higher mortality than 110 per cent., and if so, how much higher. Mr. Hunter favors the numerical method of rating risks, but I must confess that I cannot see the end of that road. Take the case of a man 34 years of age, whose height is 5 feet 8 inches, whose weight is 120 pounds, whose occupation is that of glass cutter, who has one brother dead of tuberculosis, and who has a history of one or more attacks of blood spitting more than ten years prior to his application, but who is without any distinct history of tuberculosis. Such a case is not improbable, but such thought as I have been able to give it has not indicated the numerical rating which it should receive. It is quite safe to say that there were no glass cutters in Classes 14 and 53 of the Medico-Actuarial Investigation, so that the results shown therein are not applicable to the case now under consideration. We are, therefore, driven away from the numerical method described by Mr. Hunter and are forced to adopt what he has styled the "empirical" method, in contradistinction to the numerical method.

Let us change our glass cutter and give him a favorable build and a favorable personal and family history, so that in a numerical rating of the risk he would have some credit. We must, however, assume that in the Medico-Actuarial Investigation the class of glass cutters suffered a particularly careful selection, and in my opinion it would be unsafe to place any credit on account of build and personal and family history, or otherwise, against the occupational hazard. If this view is correct, it follows that in certain cases no allowance whatever should be made on account of some favorable factors. It seems clear, also, that if absurd results are to be eliminated, there should be some arbitrary limitation of the total credits. If we said that the mortality of the very best class of risks will not be more favorable than 80 per cent. of the mortality

of average risks, then, in our system of numerical rating, we should not allow, on account of the favorable aspects of the case, a larger credit than 20.

On the other hand, if there are a number of unfavorable factors, I am not prepared to agree that in all cases the value of the risk is properly measured by adding the several ratings therefor. For example, clerks in wholesale liquor houses show a mortality of 112 per cent. of the standard. Steady, free users of alcohol show a mortality of 132 per cent. Most of us, I think, would rate this combined risk higher than 144 per cent. Proprietors, managers, and superintendents of distilleries show a mortality of 85 per cent.; but if a man engaged in that occupation were also a steady, free user of alcohol, we certainly would not put him in a class in which the expected mortality would be only 117 per cent. of the standard. In my opinion the two cases should be rated alike, but I cannot think of any satisfactory arithmetical method of rating. My present thought is that the numerical method does not afford opportunity to determine conclusively the rating of a large proportion of the risks submitted to life insurance companies, and that we still need in this division of the business the judgment of the trained mind. To be on the safe side it seems to me necessary to say that every case with an unfavorable factor which would give a higher mortality factor than, say, 110 per cent. should be referred to a medical director, and that every case with two or more unfavorable factors should also be referred, regardless of the individual rating of such factors, and also regardless in every case of any favorable factors.

Inasmuch as the laws of some states make it impracticable to charge liens or create special dividend classes, companies doing business therein are practically forced to charge an extra premium or to rate the insured at an advanced age. Several objections to both plans are set forth by Mr. Hunter, but he fails to mention what in my opinion is one of the most serious objections to either plan; viz., that each policy is treated individually and the company loses at least a part of the benefit accruing from the establishment of a special class. With regard to some of the details referred to by Mr. Hunter, I am of the opinion that it should not be made more profitable for an agent to solicit under-average business than preferred business, while on the other hand I can see that it is inadvisable to insure under-average lives upon a plan which will result in only the worst risks accepting insurance.

The advantages or disadvantages of doing an under-average business do not properly enter into a discussion of Mr. Hunter's paper; but I cannot refrain from expressing the opinion that a company has a certain advantage in educating its agency force to exercise a careful primary selection. It may of course be argued that the mission of a company is to insure lives, and that there are comparatively few men who are not insurable upon some plan; and

I am free to confess that the thought of granting insurance to all but this comparatively small number appeals to me. I do not consider it an evidence of good underwriting to secure the lowest possible death rate. This does not call for the exercise of either skill or judgment. If it is generally understood that a company's standard of selection is such that a certain mortality may be expected as long as it experiences a normal growth, those who are super-standard risks cannot complain if such standard be maintained; but if the standard be lowered, super-standard risks have, it seems to me, a right to ask that sub-standard risks be placed in a special class and charged sufficient to meet their additional mortality.

MR. HENRY MOIR:

Mr. Hunter has a happy faculty of discussing subjects which are of growing importance and regarding which previous information is meager. Our knowledge of the mortality affecting under-average lives is even yet superficial; and until this paper was submitted last May we did not have in our *Transactions* any contribution suitable for students dealing with the principles of extra rating.

The first division of the subject taken up by Mr. Hunter is the "Lien System." Personally I much regret that this system has fallen into disuse; my impression of the reason for this discontinuance is different from that of Mr. Hunter as given on page 69. According to my recollection, standard policy forms and the Armstrong legislation together made it impossible for companies operating in New York to issue a policy with a lien. It has only been within the last year or two that the laws have been so amended that it might now again be possible to issue policies with liens. One of the difficulties was automatic extended insurance which is altogether out of place in a policy contract on a sub-standard life.

I should like to bring to the attention of actuaries the lien system as applied to endowment insurance, because, in my judgment, this combination fits many forms of extra mortality. Heavy-weights at young ages may often get an endowment policy without any lien; those with a consumptive family history might have a rapidly decreasing lien; while those who are subject to a continuous hazard (as in the case of some forms of heart murmur) might have a lien which decreased in equal amounts during the entire duration of the endowment policy. This last form is one which meets many classes of impairment.

The true difficulty in the lien system as described by Mr. Hunter lies in the so-called "popular" plan used in this country of reducing the lien annually by the amount of the annual premium paid. Mr. Hunter very properly says that there is no scientific reason for this course. I tried the other for several years and found no objection on the part of agents to the issue of an endowment with a lien decreasing by equal amounts over the duration of the en-

dowment. This is simpler than the other method, and agents soon came to recognize the fact. It is easy to see that if the lien in the first year is equal to the net sum at risk (that is, something less than $1,000) and decreases by equal sums each year, we shall thereby make provision for nearly a maximum mortality. In other words, an unlimited mortality rate can be met by an insurance subject to a lien of something under $1,000 and decreasing to nil at the end of the endowment period.

But this lien system is undoubtedly most effective when applied to risks of a somewhat doubtful character, subject to mortality on the average within 50 per cent. or 60 per cent. in excess of the normal. Mr. Hunter quotes the case of a 20-year endowment with a commencing lien of $500, and shows that the percentage of extra mortality which this meets at age 40 is only 33 per cent. But if the lien existed during the duration of the endowment and decreased by $25 each year, provision would thereby be made for an extra mortality of 96 per cent.—a very different matter. There is no necessity for having the endowment run for the period of twenty years common to this country. Long term endowments maturing at ages 55, 60, or 65 can be used just as effectively and can be placed by agents, because the premium rate is usually no higher than that of a 20-payment life policy. The system entirely gets rid of the difficulty as to the reserves to be carried, because it is found that the normal reserve for a regular policy is adequate under this system, while the other difficulty of non-forfeiture values mentioned by Mr. Hunter disappears if the extended insurance is eliminated and paid-up insurance be granted as the automatic form of non-forfeiture.

During this summer I had occasion to prepare an address for the American Life Convention on "The New Mortality Tables." I interpreted this subject as involving the discussion of the M. A. Tables, and this necessitated the taking up of the question of underaverage risks. Some of this work involved a new investigation of extra mortality with the graduation of various prominent groups of the M. A. Experience, and the submission of the results in graphic form (see *Transactions of American Life Convention,* 1915). The work I did in this direction has caused me to modify my own views considerably on various points. For example, six months ago I would have subscribed heartily to the remark made by Mr. Hunter at the foot of page 75, "if the additional hazard is expected for many years to be a percentage of the regular mortality, the advance in age is the better plan." Now, as the result of the diagrams I prepared this summer, I have serious doubt as to whether this statement is correct, although on my first reading it I would not have thought of raising the question.

Continued Orally: I am much interested in the discussion by Mr. Gore to which we have just listened because his analysis of this extra mortality question and my own, as submitted to the Amer-

ican Life Convention a month ago, run along similar lines. For example, Mr. Gore gave you a comparison of the effect of a constant extra, of an age rating, and of a percentage extra. I had exactly the same thing in mind, and prepared a diagram showing the same principles in graphic form. Then, with that as a basis, I analyzed six of the principal groups in the M. A. experience, graduating the first year's risk, also the ultimate mortality of these groups, and expressing them in graphic form. Some of the results are peculiar. For example, in the case of underground miners, I find that the first year's risk at any age under 35 is greater than the ultimate mortality of the same class. The reason for that is hard to find, but in all probability it consists in poor selection, adverse to the company, because a few cases of successful fraud are quite sufficient to make the mortality appear altogether too high at a young age, and of course we know that in mining communities the medical selection is liable to be poor.

The next group is locomotive engineers, in which there seemed to be no selection at all. The first year's risk is nearly the same as the ultimate, both showing a high mortality at young ages and running into normal at older ages.

Then come "Teamsters' Mortality," "Mortality in Georgia," and "Free Users of Alcohol." In the last named group the conditions are quite different. The first year's mortality is practically identical with the mortality as given by the M. A. Table, but it increases rapidly to an ultimate rate about 40 per cent. above the normal. In other words, the selection the first year is just about as good with the free users of alcohol as normal, average mortality. But death rates become about 40 per cent. above the normal soon afterwards.

Some of the general conclusions which I reached from the study of these diagrams are interesting along the same line as the study of Mr. Gore; for example, I may read one or two of them as follows:

That no one of the present methods of treating sub-standard risks is applicable to the different circumstances;

That the extra mortality of some sub-standard classes attains an apparent maximum between ages 35 and 45;

That much of the extra mortality is at the young ages and early years of insurance—the financial effect is therefore heavier than if the excess were spread evenly over the duration of the mortality table, or came later in life.

That although premiums for sub-standard classes should be higher than normal, or the extra hazard met in some equivalent way, nevertheless the reserves for many of these hazardous groups would work out smaller than the normal reserves. This will be apparent to actuaries from the shapes of the ultimate curves. The curve for married women would however indicate an opposite tendency.

It surprised me, as Mr. Gore spoke this morning, that his analysis agreed in so many points with results I had reached by an entirely different process during the summer.

MR. D. E. KILGOUR:

The treatment of under-average lives is a subject with which we are accustomed to have Mr. Hunter deal and is one to which his various contributions have placed the Society under unusual obligation. The smaller companies, which most of our members after all represent, are deeply indebted for the valuable information thus appearing in our *Transactions* and especially for the original data so unstintingly given. The present paper, though contributing little to the practical information on the subject, deals succinctly with its theoretical aspects, and thus affords to the student and the practicing actuary the synoptic review of the essential points confronting one in the consideration of this problem.

It is a curious commentary on the training of an actuary that the more theoretical knowledge he acquires, the more practical he tends to become. I recall with what patient delving I studied the problem of surplus apportionment: how the refinements and elaborations of a complete "contribution method" appealed to me as embracing the ideal and last word in mathematical accountancy. It did not take long to discover that the various factors upon which our surplus formula was based, while theoretically independent, were in practice not so: how the simple alteration of one factor would change the final incidence, and how, indeed, almost identically the same results in the end could be obtained by a variation of the initial factors. The result was, so far as I was concerned, that elaboration gave way to brevity and theoretical complexity to practical simplicity. The desideratum indeed seemed to be a reasonably equitable and more or less easily explainable and defensible method.

In dealing practically with the problem of under-average lives I imagine one's conclusions are affected by similar considerations. To attempt to consider each individual curve of mortality, even where it is possible to isolate the curve with reasonable accuracy, is at once to introduce chaos into our method of treatment. Certainly the nearest we can attain to theoretical accuracy is to consider standard deflections from the normal curve. The practical treatment of the subject makes it necessary to give algebraic expression to standard curves more or less arbitrarily selected as types.

Mr. Hunter has alluded to Mr. King's defence of the empirical or individual method of dealing with under-average lives. The fact is that this method after all deserves little serious consideration. It may have sufficed when the under-average life was decidedly the exception and when it was possible to have each case reviewed by a trained and highly skilled official. But intuitive as

one may be, the probability to error is so much greater than when one adds to intuition exact knowledge, that the case for guess work treatment is almost entirely indefensible. If the medical examiner or medical referee could state with accuracy as a result of his observations and examination the expectancy of the applicant, we could at once determine the rating-up in age necessary. My observation has been that they are entirely incapable of supplying this information. For instance most applications call for a statement from the examiner of the exact age and the apparent age of the applicant. A review of a number of applications will show that the apparent age on the average is perhaps three to five years lower than the stated age. As this would apply to the standard application, it is quite obvious so far as even they are concerned, that the conclusions of the examiner are entirely erroneous and would result in serious loss to the company if a purely guess-work system were followed. The fact exists, too, that the great increase in under-average business which must inevitably result from the cultivation of this field, precludes the individual attention which the strictly empirical method necessitates. Nor should actuaries, of all people, be content to decide such important issues by methods which, while possibly safe, are uncertain and capricious.

The method of lien has been used for many years by Canadian companies, but except in cases where a lien is essentially appropriate, as in a class with the rapidly declining expected extra mortality, it is not, I think, now so often resorted to. The lien method is open to the obvious objection that it fails alike in protecting the policyholder or the company. The insured obtains deferred insurance, but at its inception, and perhaps when he needs it most, he is unprotected. On the other hand, the collective benefit to a company from a system of liens is very meagre. Mr. Hunter's tables afford clear demonstration of this well understood principle. Nor is the value of the method in any way enhanced by the anomalies and difficulties introduced in determining reserves and collateral values on policies so restricted. In practice, therefore, I think it may be safely said that liens will not prove a popular or profitable method of dealing with under-average lives as a whole, though in a few well defined classes, especially where there is an early termination of the risk, as under endowment contracts, a referee may be tempted still further to restrict the company's risk by the imposition of a lien.

The restriction to special dividend classes is hardly appropriate except in the case of comparatively large companies. The method possesses a distinct practical advantage and facilitates salesmanship. It is, however, at once inapplicable to non-participating business and in the case of participating business presupposes unquestionably adequate premium rates.

The use of either the third or fourth methods, viz., the changing of extra premiums and the rating-up in age, while suitable in them-

selves to large classes of risks, cannot with entire satisfaction be exclusively adopted. In saying this I am not unmindful of the findings of a committee of Ontario actuaries recently appointed to consider the whole question of under-average life rates and if possible to recommend a method for the general adoption of our companies. The committee considered at length the various methods in use and decided that for the sake of simplicity it could safely resolve the question into the problem of determining a scale of extra premiums for the classes involved and arbitrarily adjusted for main plans of insurance. A comprehensive table has been evolved and it is not improbable that these ratings will be fairly generally adopted. On the whole they do substantial individual justice, and, so far as experience can determine, are adequate to meet the expected extra mortality involved.

In Canada we have fewer companies to deal with and, except for a few recalcitrant companies on this side of the line, it is not impossible to get uniform action so far as general company practices are concerned. This has applied in the past in the adoption by nearly all of our companies of a standard scale of participating premium rates. Likewise we are hopeful that we may come to a fairly close agreement on the question of dealing with under-average lives. Even with so much brotherly love and co-operation we shall still in the field of dividend competition and elsewhere have ample scope for the exhibition of our less amiable qualities.

But while agreeing to the adoption of a method of extra premiums I must frankly confess my own decided preference to a combination of the third and fourth methods. The accident hazard is undoubtedly best met by the use of a uniform extra premium, while the presumable extra mortality resulting from conditions conducive to a general impairment of the health, by a method of rating-up in age. A uniform rating-up method for a class regardless of plan or age in my opinion too completely disregards the incidence of extra mortality. It is comparatively simple to rate a risk numerically as exhibiting Class *A, B, C,* etc., mortality where these represent multiples of the standard, say of 125 per cent., 137 per cent. and 150 per cent., etc., of the normal mortality, and by appropriate conversion tables to express these in terms of rating-up in age, with variations for plan and ages of entrance. The method is not unduly complex, as is obvious from the number of companies now adopting the plan or a variation of it. Where the expected extra mortality is not a uniform percentage of the normal, but is progressively increasing or decreasing, at any rate markedly so, no harm can result from a combination with the first two methods discussed, but these cases would for the most part prove the exception and could be left to the individual judgment of the home offices rather than to general tabulation.

Mr. Hunter has wisely refrained from being dogmatic in his treatment of this subject, and has for the most part confined him-

self to enumerating general principles and discussing their application. To this extent, therefore, his paper will tend to promote a discussion of the subject itself rather than a criticism of his treatment of it. At any rate, of the latter, I have little to offer.

MR. W. A. P. WOOD:

In his paper on "Under-Average Lives," Mr. Hunter has taken up a subject which should be of special interest to us all at the present time, coming as it does just after the publication of the results of the Medico-Actuarial Investigation. The various points brought out by him, therefore, should have our careful attention.

A short time ago I read a paper before the Insurance Institute of Toronto on "Hazardous Occupations," at which time I had a number of tables prepared on the H^m $3\frac{1}{2}$ per cent. basis. In this discussion I will again make some use of them. Although tables prepared on the American Experience would produce slightly different results, still I do not think that the difference would be of any great consequence. As a matter of convenience, therefore, I will refer to the old tables.

There are three main kinds of extra mortality, namely, increasing, constant, and decreasing. One might, of course, further subdivide the increasing and decreasing sections, because we could have many variations in the rate of increase or decrease. The fact of our knowing the ratio of the actual to expected deaths in the case of any section of under-average lives does not in itself mean that we have complete information in regard to this class. The incidence of the extra mortality is frequently of great importance. If this extra mortality does not arise until the later years of life, we would be justified in giving short term endowments at the younger ages without any extra charge. In the case of increasing extra mortality the extra premiums for endowment policies are a much smaller percentage of the extra premiums on ordinary life policies than is found to be the case in either constant extra mortality or decreasing extra mortality. This ratio is highest in the case of decreasing extra mortality. It will be found that the method of rating up the age by a uniform number of years and also the method of rating up equivalent to a certain extra percentage in mortality are cases where increasing extra mortality rates are employed, whereas, the method of charging uniform extra premiums might be considered as very nearly one in which use is made of a constant extra mortality rate. There are a number of tables showing the experience in tropical countries which would be counted as examples of a decreasing extra mortality.

In Table I below is shown a comparison of 150 per cent. mortality with that produced by a uniform rating up in the age, the rating up in one case being 17 years in order to produce approximately the same rate of mortality at age 20 as 150 per cent. mor-

tality and in the third column the rating up is 9 years for the reason that on the ordinary life plan, age 20, a rating up of 9 years produces the same net premium as 150 per cent. mortality.

TABLE I.

COMPARISON OF 150 PER CENT. MORTALITY WITH THAT PRODUCED BY A UNIFORM RATING UP OF THE AGE.

Age (x).	150% of q_x.	q_{x+17}.	q_{x+9}.
20	.009494	.009464	.007433
25	.009945	.010734	.008504
30	.011585	.013702	.010082
35	.013161	.017549	.011556
40	.015459	.023990	.015220
45	.018288	.034636	.019734
50	.023925	.049889	.027539
55	.031550	.074940	.040410
60	.044517	.114686	.057340
65	.065147	.171352	.091195

It will be noticed from the above table that although the rates of mortality on the 150 per cent. and the rated-up 17-year plans start off practically the same at age 20, still at age 65 the mortality corresponding to the rated-up age is nearly three times that shown by 150 per cent. mortality. It will also be noticed that the 9-year rating up in age has a much lower rate of mortality for a number of years than 150 per cent. mortality, and it is not until after age 40 is passed that the former rate becomes as great as the latter. At age 65 the rated-up 9-year method shows nearly 50 per cent. heavier mortality than that found in the other plan. These are given merely as examples of different kinds of increasing extra mortality.

Mr. Hunter has mentioned the four chief methods of treating under-average lives. In dealing with the lien system another plan might be mentioned, namely, that in which the lien is level for a definite number of years, usually 5 or 10, and runs off by a certain percentage each year for 5 or 10 years until it is extinguished. For example, one might have a lien of 50 per cent. level for 10 years and decreasing 5 per cent. each year thereafter until it is cancelled. In the case of certain impairments or defects in family history, such a lien seems to be quite suitable. In this way we are able to take into consideration the effect of medical selection during the early policy years and also an expected extra mortality decreasing as the age increases until this extra mortality vanishes. For example, an applicant, age at entry 25, who is underweight and with a family history showing tuberculosis, might be considered with a lien of 40 per cent. level for 10 years running off in the following 10, or in some cases the following 5 years. We expect the mortality to be more favorable for a number of years after

the medical examination. Therefore, in cases where we expect decreasing extra mortality as the age increases, instead of having the lien decrease from commencement, we are able to have a somewhat smaller lien and make it level for a number of years and then have it decrease. We have two forces at work here, one tending to create a condition requiring an increasing lien, and the other, a decreasing lien. We will probably be not far wrong in having the lien level for 10 years and running off in the following 5 or 10 years. In obtaining a rating up equivalent to a lien, one must not overlook the incidence of the extra mortality. In the tables in Mr. Hunter's paper showing the percentage of American mortality borne by the liens, he has assumed the uniform extra percentage of q_x throughout for each age. Therefore, it would appear as though one might not be justified in using such a table when the extra mortality experienced decreases from age to age.

The danger has been pointed out of allowing the applicant the option as to whether he will take a policy with a lien or extra premium. There is always the danger of selection against the companies where this option is allowed, and if it is a case in which the company would prefer the lien, I think the extra premium should be something higher than the amount deduced from the mortality tables as the mathematical equivalent of the lien.

Mr. Hunter has stated that the device of placing policyholders in a special class as to dividends is an equitable way of treating persons engaged in various occupations. This is correct in theory, but most companies have not a sufficiently large number of persons insured in each of the special mortality groups to bring out well-adjusted results. With small groups it might easily happen that those in the 115 per cent. to 140 per cent. mortality group would show heavier mortality than those in the 140 per cent. to 180 per cent. group. A company, therefore, could not employ this method unless it has a considerable number of policies in each group.

Table II shows the net extra annual premiums equivalent to various ratings up in age. From this table it will be seen that the extra premiums for ordinary life and life 20 payments lie fairly close together, but that they are both much larger than the extra premiums for 20-year endowment, the former being four or five times the latter for the younger ages. Thus for age 20, a rating up of 5 years on the ordinary life plan means an extra premium of $1.91; on the life 20-payment plan, $2.16, and on the 20-year endowment 44 cents, i. e., the life extra premium is between four and five times the endowment extra premium.

Table III shows the net extra annual premiums required to provide for the extra mortality exhibited in 125 per cent., 150 per cent., 175 per cent. and 200 per cent. mortality. Here the difference between the extra premiums on the life and endowment plans is not as great as is found in Table II. For example, at age 20, 150 per

cent. mortality shows an extra premium of $4.01 on the ordinary life plan; $4.78 on the life 20-payment plan, and $2.10 on the 20-year endowment, i. e., the extra premium on the life and life 20-payment plans is about twice as great as is found on the 20-year endowment plan for this particular age. Both Tables II and III show the extra premiums produced by rates of mortality increasing as the age increases. From a consideration, however, of Tables I, II, III and IV, it would seem as though the rating up a definite number of years is best adapted to cases where the extra mortality is comparatively small for a number of years and gradually increases throughout life, the rate of increase being much more rapid as the life reaches the older ages. The method of adding a constant percentage to the rate of mortality would appear to apply to cases where there is an increasing extra mortality but where this extra mortality does not show as much of an increase from age to age as is produced by the uniform rating up in age method. In Table IV, it will be noticed that 150 per cent. mortality on the ordinary life at age 20 means 9 years rating up, for age 40 a rating up of 6 years, while on the 20-year endowment plan the corresponding ratings up are 15 years and 7 years, respectively.

We have referred above to the case of increasing extra mortality. Let us now consider the effect of a constant addition to our q_x. Messrs. White & Whittall, in their paper found in Volume 24 of the *Journal of the Institute of Actuaries,* show that a constant extra mortality produces extra premiums which are much more nearly uniform for different ages than those produced by increasing extra mortality experiences. We find also that for age 35 the extra premium on the 20-year endowment plan is about 80 per cent. of that on the ordinary life plan, whereas, the corresponding ratio is 40 per cent. for the increasing extra mortality and 95 per cent. for the decreasing extra mortality. If we are dealing with hazardous occupations in which the accident hazard predominates, we would come nearer the truth by using the uniform extra premium method than by employing either the uniform rating up in age method or the method of charging a premium corresponding to 125 per cent., 150 per cent., etc., mortality. If one were considering the adoption of the uniform extra premium method it would be better to charge a lower amount for short term endowments, say up to 20-year endowment. Theoretically, one might prepare a different extra premium for each plan of insurance, but I think for the sake of simplicity it is better not to do so. If one wants to employ the uniform extra premium method or some modification thereof, the question arises what extra premium should be charged. The Medico-Actuarial Experience shows the percentage of actual to expected deaths. If those engaged in any particular occupation show 150 per cent. mortality, we might take an average age, say 30 or 35, and find the difference between the regular ordinary life premium and that obtained by using 150 per cent. mor-

tality. This could then be used as a basis for arriving at the extra premium to be charged. The aim is to arrive at an equitable average extra premium.

As accidents are the chief cause of the extra mortality experienced in most occupations, I believe if one were going to adopt only one method, the modified uniform extra premium plan would be the most satisfactory. It is also a question if one would not be justified in employing this method for all hazardous occupations on account of its simplicity and owing to the fact that we are unable in any event to obtain absolute accuracy. In the case of medical impairments, I do not believe this method could be considered as satisfactory. One would then have to resort to one of the other methods.

The question of reserves and surrender values on these under-average lives has been referrd to by Mr. Hunter. In this connection I think that possibly some tables which I have prepared in connection with my other paper may be of some interest. In Table V will be found the reserve on a 15-year endowment on the H^m $3\frac{1}{2}$ per cent. Table and the corresponding reserve where 150 per cent. mortality has been experienced. An additional column is inserted showing the funds which would arise using the normal premium with 150 per cent. mortality and, of course, assuming $3\frac{1}{2}$ per cent. interest. This latter column is an indication of the loss sustained from unfavorable mortality as compared with a normal policy. Table VI shows the same three columns for the ordinary life plan and, in addition, the reserve for age 42, i. e., a policy rated up 7 years. If you refer to Table IV you will notice that at age 35, corresponding to 150 per cent. mortality, the equivalent rating up in age is given as 7 years. It will be observed that the reserves on the policy rated up 7 years are considerably higher than those brought out by using 150 per cent. mortality. It would look, therefore, as though the allowing of surrender values at the rated-up age is very generous to the assured if the mortality experienced has the same incidence as is shown by the 150 per cent. mortality table. The reserves, of course, for age 42 correspond to those deduced from a mortality experience where the extra rate of mortality is comparatively low in the early policy years and increases more rapidly than is found in 150 per cent. mortality. One should note also in the fourth column of this same table the small amount of the fund compared with the reserve on the policy, when no extra premium is charged.

In dealing with under-average lives one should take into account the class of business being considered. Accordingly, as stated above, I would not think it advisable to treat medical impairments in the same way as hazardous occupations. It must be remembered that we cannot expect to get absolute accuracy in regard to the extra premiums chargeable on under-average lives and we should be satisfied if we are able to deal in a comparatively equitable way

TABLE II.

EXTRA ANNUAL PREMIUMS (NET) EQUIVALENT TO VARIOUS RATINGS UP IN AGE.

H^m $3\frac{1}{2}$ Per Cent.

Age.	Years. 2	3	4	5	6	7	8	9	10	11	12	Age.
Ordinary Life.												
20	.71	1.08	1.48	1.91	2.36	2.84	3.34	3.85	4.39	4.94	5.53	20
25	.93	1.43	1.94	2.48	3.03	3.62	4.23	4.87	5.55	6.25	7.00	25
30	1.14	1.75	2.39	3.07	3.77	4.52	5.29	6.11	6.96	7.88	8.85	30
35	1.45	2.22	3.04	3.89	4.81	5.78	6.82	7.92	9.09	10.32	11.61	35
40	1.89	2.93	4.03	5.20	6.43	7.72	9.07	10.50	12.02	13.63	15.35	40
45	2.52	3.87	5.30	6.82	8.43	10.15	11.99	13.94	16.02	18.24	20.59	45
50	3.33	5.17	7.12	9.20	11.42	13.77	16.28	18.96	21.81	24.84	28.06	50
55	4.57	7.08	9.76	12.61	15.64	18.86	22.28	25.90	29.77	33.91	38.36	55
60	6.25	9.67	13.29	17.16	21.30	25.75	30.58	35.85	41.57	47.74	54.33	60
Life 20 Payments.												
20	.82	1.24	1.68	2.16	2.66	3.18	3.72	4.27	4.83	5.40	5.99	20
25	1.02	1.56	2.11	2.67	3.24	3.83	4.45	5.09	5.75	6.44	7.15	25
30	1.16	1.78	2.42	3.08	3.77	4.48	5.21	5.96	6.76	7.59	8.48	30
35	1.40	2.13	2.88	3.68	4.51	5.40	6.34	7.33	8.37	9.46	10.59	35
40	1.72	2.66	3.65	4.69	5.78	6.91	8.09	9.34	10.64	12.03	13.52	40
45	2.22	3.40	4.65	5.95	7.34	8.83	10.41	12.10	13.90	15.83	17.89	45
50	2.88	4.46	6.15	7.95	9.88	11.94	14.15	16.52	19.07			50
Endowment 20 Years.												
20	.16	.23	.33	.44	.58	.73	.89	1.04	1.21	1.38	1.56	20
25	.29	.45	.60	.77	.94	1.12	1.31	1.53	1.76	2.01	2.28	25
30	.35	.54	.76	.99	1.24	1.51	1.79	2.09	2.43	2.81	3.23	30
35	.52	.80	1.10	1.44	1.82	2.24	2.72	3.25	3.83	4.47	5.15	35
40	.80	1.28	1.81	2.39	3.03	3.71	4.45	5.25	6.12	7.08	8.15	40
45	1.32	2.06	2.86	3.73	4.69	5.76	6.94	8.23	9.66			45
50	2.03	3.21	4.50	5.93								50

TABLE III.

EXTRA PREMIUMS (NET) ON THE BASIS OF H^m $3\frac{1}{2}$ PER CENT. REQUIRED TO PROVIDE FOR THE MORTALITY STATED.

Age.	Ordinary Life. % 125.	% 150.	% 175.	% 200.	Life 20 Payments. % 125.	% 150.	% 175.	% 200.	Endowment 20 Yrs. % 125.	% 150.	% 175.	% 200.
20	2.05	4.01	5.91	7.76	2.49	4.78	6.92	8.94	1.04	2.10	3.16	4.23
25	2.28	4.45	6.55	8.58	2.62	5.02	7.27	9.39	1.16	2.33	3.51	4.70
30	2.61	5.09	7.48	9.79	2.83	5.43	7.86	10.16	1.36	2.73	4.12	5.52
35	3.01	5.87	8.62	11.28	3.08	5.91	8.56	11.09	1.62	3.26	4.91	6.58
40	3.55	6.91	10.14	13.25	3.41	6.56	9.53	12.37	1.99	4.01	6.05	8.11
45	4.32	8.41	12.33	16.13	3.95	7.65	11.15	14.54	2.63	5.28	7.97	10.68
50	5.38	10.49	15.39	20.13	4.79	9.31	13.67	17.91	3.60	7.25	10.94	14.66
55	6.92	13.50	19.83	25.98	6.12	12.01	17.76	23.41	5.16	10.39	15.65	20.96
60	9.17	17.94	26.41	34.67								

with these cases. The tables which are shown here, and which have been referred to above, are submitted merely with the idea that they may assist us in arriving at a proper decision in regard to the method or methods to be employed in the case of under-average lives. For example, Tables I, II and VI would lead us to believe that in any case where the extra mortality is comparatively uniform throughout life, we would not be justified in employing the uniform rating-up in age method and allowing the surrender values at the rated up age.

TABLE IV.

NUMBER OF YEARS RATING UP REQUIRED TO PROVIDE FOR THE EXTRA MORTALITY STATED.

Age.	Ordinary Life.				Life 20 Payments.				Endowment 20 Yrs.			
	25%.	50%.	75%.	100%	25%.	50%.	75%.	100%.	25%.	50%.	75%.	100%.
20	5	9	13	15	6	10	13	16	9	15	19	22
25	5	8	11	14	5	9	12	15	7	12	16	18
30	4	8	11	13	5	8	11	14	6	11	14	16
35	4	7	10	12	4	8	10	12	5	9	12	14
40	4	6	9	11	4	7	9	11	4	7	10	12
45	3	6	8	10	3	6	8	10	4	7	9	11
50	3	6	8	9	3	6	8	10	3	6	8	10
55	3	5	7	9	3	5	7	9	3	5	7	9
60	3	5	7	9								

TABLE V.

15-YEAR ENDOWMENT—AGE 35.

Premium for Normal Mortality, $55.58.

Premium for 150 Per Cent. Mortality, $58.41.

At End of Year.	Reserve on Normal H^m 3½% Basis.	Reserve on 150% Mortality Basis.	Fund using Normal Premium with 150% Mortality.
1	49.18	47.93	44.96
2	100.23	97.73	91.64
3	153.24	149.54	140.17
4	208.39	203.55	190.73
5	265.80	259.93	243.48
6	325.68	318.94	298.67
7	388.18	380.84	356.54
8	453.42	445.68	417.14
9	521.49	513.64	480.62
10	592.55	584.87	547.11
11	666.75	659.54	616.74
12	744.30	738.00	689.83
13	825.48	820.59	766.70
14	910.61	907.81	847.80
15	1,000.00	1,000.00	933.44

TABLE VI.

ORDINARY LIFE—AGE 35.

Premium for Normal Mortality, $20.76.
Premium for 150 Per Cent. Mortality, $26.63.
Premium for Normal Mortality, Age 42, $26.54.

At End of Year.	Reserve on Normal H^m 3½% Basis.	Reserve on 150% Mortality Basis.	Regular H^m 3½% Reserve Age at Entry 42.	Fund using Normal Premium with 150% Mortality.
1	12.82	14.60	16.92	8.44
2	25.88	29.42	34.23	16.78
3	39.18	44.45	51.95	25.01
4	52.77	59.77	69.90	33.19
5	66.68	75.44	88.01	41.32
6	81.03	91.60	106.32	49.56
7	95.86	108.35	124.87	57.97
8	111.16	125.63	143.68	66.44
9	126.81	143.30	162.83	74.81
10	142.83	161.34	182.36	83.02

MR. M. M. DAWSON:

It is certainly most gratifying to find a paper upon this subject attracting so much interest; and it is a great contrast with the general attitude not many years ago. I well remember when Mr. Frankland was brought from far New Zealand to be assistant actuary of the great company with which Mr. Hunter is connected for the express purpose of assisting in plans for insuring underaverage lives; it is not many years ago but there has been a very great change of attitude since then.

Mr. Frankland, impressed with the difference in agency methods, did not recommend the "rating-up" in age method, with which he was best acquainted, but instead helped to bring the "lien" plan into practical shape.

It seemed to me at the time—and it still seems to me—that the other plan could have been used to better advantage then, as now; and, very particularly, that a study of Australian agency methods, in this regard, might have afforded a better solution than the argument, "If you prove yourself a good risk by living so many years, there is no lien," which enabled a lien policy to be *placed,* though often and even usually, it had not been applied for.

That agency method has been described to me by our fellow-member, Mr. Richard Teece, actuary and manager of the Australian Mutual Provident Society, about as follows: Medical examiners, employed by the company, accompany agents frequently if not always; examination is made usually, if not always, before rate is named or application taken. Examiner, noting any defects, consults company's instructions, thus determines the rated age, and advises agent who quotes the rate. Usually, though by no means always, applicant is accepted and policy issued at the rated age.

Mr. Teece kindly furnished me a copy of the instructions to medical examiners in use some ten years ago. It shows ratings up to seven years in age, lives rarely being accepted if the impairments called for a higher rating; a large number of the lives accepted by American companies subject to lien for more serious impairment would not have been accepted.

Up to twenty years ago or thereabout—I have no reason to suppose that the condition has changed—about one-third of the lives were rated-up. The experience of the company regarding them had been very good, only about 67 per cent. of the expected deaths, computed at the rated-up ages, a percentage only 2 per cent. higher than that on lives accepted as first-class, computed at the true ages.

This large proportionate volume of rated-up lives signifies, in part at least, that many were rated up from one to three years for slight impairments, whom American companies would accept as first-class.

It has been, and remains, my personal opinion that the agency method pursued by Mr. Teece's company, could, to advantage, be introduced in this country and would be entirely successful and satisfactory.

The various expedients here discussed for dealing with peculiar cases of impairment involving increasing, decreasing or stationary additions to the mortality rates are most interesting and may be very useful in dealing with seriously impaired lives; for the general run of slight impairments no other system has made so good a showing as the rating-up as per the Australian experience, nor is any other very likely to approach it.

The statement has frequently been made that the Armstrong laws prohibited liens for impairment. They did render special dividend classes impracticable, unless dividends were declared annually; but the lien plan was affected only to the extent that it would have been necessary for the Superintendent of Insurance to approve a standard form for such a policy. The automatic extension provision of the law did not necessarily prevent the use of the plan, for such extension would, under such a standard policy, also have been subject to the lien.

Undoubtedly, however, the rating-up plan and extra premiums were encouraged by these laws; and the great extension of the insurance of under-average lives appears to prove that this was the wiser course, in any case.

An expedient for dealing quickly, effectively and economically with certain phases of insuring impaired lives which arise in connection with proposed reinsurance of lives already subject to adverse selection, has been evolved in our office work, which may be of interest. We have used it only in a rough way to determine liens to be imposed and the like; but in principle it is, when more narrowly considered, well adapted for more general use in the frequent case when the impairment is deemed likely to improve grad-

ually, if not fatal, or at least to exercise a steadily proportionate effect upon the prospect of surviving one year and consequently a steadily diminishing effect upon the probability of dying within the year.

As first employed by us, it was accepted as a rough and ready means of dealing with irregular percentage increases in the rate of mortality; it really provides, however, for impairments which are constant percentages of the tabular probabilities of surviving one year. The exact method is developed as follows:

Let $p_x^{(I)}$ be the probability of the impaired life, aged x, surviving one year and r be the percentage diminution of the tabular probability of surviving one year; so that $p_x^{(I)} = (1-r)p_x$.

Then, obviously:

$$
\begin{aligned}
{}_np_x^{(I)} &= [(1-r)p_x][(1-r)p_{x+1}]\cdots[(1-r)p_{x+n-1}] \\
&= (1-r)^n \cdot {}_np_x.
\end{aligned}
$$

Since

$$a_x = \sum_{t=1}^{t=w} v^t \cdot {}_tp_x,$$

it follows that

$$
\begin{aligned}
a_x^{(I)} &= \sum_{t=1}^{t=w} v^t \cdot {}_tp_x^{(I)} \\
&= \sum_{t=1}^{t=w} v^t \cdot (1-r)^t \cdot {}_tp_x.
\end{aligned}
$$

This may be transformed, since $V^n = (1-d)^n$ as follows:

$$
\begin{aligned}
a_x^{(I)} &= \sum_{t=1}^{t=w} (1-d)^t(1-r)^t{}_tp_x \\
&= \sum_{t=1}^{t=w} \{1-(d+r-rd)\}^t{}_tp_x
\end{aligned}
$$

which, putting $d' = d + r - rd$ and using $v^{(i')t}$ as an expression for the value of one, dicounted t years at the changed rate of interest

$$i' = \frac{d'}{v^{(i')}},$$

becomes:

$$a_x^{(I)} = \sum_{t=1}^{t=w} v^t \cdot {}_tp_x = a_x^{(i')},$$

i. e., the value of an annuity by the standard table but at the changed rate of interest.

And since $A_x = 1 - d(1 + a_x)$, it follows, since the impair-

ment affects only elements in this equation in which mortality is involved, that

$$A_x^{(I)} = 1 - d(1 + a_x^{(i')}) \quad \text{where} \quad d = \frac{i}{1+i}$$

in which i is the original rate of interest.

Therefore, by looking up in Orchard the net single premium at the original rate of interest corresponding with the annuity at the changed rate of interest, we obtain the net single premium at the original rate of interest for the insurance of the impaired life.

The net annual premium may then be computed by the formula

$$P_x^{(I)} = \frac{A_x^{(I)}}{1 + a_x^{(I)}}.$$

This method is also readily adapted to the computation of term and endowment premiums.

MR. J. B. MCKECHNIE:

This paper is of a very practical nature and comes at an opportune time when many companies, as Mr. Hunter points out, are taking advantage of the Medico-Actuarial Investigation to remodel their plans of insurance for impaired lives.

In connection with the lien method, the author refers to the British plan of reducing the lien uniformly each year through a period equal to the expectation of life and to the American plan of reducing the lien by the amount of the premiums paid. The plan usually followed by Canadian companies is to reduce the lien uniformly through a period of, say, 10, 15 or 20 years.

The author refers to the fact that the prominent companies discontinued the lien method owing to the very small extra mortality taken care of by the lien. I was under the impression that the method was discontinued owing to the impossibility of using it under the New York insurance law, and I would like to enquire from Mr. Hunter whether this fact had any bearing on the matter. It is true that the lien method does not cover a very great additional mortality, but I think that Mr. Hunter's percentages on page 69 are too low. The lien method is applicable only to lives on which the extra mortality falls in the early years of the insurance, and I think that if Mr. Hunter had allowed for this in his calculations he would have found that the lien method was much more effective than indicated by him.

The most important matter referred to in the paper is, in my opinion, that dealing with the question of how great a departure from the normal mortality should be allowed before a life is rated up. Mr. Hunter just touches on this problem, but I judge that he considers it quite safe to accept lives showing an extra mortality of 25 per cent. on endowment 10, 15 and 20 plans.

The medical director usually looks upon the actuary as being, not only infallible in matters of this kind, but also as one who hews right to the line. When therefore a life is presented for consideration showing a mortality of, say, 125 per cent. of that by the Medico-Actuarial Table, he expects the actuary to support him in his recommendation that the life should be rated up. The actuary knows, however, that other things have to be considered. Mr. Hunter in a previous paper (*T. A. S. A.*, XII, p. 8) referred briefly to the same point when he said "Thus if a company had the opportunity of either accepting or rejecting 1,000 lives for $2,500,000 of insurance, the actuary could determine theoretically to what extent the placing of such insurance on the books would decrease the fixed charges, and that would be the measure of the maximum extra mortality which the company would be justified in standing."

The matter, however, is not a very simple one, particularly in the case of Canadian companies, all of which issue both participating and non-participating insurance, and of which out of a total of 25 companies only one is purely mutual. Moreover, in the charters of the majority of the companies provision is made that the profit or loss in the non-participating section is taken wholly by the shareholders. It would seem therefore that in companies of this nature a life could be profitably accepted in the non-participating section, even though it is subject to a mortality of 100 per cent. of that by the table by which the premiums were calculated (equivalent to say 150 per cent., of that by the Medico-Actuarial Table), the shareholders looking for their profit to interest and loading.

In the participating section, however, you cannot go this far because it will be inadvisable to introduce into this section a policy which is likely to reduce the average profit per $1,000 of insurance. You can, however, without reducing the average profit accept at the regular premium a life which will be subject to an extra mortality equivalent to the saving in the fixed charges, as pointed out by Mr. Hunter in his former paper. The amount of this saving will, of course, vary with each company, but should be capable of being determined approximately.

It should be kept in mind that the above are the limits to which a company should go. If a policy can be placed on terms more favorable to the company it should be done. The proper plan to follow appears to me to be as follows. The sub-standard life should be rated up as much as the additional mortality requires. This extra rating, however, should be reduced to a point where the policy is likely to be accepted by the applicant, provided, however, that the reduction should not go beyond the limits referred to above.

Some objection may be taken to the fact that the rating given to the substandard life is made to depend partly on competition. In reply to this it may be pointed out that the above plan will bring the maximum profit to the policyholders, and can on this ground, if on no other, be justified.

MR. R. HENDERSON:

Mr. Hunter's paper discloses the familiarity with his subject which was to have been expected in view of his wide experience and natural qualifications, and I shall only venture to comment on a few points upon which my own experience appears to have thrown a special light.

On page 66 the following statement is made:

"It is proper, therefore, that the relative mortality for the various types of lives should be based on the company's experience by amounts insured."

The context does not appear to me to fully support this statement. It is an admitted fact that a mortality experience taken out by lives is less liable to fluctuation than one covering the same amount of material taken out by amounts. If, therefore, our object is to determine as closely as possible the relative mortality of a small class of risks, as compared with the company's general experience, and if our principal difficulty is, as is usually the case, the scantiness of our material, it would appear that that difficulty can be best overcome by basing our experience on lives rather than on amounts, provided, of course, that the standard with which the comparison is made is also an experience by lives. Of course, if we had any reason to believe that the amount insured had a special influence in the particular class with which we are dealing, then we should investigate that influence.

On the general question of the use of special classes as to dividends, perhaps a word of warning would not be out of place, in view of the possibility that this may very readily be carried to excess and special classes constituted in which there is practically no chance of a sufficient volume of business ever being secured to constitute a reliable basis of dividend apportionment. The size of the company has an important bearing on the possibility of adopting this scheme, but it may readily happen that even a large company will fall into error in this direction.

With respect to the general question of reserves to be held on substandard business, there is one point which I think deserves a little more emphasis than is given by the author in his paper. I refer to the fact that prior to the amendment of insurance laws generally, which took place about 1906 or 1907, a wide margin was available to the companies between the reserve calculated upon a particular policy and the surrender value allowed. By the amendment then made this margin was reduced to very narrow limits, with the consequence that if the reserve is calculated on too stringent a basis the surrender value will necessarily be higher than is strictly just. It seems to me that this is the case where the valuation is made on the basis of the rated up age or of a rated up age equivalent to the extra charge. For this reason the company with which I am connected has recently, with the approval of the Insurance Depart-

ment, changed its basis of valuation for policies issued with an extra premium either for occupation or physical impairment and has adjusted its tables of surrender values accordingly.

On page 81 under the heading of "Special Problems" the following statement is made:

"When a company is issuing policies with liens, a request is occasionally made to give the applicant the option of taking that form or one with an extra premium equivalent to the lien. If a company grants such requests, it is likely to suffer financially because the insured who believe in themselves will generally be contented with the lien, while those who fear that their lives will be cut off early will prefer to pay the extra premium."

I feel especially qualified to comment on this statement, because, so far as my knowledge of history goes, this statement is made by Mr. Hunter on a purely theoretical basis with very little of his own experience back of it; whereas the company with which I am connected did exactly this thing for a period of six and a half years and only discontinued it because it appeared necessary to do so under the standard policy laws going into effect in 1907. Possibly our experience might have been different with this option if we had been attempting to cover very high rates of extra mortality. Our ratings at that time were confined to relatively moderate impairments, a very large percentage of them being less than 150 per cent. of the normal, and our experience on this class of risk was that there was no selection exercised by the applicant against the company in the way indicated by Mr. Hunter, for the very simple reason that in practically every case where the option was granted the extra premium was paid. In fact, to such an extent was this true that very shortly after we started granting the option we found it advisable, in order to expedite the placing of the business, to issue the policies in all such cases with the extra premium at the rated up age with the understanding that they could if desired be returned for change. But practically none were ever returned.

MR. J. D. CRAIG:

Mr. Hunter sets forth four methods of treating under-average lives which he says are the four principal methods, but that there may be combinations thereof. There is one method not mentioned by Mr. Hunter which can hardly be called one of the four outlined or a combination thereof, but which produces substantial justice and at the same time eliminates to a great extent many of the special questions discussed relative to proper reserves, cash values and other concessions, as well as commissions.

The method is simply the straight calculation of premiums upon mortality tables which provide for the increased hazard and then make all other factors correspond. It differs from the method of advancing age, as brought out by Mr. Hunter at the top of page 78,

by stopping at the adoption of the premium of $23.21 as worked out from a table where the mortality is 150 per cent. of the American Table, without trying to follow it further by assuming that because this premium of $23.21 corresponds to a similar premium computed on the American Table at an age 8 years older, the reserves and cash values will also follow. Such a method provides for the extra mortality just as it is expected to be incurred and frankly states the problem in its simplicity.

Mr. Hunter recognized the principle of different classes where he referred to the possibility of establishing Intermediate, Hazardous and Extra Hazardous classes of risks with varying mortality, and then stated:

"While a system of this kind does not discriminate with exactness between the different individuals of each class, it gives as much justice as the present practice of granting insurance at the same rates of premium alike to those who are exceedingly good, those who are very good, and those who are average lives."

While the different percentages of mortality would determine the various classes, the period at which the extra mortality is expected would be a material factor. In one case the extra mortality might be expected to remain constant, in another it might be expected to decrease, while in still another it might be expected to increase. When the extra percentages likely to be incurred are considered in connection with the future course of these percentages, it might be better to construct a substandard table based on the combined experience under the various classes of substandard risks and then base all calculations directly thereon. While Mr. Hunter points out that the mortality on the grade of lives accepted at the regular premium probably ranges from 60 per cent. to 115 per cent. or 120 per cent. of the probable, the combination of all these risks in one class has produced no criticism in the past, but on the contrary has followed the true principle of life insurance where those who live long pay more than those who do not, and if such variations as those mentioned are successfully and satisfactorily combined for standard lives, why cannot equal variations in the rates on substandard lives be combined with as satisfactory results?

In fact, such a table has been used in one company and policies issued with premiums, reserves and surrender values all based on it. One of the most remarkable results is the uniformity in the ratio of probable claims to actual experienced among the different groups of policies in this one class from the years 1899 to 1914 inclusive. Thus liquor dealers have a percentage of 69; hazardous occupations other than liquor dealers, 68 per cent.; overweights, 69 per cent.; underweights, 63 per cent.; family history, 65 per cent.; personal history, 60 per cent. Impaired hearts constitute the only group with an exposure sufficient to give a reasonable experience which shows a percentage greatly different from any of the others, the percentage being 91 which is only 22 points higher than the percentage for liquor dealers and overweights.

When this branch was first started, policies were issued with a regular premium, but with a lien sufficient to provide for mortality practically double that called for by the standard table. The policies were made participating with the proviso that dividends would be applied toward cancelling the liens. In 1907, when stock companies were prohibited from issuing both non-participating and participating policies, a new table was prepared, premiums based thereon adopted and the lien system discontinued.

As a practical proposition, it might be interesting to note that under the lien system discrimination against the company was more pronounced. If the insured felt as though an injustice was done in offering the special policy, he refused to accept it, with the result that over 50 per cent. of the policies issued were returned as "not-taken." When the change was made in 1907 and the special premium charged for the full amount of insurance without any lien, the ratios of "not-taken" policies immediately fell off to a trifle over 30 per cent.

MR. ARTHUR HUNTER:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

In the first place I wish to thank most sincerely the members of the Society who have been good enough to prepare discussions on my paper. I appreciate greatly the time they have spent, not only in analyzing my paper, but in making suggestions of other ways of handling under-average lives. This is particularly thoughtful at the present time, because it is my duty to prepare the chapter on under-average lives for the Text-Book, and in order to state the matter in a broad manner, views of such prominent members of the Society as have discussed the subject to-day must be obtained. After the paper was finished I was conscious that the question of insuring under-average lives had not been adequately covered. To do so would have greatly increased the length of my paper, and would have brought in so many details that the main features might have suffered. I especially wished to deal with the changes in the various plans which would result from having to cover the three main types of extra mortality, which may be stated to be (*a*) a constant for many years (the accident hazard); (*b*) a decrease towards normal from the early policy years; (*c*) an increase for several years towards a maximum, then a gradual decrease towards the normal. No system yet suggested can adequately cover these three conditions, together with other variations of them. A company doing a small amount of business each year, or one which accepts a small amount of under-average business as an incidental part of its total production might be able to use three distinct plans for treating under-average lives, thereby covering the main curves of extra mortality; but if a company did a large under-average business a simple program would be necessary—otherwise the agents

would not know on what plan to canvass risks. An agent, for example, would be seriously handicapped if he could not give a substandard applicant some idea of the plan on which the policy would be issued, i. e., whether it would be charged with a lien, with an extra premium, or be advanced in age. From the standpoint of the production of business, two of the requisites of any plan, in my judgment, are salability and simplicity. It is our duty as actuaries to try to develop for our companies plans which will meet these conditions, and not one which might be scientific to the last degree but so complicated that the agents could not succeed in selling policies. That viewpoint was in mind in writing this paper.

Before taking up the criticisms and suggestions made by the various speakers, it seems advisable to refer in a general way to the numerical basis of rating risks. In May of 1911 (*T. A. S. A.*, XII, 7) I made the following statement: "I do not for a moment desire it to be understood that a numerical method is a cure for all ills. The automatic application to *all* cases of any numerical rating system would undoubtedly work harm in individual instances, because it does not make due allowance for the applicant being a better or a poorer risk than the average of the group which forms the basis for the mortality used in the numerical rating. At the same time it is of great assistance to the medical man, as it reduces error, increases the time the medical man can give to other important matters, makes the work easier for him, and results in a more uniform selection." On page 295 of the same volume of the *Transactions,* I asked the question: "Are we to be contented with the old method which is partly based on impressions and is at the best more or less arbitrary; or are we willing to accept a scientific method *with limitations* which can be largely removed by the use of judgment?" and on page 298, "I do not wish it to be understood that the numerical system, such as outlined, has attained its highest development, nor that it is satisfactory in every respect." In the paper now under discussion the following statement is made: "Freely admitting that the system is not a perfect one, and acquiescing in the opinion of one of my critics, that it requires men of unusual experience and unusual judgment to use it, and that there are pitfalls for the unwary and inexperienced, I believe that the adoption of such a method is better than to cling to the old plans." I fully agree with Mr. Rhodes that "if absurd results are to be eliminated there should be some arbitrary limitation of the total credits." The practice of the New York Life is not to grant sufficient credits to offset a marked impairment. If, for example, a man has albuminuria, or is greatly overweight, or has some other marked impairment, however many credits he might obtain from favorable features, he would still be treated as an under-average life. While he would obtain certain advantages from the favorable features, these would not be sufficient to result in his receiving a policy without an extra premium.

With regard to Mr. Dawson's statement that in the years immediately following the amendments to the New York law of 1906

the lien plan was not affected to the extent "that it would have been necessary for the Superintendent of Insurance to approve a standard form for such a policy," the fact is that the Superintendent of Insurance was not in favor of the issue of such policies. The companies which were considering a continuance of the plan for the issue of policies with lien were informally told that it would be very difficult to have an appropriate standard form approved by the Superintendent. There would probably have been a marked decrease in business issued with lien even if standard forms had been provided, because it did not then seem feasible to make separate dividend classes, and the companies which did the largest business on underaverage lives had a separate classification with regard to dividends. That there are advantages and disadvantages to the lien system may be seen from the fact that two eminent actuaries disagree—Mr. Moir regrets that "the lien system has fallen into disuse"; while Mr. Gore has "no enthusiasm for the policy lien plan."

I am especially indebted to Mr. Gore for his very able discussion of my paper. There are many suggestions of great value to all of us, and I wish that it were possible to comment on a number of them. His method of showing the yearly extra mortality covered by a lien brings out clearly the fact that liens are best adapted to the class of risks with a high extra mortality in the early policy years, gradually decreasing to the normal. I have been of the opinion for some time that there is no class of under-average risks which shows a steadily increasing ratio of the normal mortality: there are undoubtedly types which show an increasing ratio for a number of years, but thereafter the tendency is towards a reducing ratio in the later years of life. Mr. Gore brings this to our attention by giving the relative mortality after attained age 60 in a large group of impaired risks taken from the Medico-Actuarial experience. He shows, and Mr. Moir confirms this statement from a different set of figures presented to the American Life Convention, that the mortality under impaired cases approaches the normal after the policies have been in force for a number of years. This point is also brought out in the experience of a large class of underaverage lives insured by the New York Life during the years 1896 to 1906 inclusive. Underaverage lives of every type were put into this class, yet the percentage of the mortality to the normal has gradually decreased until in 1915 the ratio was markedly lower than in 1906. Too short a period has elapsed, however, to determine whether or not these lives would be practically normal after the lapse of say twenty-five years from date of entry. The principal strain from this business comes in its early years, and as the lien system was generally used the company was properly protected.

Mr. Gore favors the granting of the same surrender options and dividends to under-average risks as to standard lives at the true age. Under the plan of advancing the age of the insured, however, it seems

an advantage from the agency standpoint to pay the surrender values and dividends at the attained age. Where the extended insurance is the automatic non-forfeiture feature there is a certain offset to the larger cash surrender values granted at the rated up age.

We are all interested in Mr. Rhodes's statement of his conception of an under-average life. Many of us would agree with him that it is an excellent statement of the case, but if one were to attempt to write a paper on under-average lives, starting with such a conception of an average life, it would be very difficult to make any headway. Whether one believes in the numerical system or not, it seems to me that as the basis of it can be readily understood, a discussion of any particular phase of the under-average business can be more easily understood by all actuaries through such a system of rating. Mr. Rhodes's clear statement of the advantages and disadvantages of under-average business must be of great value to companies considering this matter seriously. It does not come within the province of my paper, however, to discuss this phase. The fact that the New York Life has made a success does not mean that new companies can also find the under-average business profitable. When it is remembered that the two companies organized in recent years in this country to insure only under-average risks were reinsured, we can appreciate the dangers that lie in this business. The company, however, which can surmount them satisfactorily has decreased waste, and has given protection to people who need it most and whose families would otherwise go without the benefits of life insurance.

Mr. Wood thinks that in occupations in which the accident hazard predominates it is better to charge a uniform extra premium, although theoretically a different extra premium for each plan of insurance should be prepared. In this connection we are again faced with the clash of theoretical and practical considerations. In order to be correct theoretically, it would be necessary to have a different extra premium not only for each plan of insurance, but probably for groups of ages at entry. It is not generally considered feasible to adopt such a method, but to charge an extra premium irrespective of either plan of insurance or age of applicant, although some companies charge about one-half the extra premium on short term endowments that they do on ordinary life and limited payment policies. If the advance in age method is used, a uniform advance irrespective of age or plan results in a smaller extra premium in dollars and cents under the endowment than under the life plans, the shorter the endowment period the smaller being the extra premium. An actuary can determine from the experience of his company what the average age and average plan of insurance are, so that a uniform advance in age can be charged for each occupation, which will provide enough extra premium to meet the occupation hazard. By charging the same advance in age under all plans the policyholders on the life plans are theoretically charged slightly too much, and those on the endow-

ment plans slightly too little. This is generally justified on the ground that the selection exercised by the latter in favor of the company results in a lower mortality which more than compensates for charging lower extra premiums on the endowment plan than are theoretically called for. The advance in age method should not be applied rigidly at the older ages. In the company with which I am connected the extra premium corresponding to the advance at age of entry 40 is the *maximum* which can be charged at the older ages for the specified occupation.

With regard to the question of giving policyholders the option of taking a policy with an extra premium or with a lien, my statement that there would be a selection against the company was based partly on theoretical grounds, but partly on experience which, however, was not extensive. The experience to which Mr. Henderson refers, in which there was no selection against the company, may be due partly to the option being permitted only in cases with slight impairments, but largely to agents not submitting both offers to the applicants. Is it not probable that as the agents received a commission on the extra premium, they did not submit the policy with the lien to the applicant?

With regard to Mr. Craig's statement that there was a discrimination against the company under their lien system, which he thought was due to anti-selection resulting from over 50 per cent. of the policies being returned as "not taken"—is this experience an individual one, and might it not have been avoided by liens which ran from say $100 to $900 per thousand? The experience of a company with which I am familiar shows that a large percentage of the policies with light liens were placed, but only a small percentage of those with very heavy liens. Under the latter the mortality was distinctly less than anticipated and provided for, showing that the small percentage of the policyholders who accepted contracts with heavy liens had faith in themselves and did not therefore select against the company.

Mr. Kilgour expresses himself as being in favor of using either the advance in age or the extra premium, depending upon the incidence of extra mortality. That practice is being followed by several of the smaller companies which generally charge an extra premium where the hazard from occupation is approximately a constant, while an advance in age is charged on occupations involving other than the accident hazard, and in cases with medical impairments.

In conclusion, I should like to repeat the statement made in my paper,—i. e., there is no system for treating under-average lives which is satisfactory in every particular, and that, accordingly, the method should be adopted which gives justice in general to all policyholders, and which is adapted to the conditions under which the company does business.

VALUATION OF THE DEATH BENEFITS PROVIDED BY THE WORKMEN'S COMPENSATION LAW OF NEW YORK—EDWARD OLIFIERS.

VOL. XVI, PAGE 83.

WRITTEN DISCUSSION.

MR. E. E. CAMMACK:

We are indebted to Mr. Olifiers for his demonstration of a convenient method of calculating with a close degree of accuracy the valuation of pensions to widows and children included amongst the benefits provided in the workmen's compensation law of the State of New York.

The exact valuation of these pensions based upon the Dutch State Insurance Fund Remarriage Table and the Danish Female Survivorship Annuitants' Table, even though the "equal age" method of Makeham's law is applicable, is a laborious process, involving the computation of from five annuities in the case of a widow with two children under the age of 18 to nine hundred and fourteen annuities in the extreme case of a widow with nine children.

The principle involved in the method explained in the paper for the calculation of these benefits is that life annuities can, if the force of mortality is a constant, be replaced by annuities certain computed at a rate of interest modified according to the number of lives involved in the annuity.

This is easily demonstrated as follows:

If the force of mortality $\mu_x =$ some constant, $-\log_e s$, then since

$$\begin{aligned} d \log_e l_x &= -\mu_x dx \\ d \log_e l_x &= \log_e s dx \\ \log_e l_x &= \int \log_e s dx \\ &= x \log_e s + \log_e k. \end{aligned}$$

Therefore

$$l_x = ks^x.$$

Now if it is assumed that the force of mortality up to age 18 is a constant, then where $y_1, y_2, y_3, \cdots, y_n$ are the ages of n children all under the age 18, of whom y_n is the oldest, and where x is the age

of a widow, then

$$\bar{a}_{y_1y_2y_3\cdots y_n\overline{18-y_n}|} = \frac{1}{l_{y_1y_2y_3\cdots y_n}}\int_0^{18-n} v^t l_{y_1+t\cdot\, y_2+t\cdot\, y_3+t\cdots y_n+t}\mathrm{d}t$$

$$= \frac{1}{k^n s^{y_1+y_2+y_3\cdots+y_n}}\int_0^{18-n} v^t k^n s^{y_1+y_2+y_3\cdots+y_n} s^{nt}\mathrm{d}t$$

$$= \int_0^{18-n} v^t s^{nt}\mathrm{d}t$$

$$= \bar{a}'_{\overline{18-n}|}$$

where a' is calculated at a rate of interest so changed that "v" becomes "(vs^n)" and

$$\bar{a}_{xy_1y_2y_3\cdots y_n\overline{18-y_n}|} = \frac{1}{l_x l_{y_1y_2y_3\cdots y_n}}\int_0^{18-n} v^t l_{x+t} l_{y_1+t\cdot\, y_2+t\cdot\, y_3+t\cdots y_n+t}\mathrm{d}t$$

$$= \frac{1}{l_x}\int_0^{18-n} v^t l_{x+t} s^{nt}\mathrm{d}t$$

$$= \bar{a}'_{x\overline{18-n}|}$$

where a' is calculated at the same changed rate of interest.

Advantage has been taken of this principle by assuming that the force of mortality for the lives of children up to age 18 is a constant, and the error involved in this assumption is small. For the value of "s" .99486 has been taken, which corresponds to an assumption that the force of mortality is a constant of .00516. It is difficult to see why this force of mortality has been assumed. In the Danish Female Survivorship Annuitants' Table, the force of mortality gradually increases from .00518 at age 0 to .00531 at age 18. Obviously the force of mortality assumed has been too low. Better results would have been attained if the force of mortality of .0052 (i. e., between the actual limits of .00518 at age 0 and .00531 at age 18) had been taken. This would have practically eliminated the differences shown on page 89 between the correct values of the annuities calculated by the "equal age" method and the approximate values.

The value of "s", corresponding to an assumption of a constant rate of mortality on the lives of children up to age 18 of .0052, would be .99481, and I show in the table below a comparison of five annuity values calculated on this assumption with the correct values as given by Mr. Olifiers.

The author states that for benefits in connection with a widow with less than four children his values are exact, so that he evidently does not intend to apply the above principle to the calculation of joint annuities involving the lives of a widow and one child. The

substitution of temporary annuities on a widow's life at a modified rate of interest for joint annuities on the lives of a widow and one child will, however, be found convenient.

VALUE OF JOINT TEMPORARY ANNUITIES.

Lives Involved.	Term.	Correct Value.	Approximate Value.	Difference.
Five children aged 1, 2, 3, 4, 5	13	9.007	9.005	+.002
Children aged 1, 2, 3, 4, 5 and widow aged 20	13	8.735	8.734	+.001
Children aged 1, 2, 3, 4, 5 and widow aged 40	13	8.629	8.630	−.001
Children aged 1, 2, 3, 4, 5 and widow aged 20 (with allowance for remarriage)	13	5.283	5.283	.000
Children aged 1, 2, 3, 4, 5 and widow aged 40 (with allowance for remarriage)	13	7.769	7.768	+.001

The formulas given on pages 91 and 92 for the calculation of benefits where the lives of a widow with from one to seven children are involved will be found very useful to those actuaries who have the actual computation to make of the values of pensions as provided in the New York compensation law. In the absence of a complete set of tables such as are outlined in the paper, all that is required for the calculation of benefits by means of these formulas will be

(1) A table of temporary annuities certain for terms of from 1 to 18 years, calculated at three rates of interest so taken that these annuities may be substituted for joint life annuities involving 5, 6 and 7 lives respectively.

(2) A table of temporary life annuities for the same terms, calculated at five rates of interest so taken that they may be substituted for joint life annuities involving the lives of a widow and 1, 4, 5, 6 and 7 children respectively, and similar tables where the lives of a widow and 4, 5, 6 and 7 children are involved in which allowance is made for the remarriage of the widow.

It would be interesting to know whether Mr. Olifiers has computed such a set of tables as he has outlined.

MR. H. R. STEPHENSON:

The New York workmen's compensation law provides upon the death of an employee arising out of his employment, that pensions are to be paid to certain of his relatives who survive, the amount of each pension being a definite percentage of the average wages of the deceased employee.

In this paper, Mr. Olifiers has presented a method for valuing the benefits payable to the first or preferred class of beneficiaries, consisting of the widow (or dependent widower) and the children under eighteen years of age.

The mathematical expressions for these benefits are complicated owing to the provision that the total amount of all the pensions must not exceed $66\frac{2}{3}$ per cent. of the average wage of the deceased employee. In fact, as the number of beneficiaries increases, the expressions become so cumbersome that it is impossible to employ them in practice without resorting to some simplifying process.

The suggestion made by Mr. Olifiers is, in effect, to assume that the force of mortality is constant during the first eighteen years of life. The inclusion of a child's life in a temporary annuity thus has the same effect as an increase in the rate of discount. In this way the individual *ages* of the children involved in the annuities are lost sight of, and regard is had only to the *number* of children. A joint temporary annuity on the children's lives becomes simply an annuity certain at a rate of interest increasing with the number of lives involved. By this expedient the formulas are reduced to workable size and the calculation of the various benefits is made practicable.

The error caused by this assumption, as applied to the Danish Female Survivorship Table, is very small, but that table is peculiar in that the rate of mortality constantly increases from age 0 to age 18. In the case of a table which exhibits the usual high mortality in the earliest years of life, and when lives aged one or two years are involved in the annuities. a much larger error is introduced. This may be corrected by bringing into the calculation of the annuity the actual age of the youngest child in such cases.

This way of simplifying the formulas would seem to be justified by the reasonably close agreement of the results with the exact values in the examples shown—the more so, bearing in mind the very considerable assumptions which must always be made in fixing the mortality and interest bases for the valuation.

For convenience in valuation, Mr. Olifiers recommends the use of a series of tables containing successively the reserves in respect of the various beneficiaries. Such tables appear to be admirably adapted for office use. On account of the ever-diminishing extra reserve required in respect of each additional child after the limit of $66\frac{2}{3}$ per cent. is reached, there is a limit to the number of tables needed. Mr. W. W. Greene has pointed out that it is unnecessary to take account of more beneficiaries than a widow and nine children.

As regards the death benefits payable to the second or non-preferred class of beneficiaries, my reading of the act is that, if on account of the number of beneficiaries of the first class it is found necessary to scale down or postpone the pensions of beneficiaries of the second, these latter still retain a reversionary interest up to the

amount of the maximum pensions to which the act entitles them. If this interpretation is correct, the valuation formulas for benefits of the second class become extremely complicated when many beneficiaries are involved. Moreover, the expedient of valuing the joint life annuities as single life annuities at an increased rate of interest does not effect such an economy of labor in the case of adult lives, since individual ages cannot be disregarded. Also, the pensions to adult beneficiaries are payable only so long as they remain dependent. No doubt the actuaries who undertake the practical valuation of these benefits will find ways and means of overcoming the theoretical difficulties and of attaining satisfactory results. A description of how an actual valuation was accomplished would make an interesting contribution to the *Transactions* of the Society.

As regards reserves in respect of employees still alive, a surplus must be maintained, as provided in the act, to guard against the catastrophe hazard. I presume no reserve is required in addition to this.

INCOME TAX ON ANNUITIES (FEDERAL INCOME TAX ACT OF 1913)—

JOHN S. THOMPSON.

VOL. XVI, PAGE 95.

WRITTEN DISCUSSION.

MR. E. E. RHODES:

The various rulings quoted by Mr. Thompson, taken in connecnection with Article 5 of the Income Tax Regulations issued by the Treasury Department January 5, 1914, show that the Treasury Department has not had any fixed opinion regarding the correct interpretation of that part of the Federal income tax law which relates to insurance and annuity contracts, and I am satisfied that the ruling of February 12, 1915, is not the last word upon the subject.

The trouble regarding insurance and annuity contracts arises from the proviso in Paragraph B of the act, which reads as follows:

"Provided, That the proceeds of life insurance policies paid upon the death of the person insured or payments made by or credited to the insured, on life insurance, endowment, or annuity contracts, upon the return thereof to the insured at the maturity of the term mentioned in the contract, or upon surrender of contract, shall not be included as income."

The particular words that are troublesome are "payments made by or credited to the insured." It is probable that the proviso would be construed to have the usual and primary office of a proviso, i. e., to limit the generality of Paragraph B and exclude from its operation items which would otherwise be within its terms, its purpose being only to enumerate, so far as these matters are concerned, what part of the amount received shall not be included as income. It seems clear that, excepting policies payable at the death of the insured, regarding which there is no question, the only amounts exempt under the express terms of the proviso are "payments made by or credited to the insured" upon the "return" thereof to him. It may have been the intent of Congress to include under the term "insured" all owners or beneficiaries of policies and annuities, including owners by assignment; but it is to be noted that the Treasury Department in its various rulings has referred to the insured as "the person making the contract."

With regard to the proper interpretation of the words "payments made by or credited to the insured" I may say that I have always held to the views of Mr. Hurrell quoted by Mr. Thompson. These views, in my opinion, obtain additional weight if we further

analyze the language used by Congress. Eliminating that part of the law which does not relate to annuities, we have: "Provided, That . . . payments made by or credited to the insured on . . . annuity contracts, upon the return thereof to the insured at maturity of the term mentioned in the contract, . . . shall not be included as income." This clause can be divided into two parts, as follows: "Payments *made by* the insured on annuity contracts upon the return thereof to the insured at maturity of the term mentioned in the contract, shall not be included as income," and "Payments *credited* to the insured on annuity contracts upon the return thereof to the insured at maturity of the term mentioned in the contract, shall not be included as income." It must be presumed that Congress was familiar with the general practice of companies issuing annuity contracts on a non-participating basis, and knew that the only "payments credited to the insured on annuity contracts" were the annuity payments falling due thereunder. Congress therefore intended to provide that not only should so much of the annuity payments as represented a return of the premium be not taxed, but that, also, all payments made under the contract should not be taxed.

It is clear that the purchase price of an annuity does not represent all that the holder of an annuity contributes. As was stated by Mr. Strong, on pages 131–132 of Volume 15 of the Society's *Transactions,* he contributes also a certain risk each year that he will not get all his money back; and until both the purchase price and the risk have been repaid he cannot be said to have received anything more than a return of what he has given. From an actuarial point of view the yearly annuity paid as long as the annuitant lives is the equivalent of the payments made for the contract and the risk referred to above, so that in this sense there cannot be said to be any profit in the contract to the annuitant. Insurance companies will pay an income tax each year on the premiums paid by the annuitant and also on the interest earned on the invested reserve. The only item for which a company will in the long run get any credit, aside from the small expense incurred, will be the annuity payments made. It is clear that both the company and the annuitant cannot profit by the issue of the annuity contract. It may safely be assumed that so long as a company is issuing annuity business it is not experiencing a loss thereon. This being the case, the annuitants as a class do not make a profit on the transactions and should not, therefore, be individually taxed. It is true that the company in its income tax return gets credit in the year in which an annuity contract is issued for the required legal reserve thereon; but it will not be forgotten that such reserve decreases each year thereafter, and is finally eliminated altogether by the death of the annuitant. The yearly decrease and the final elimination of the reserve by reason of the death of the annuitant is covered back into the taxable income of the company,

so that the company really receives no credit by reason of the reserve. The initial credit on account of the reserve which the company receives is in subsequent years exactly offset by the decreases in the reserve which are included in income.

Admitting, however, for the sake of argument, that some part of an endowment or annuity contract may be subject to the income tax, it seems very clear that no part of such tax is required to be withheld at the source. Paragraph D of the law provides that "All persons, . . . and insurance companies, except as hereinafter provided, in whatever capacity acting, having the control, receipt, disposal, or payment of fixed or determinable annual or periodical gains, profits and income of another person subject to tax, shall in behalf of such person deduct and withhold from the payment an amount equivalent to the normal income tax upon the same." Paragraph E contains the following provision: "All persons, . . . and insurance companies, in whatever capacity acting, . . . having the control, receipt, custody, disposal, or payment of interest, rent, salaries, wages, premiums, annuities, compensation, remuneration, emoluments, or other fixed or determinable annual gains, profits, and income of another person exceeding three thousand dollars for any taxable year," are required to deduct and withhold therefrom the normal tax. It will be noted that the controlling words are "fixed or determinable annual gains, profits, and income." It is true that the word "annuities" appears in the provision quoted from Paragraph E, but the word as there used applies, in my opinion, and it is Mr. Hurrell's opinion also, to annuities payable under the provisions of wills or deeds and not to such annuity contracts as we are now considering. Apparently the Treasury Department has recognized such a distinction, for in Article 63 of the Regulations issued under date of January 5, 1914, the expression "taxable annuities" is used.

It has been held that the law does not require that the tax on the income of agents compensated on a commission basis be withheld at the source, for the reason that such income is not fixed or certain, and is indefinite or irregular as to amount or time of accrual. This can only be the case for the reason that the continuance of the insurance contract itself is indefinite or not fixed or certain. There are several contingencies which may operate against the payment under a life insurance, endowment, or annuity contract, and since these contingencies are sufficient to relieve an insurance company from withholding at the source the tax due on the compensation of the agent, it follows by clear analogy that the tax should not be withheld on payments made under the contracts themselves.

What I have said above does not apply, of course, to the income derived from the proceeds of policies left with the company at a guaranteed rate of interest, or settled in a specified number of equal annual instalments certain. No exception can be taken to Mr.

Thompson's method of treating such cases. In my opinion, however, the principles which are applicable to contracts not involving life contingencies are not at all applicable to contracts involving life contingencies, assuming that the latter are taxable. What certainty is there that under an annuity contract the payments made by the company will exceed the premium? If there be no excess there is no tax, even under the construction of the law adopted by the Treasury Department. The courts have at times admitted mortality tables as evidence to assist juries in fixing the amount of damages, but it is difficult to believe that mortality tables will be received as evidence to determine a person's taxable income. I do not question the correctness of Mr. Thompson's actuarial reasoning, but I do question very seriously its applicability to the income tax law.

Mr. Thompson's argument is based upon the theory that whether or not any part of the annuity received during the year is taxable income, depends upon the rates of interest and mortality assumed by the company. This involves two propositions: (1) that a computation based upon assumed probabilities is a correct method of ascertaining income for the purpose of taxation, to which I have already referred; (2) that the annuitant, notwithstanding his absolute payment to the company, retains virtual ownership of the fund which produced the income.

The latter proposition assumes that the relation between the annuitant and the company is that of trustee and *cestui que trust.* But the character of the transaction shows that this is not so. It is purely one of legal contract, creating the relationship of debtor and creditor. The company's obligation to pay, and the annuitant's right to receive, are absolute, depending only upon the contingency that the life of the annuitant shall continue until the due-date each year. He has no concern with the income received by the company upon the premium he has paid as a consideration for his contract, nor is he concerned whether the mortality turns out to be the same, or greater or less than was assumed. Instead of receiving an estimated interest return of 3 per cent., the company may receive only 2 per cnt., or nothing, or it may receive 6 per cent., or it may lose both principal and interest. The rights and obligations under the contract cannot be affected. Neither the fact that these factors were considered by the company in fixing the premium rate for which it was willing to assume the contractual obligation to pay, nor the fact that it afterwards found that it actually received a greater or less income, or had a more or less favorable mortality experience than that upon which it based its estimate, is of any concern to the annuitant. The situation is much the same as the case of an owner who contracts for the erection of a building. Whether his contractor makes a gain or loss on the contract, or whether the contractor's estimate of cost may or may not have been found to be too high or too low as the work proceeds, is a matter of no consequence to him.

It is interesting to compare the present value of the tax under the two methods. Let us take first the case cited by Mr. Thompson, viz., that of an annuity for $15,000 issued at age 69 on the life of a man. Assuming that there is no exemption, the present value of the 1 per cent. tax on the interest portion of the annuity payment is $211.58, while the present value of the 1 per cent. tax on the whole annuity payment, after the consideration is repaid, is $323.07. Assuming an exemption of $3,000, the corresponding figures are $108.20 and $258.45. If we have an annuity of $30,000 on the same life the present value of the tax by the two methods, assuming an exemption in each case of $3,000, would be $419.54 and $581.52. If we take an annuity for $15,000 issued at age 49 on the life of a man, the present value of the 1 per cent. tax on the interest portion of the annuity payment is $739.06, and the present value of the 1 per cent. tax on the whole annuity payment after the consideration is repaid, is $634.95, assuming no exemption in either case. Assuming an exemption of $3,000 in both cases, the corresponding figures are $706.21 and $507.96. It is clear therefore that in some cases Mr. Thompson's method would be more favorable to the government, while in other cases it would be more favorable to the taxpayer. The above comparisons are made upon the basis of McClintock's Table with 3 per cent. interest.

I cannot follow Mr. Thompson in what he says regarding a deferred annuity. He says that the interest earned by a company upon its annuity reserves is held for the benefit of individual annuitants. The statement of course is true when applied to annuitants as a class, but the individuals of the class have no interest in the fund except upon the contingency of survivorship. Inasmuch as the reserve at the completion of the deferred period includes the accretion arising from interest as well as from the "benefit of survivorship," I cannot see why the interest taxable in the first annuity payment should be any different in the case of a deferred annuity from that in the case of an immediate annuity. A deferred annuity really consists of an n-year pure endowment which at the end of the term is to be converted into a life annuity beginning at the end of $n+1$ years; that is to say, the pure endowment is to be paid in the form of an annuity, and if any tax is to be paid it should be only on the annuity. To tax the pure endowment and also the annuity is double taxation. It is quite clear to me that no income tax can possibly arise during the deferred period. A possible future payment conditioned upon the uncertainty of survivorship certainly cannot be included in taxable income. The increase during the deferred period in the reserve on a deferred annuity does not, from a legal point of view, represent *income* which has "arisen" or "accrued" during the deferred period. The annuitant acquires no property right to the reserve accretion. He has at best only a contingent interest therein, which in event of his death during the deferred period will be entirely destroyed. The above

remarks apply with even greater force to the position taken by Mr. Thompson regarding continuous instalment settlements, because in connection therewith he would deduct from the instalments certain a tax on the interest accretion to the deferred annuity reserve during the deferred period.

I have not referred to the practical difficulties which would arise if it shall finally be determined that life annuities are taxable and Mr. Thompson's views regarding the amount of tax be adopted, because in the eyes of the law the taxpayer must be presumed to have sufficient knowledge of his income to enable him to make a correct return. This imputes more actuarial skill and knowledge than one person in a thousand possesses.

The difficulties involved in the ruling of February 12, 1915, appear more clearly, perhaps, in case of surrender values and endowment policies than in case of annuities. Take for example a 20-year endowment policy issued this year at age 35 with a premium rate of $49.85 per thousand. The twenty contract premiums would amount to $997 and the policy would yield at maturity $1,000. Under no possible construction could the "payments made by or credited to the insured" be less than the contract premiums, so that the taxable income according to the ruling would be $3 per thousand. It cannot however be maintained that all of "the amount by which the sum received exceeds the sum paid and coming into the hands of the person making the contract and payment," in case of contracts issued prior to the enactment of the income tax law, is taxable income. It would seem to be entirely correct to say that each of the twenty premiums paid contributed to the final result, so that in the case of policies issued before the enactment of the law, some part of the gain is not taxable. If it be argued that in order to ascertain what part of the gain is taxable, the payments made by or credited to the insured from the time the law went into effect should be ascertained, and that if the increase in the value of the policy during that time exceeds such payments the difference would represent the gain attributable to the period elapsed since the enactment of the law, we would get absurd results. For example, if the policy in question was issued nineteen years before the enactment of the law we would have an increase in value of $71.09 during the twentieth year, on the basis of the American Experience table and 3 per cent. interest. The payments made by or credited to the insured would be $49.85, so that the assumed gain for the twentieth year would be $21.24, which is considerably more than the entire gain arising during the complete history of the policy. In other words, a 20-year endowment policy maturing one year after the enactment of the law would be taxed seven times as much as a precisely similar policy maturing twenty years after the enactment of the law. Still further difficulty would arise if, as frequently happens, a policy should be assigned after the enactment of the law. In that case, would the assignee be chargeable with the entire

gain accruing from the time the law was enacted or only for that part of the gain which arose subsequent to the date of the assignment?

MR. WENDELL M. STRONG:

Mr. Thompson's paper has presented to us the actuarial theory of the separation of the interest element from the other component elements of annuities—both of annuities certain and those depending upon a life. It is well to have this set forth in the *Transactions* of the Society.

Mr. Craig's invitation to discuss this paper has given me the opportunity of presenting to the Society the convictions to which I have come in reference to the income tax law and annuities, without the necessity of bringing my remarks up to the standard of a paper. In speaking, I wish to go back a step further than Mr. Thompson's paper, to the fundamental question of the whole matter; that is, whether the annuities issued by life insurance companies are taxable under this law: I believe they are not. Whether in this belief I am correct or not, rests upon the interpretation of the latter portion of the first paragraph of Section B of the law, which reads as follows:

"*Provided,* That the proceeds of life insurance policies paid upon the death of the person insured or payments made by or credited to the insured, on life insurance, endowment or annuity contracts, upon the return thereof to the insured at the maturity of the term mentioned in the contract, shall not be included as income."

This would be an absolutely clear statement that the annuities issued by life insurance companies were not to be included in taxable income at all, were it not for the confusing effect of the little word "by," to which Mr. Craig called attention in his brief discussion of Mr. Rhodes's paper upon the income tax and which, at first reading of the sentence, might be easily overlooked. The effect of this word "by" is not that of giving a totally different meaning to the sentence, as the Treasury Department has ruled, but rather to introduce a confusing effect into a sentence that, if the word were omitted, would be absolutely clear and logical and in accordance with the context; it would also state without question that annuities were not to be included in income. The Treasury Department has interpreted the clause rendered obscure by the word "by" to mean that so much of the payments to the annuitant as are a return of the amount paid for the contract, that is, a return of the principal, is not income, but that the remainder is income. If Congress had intended this, it could have very easily so stated directly and in a way not open to different construction. Furthermore, it would seem that it would have so stated, for why should Congress adopt a confusing and indirect method of statement when a direct statement would have been easier and would have left no doubt as to the meaning? For instance, the clause could be altered thus, to make it state what the Treasury Department claims that it states:

"*Provided,* That the proceeds of life insurance policies paid upon the death of the person insured shall not be included as income, and only payments made to the insured on life insurance, endowment or annuity contracts in excess of the amount paid for the purchase of such contracts, shall be included as income."

A further difficulty that meets us immediately, if the Treasury Department's interpretation is accepted, is that the provision is superfluous because already fully covered by the law. The principal of a note received back when the note is paid is not included in income; the amount paid for a bond when received back in the sale of the bond, is not included in income, and, in general, money paid out for anything and received back in settlement, is not included in income, and nowhere did Congress find it necessary to so state. Would Congress, therefore, have inserted in the law an indirect and confusing sentence for the purpose of covering something which, both by the letter and spirit of the law, was already unquestionably covered? This consideration alone is enough to make such an interpretation improbable, if any interpretation which could add to the law is to be found.

Not only is another interpretation to be found but it is the more natural one. The word "by" is used with the words "or credited to." These words make the expression practically all inclusive as has been shown by Mr. Hurrell (see quotation, page 98) so that all payments made to the insured are covered by these words. It would seem as if too much attention had been given to the words "upon the return thereof to the insured at the maturity, etc." These words seem to me merely a formal method of stating "upon payment to the insured at the time provided for by the contract." In other words the word "thereof" does not seem to me to have the special significance which has been attributed to it of limiting the payments excluded from income to payments "made by or credited to the insured" but rather to be a word used unfortunately in adding formality to the statement. Here again if the intention of Congress had been to limit the exclusion from income as claimed, a different and unquestionable form of statement would have been used. This consideration of the word "thereof," however, is not at all essential when we consider the meaning of the words "or credited to."

The context as well as the words of a single provision of a general statute tend to show the meaning of that provision. A meaning that is not out of harmony with the context is always to be looked for and any one claiming that a provision has a meaning out of harmony with the context has the burden of proof to show that he is correct. When we go to the context here, we find, immediately preceding the word "Provided," a blanket statement of what is taxable, which may briefly be said to include all income or profits; immediately following this comes the clause previously quoted, the word "Provided" with which it begins indicating that certain exceptions or exclusions were to follow, and that is what happens

with the first half of the clause quoted, for there the proceeds of life insurance policies on the death of the insured, which might well be income to the recipients, are excluded. Then comes the question of surrender values, endowments and annuities, etc., that is, settlement during the lifetime of the insured, in the latter part of the clause, and this part is, if the Treasury ruling is to be followed, not an exception or exclusion but simply an unnecessary declaration. In one respect the context is particularly striking. The first statement following "Provided" is a positive and unqualified exclusion from income of the proceeds of life insurance policies on the death of the insured. Immediately following this, in the closest co-ordination with it in form of statement as well as substance—the two parts being connected by the word "or" and both preceding the words "shall not be included as income"—are the words "payments made by or credited to the insured." The writer of the sentence evidently put them on a par with the words "proceeds of life insurance policies" as both being positive exclusions from income. The Treasury interpretation, however, makes the second of these a mere declaratory statement and one, moreover, which is unnecessary, since the return to the insured of the amount paid by him would not, in any case, be income. If this interpretation is correct, we have here a badly mismated team—an important positive provision of the law in double harness with a mere restatement of something already covered. That Congress should so mismate its team of exclusions, putting the apparent beside the real, on an equality with the real and in the guise of the real, is not inherently probable.

The burden of proof that annuities are taxable is also thrown upon the Treasury by the rule that tax laws must be constructed strictly as against the taxing authority. In the opinion in the case of The Mutual Benefit Life Insurance Company *vs.* Herold (*T. A. S. A.*, XIII, 300), the District Judge quoted the following enunciation of the rule by the United States Supreme Court:

"Keeping in mind the well-settled rule that the citizen is exempt from taxation, unless the same is imposed by clear and unequivocal language, and that, where the construction of a tax is doubtful, the doubt is to be resolved in favor of those upon whom the tax is sought to be laid."

A point that is raised on the other side is that in Section E, relating to the deduction of the income tax at the source by those having "the custody, disposal, or payment of interest, rent, salaries, wages, premiums, annuities, compensation, remuneration, emoluments, or other fixed or determinable annual gains," annuities are included in taxable income. The context, however, would indicate that the annuities considered are not the sort of annuities we have been considering, that is, annuities issued by life insurance companies, but rather such annuities as might come from devise, bequest, etc., so that the law draws a distinction between such annui-

ties, which are different in character and usually for a different purpose from the annuities issued by life insurance companies, and the annuities issued by life insurance companies. Again, I would refer to the quotation from Mr. Hurrell made in Mr. Thompson's paper, page 98.

The whole of the provision regarding exclusions which we have been considering would be so clear and logical, the sentence containing it so consistent in its construction, the resulting provision so in accordance with the evident intention of the authors of the law, if the word "by" were omitted, that it seems to me that this word "by," which has caused all the confusion, crept in by error and remained because unnoticed. Be that as it may, the word is there and the question is whether it changes the meaning of the provision and prevents the total exclusion of annuities from income. Notwithstanding the Treasury interpretation, I do not believe it does, for the reasons above advanced. To recapitulate, these are;

1. That such interpretation would render the greater part of the clause quoted a mere declaratory statement which was unnecessary;
2. That the words "credited to" contradict such an interpretation;
3. That such an interpretation makes the statement interpreted out of line with both the context and the apparent intention of the provision;
4. That, so far as ambiguity exists, the rules of interpretation require the ambiguity to be resolved against the imposition of the tax.

ORAL DISCUSSION.

Mr. Miles M. Dawson: It is, of course, impracticable for us to settle what would be the determination of the courts as to the meaning of this tax law, and while personally I should be inclined to join the last speaker (Dr. Strong) and Mr. Rhodes in their view of what the law is, I think it is more important for us, as an actuarial society, perhaps, to consider what would be the effect if the ruling of the Treasury Department were sustained by the courts; because obviously, as an actuarial society, we will have nothing to do whatever if the view which has been expressed by Mr. Rhodes and Dr. Strong should prevail.

The paper which Mr. Thompson presented was an exceedingly interesting one. It adopted, for the purpose of making known to the younger members of our society—and perhaps the older as well, for we did not have to deal with it prior to the passage of this income tax act—the methods of dealing with the separation of simple annuities (the whole amount of life annuities is income under the British tax act) which have been in use in Great Britain, under the income tax law there, for a good many years, with certain suggestions of possible modifications, and with very valuable suggestions as to the possible application of these principles under our income

tax act. But it seems perfectly clear to me that, if the Treasury Department adheres to the ruling which you will find in Mr. Thompson's discussion, that particular method of dealing with the matter which was described in Mr. Thompson's paper will not actually come into play; but instead no tax upon the annuity payments can be levied until there has been a return of the principal deposited or paid by the purchaser of the annuity, and after that has taken place all the annuity payments will be treated as income.

I put forward this view with a great deal of diffidence, but it seems to follow almost absolutely to a certainty from the construction which the Treasury Department has put upon the law.

MR. JOHN S. THOMPSON:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

It was not the intention of the writer to enter into the argument as to whether a life annuity should be excluded wholly or partially from the return showing the annuitant's taxable income under the Federal income tax act. Recognizing, however, that the Internal Revenue authorities had more than tacitly admitted that life annuities were in part or under certain circumstances taxable, the writer desired to present the analysis set forth in the paper as a substitute for empirical methods which are immediately suggested by the form in which the Treasury Department's rulings have been offered. In other words, accepting the obvious element of doubt as to the interpretation of the law, the idea was to suggest a practical method of giving effect to the spirit of the rulings. It was felt that if a certain group of annuitants holding contracts issued at various ages, have paid single premiums based on uniform assumptions, they all take a similar risk with regard to the outcome of their contracts and therefore should be treated similarly in taxation, admitting taxation to be proper to a certain extent and under certain circumstances. This sort of equity might not be a consequence of some unscientific methods which are probably now being used in connection with the Department's rulings. There is, therefore, a fundamental difference between the point of view of the gentlemen who have discussed the paper and that of the writer. Their analysis, however, of the pertinent clauses of the Federal income tax act are most enlightening and most conclusive and must be regarded as valuable contributions to the discussion of that phase of the subject.

With regard to the deferred annuity it seemed that the method suggested was a logical sequence of that adopted in connection with the life annuity. The rate of income at age 77 to a male annuitant who at age 20 took a fifty-seven-year deferred annuity of $1,000 per annum, paying therefor the sum of $262, is quite different from that at the same age to one who at age 75 took out a two-year

deferred annuity for the same amount at a single premium of $4,633 (the figures are net, based on McClintock's Male Annuitants Table, interest 3 per cent.) because the former has taken a much greater risk.

It is, of course, true that the annuitant is at no time the owner of his annuity reserve and could not even obtain any part of it as a surrender value, but he is one of a large group who share it under certain contract conditions and hence considerations based on the idea of "benefit by survivorship" should be applicable.

The annuitant's idea as to the constitution of his successive instalments is doubtless very indefinite as regards their division into principal and interest; he does not consider the return all interest because the prospective applicant at age 80 will complain of an 18 per cent. "return" upon the money paid in as emphatically as one applying at age 55 will complain of a 7½ per cent. "return," and he certainly does not consider the successive instalments solely a return of principal. But the distinction exists and the annuitant would, no doubt, admit on general grounds the justice of a tax on a portion of the annuity.

The practical difficulties are serious but not insurmountable. They are small compared to the problems confronting the attorney who is called upon to pay the inheritance taxes on the proceeds of a large estate. The complexity of the interests involved in some trust bequests is notorious. Their analysis, if necessary, must be made by a competent actuary to the satisfaction of the authorities who levy the tax and used accordingly in the return. The assumption of actuarial knowledge by the attorney for or the beneficiary under a trust estate has never been a bar to the collection of the tax, notwithstanding the fact that before the tax is determined the death or marriage of one of the beneficiaries may have indicated that the contingent value of the interest at the testator's death was quite different from the actual value of the interest as determined by experience.

A STAFF PENSION FUND—DWIGHT A. WALKER.

VOL. XVI, PAGE 109.

WRITTEN DISCUSSION.

MR. WILLIAM A. HUTCHESON:

As Mr. Walker states, the subject of staff pension funds is one which is much before the American public at the present time, and any information, therefore, which we can get on the subject is of interest to the Actuarial Society. Our thanks are therefore due to Mr. Walker for his thorough paper on the railroad staff pension fund described therein.

His paper, first of all, gives a statement of the benefits, then explains the data used for the various rates of mortality, disability, et cetera, and then gives a statement of the notation used and the values of the contributions and benefits, followed by a set of tables required in connection with the ascertainment of the contributions. The formulae therein given and the set of tables will be of decided use to anyone studying pension funds, for it gives a method in which such tables can be computed.

The experience derived from one pension fund can seldom be used without modification in connection with any other fund, because the benefits as well as the withdrawal, disability and mortality rates generally vary in each case.

Mr. Walker explains the various sources from which he derived the rates of death, invalidity, et cetera, used in his investigation, but we are not in a position to judge whether or not these rates were applicable to the employees of the railroad in question, which, by the way, is referred to only as "one of the large railroads of America," and some years will have to elapse before it can be proved whether or not the rates assumed were applicable to this particular case.

The particular rates assumed, however, are not, in my opinion, the important part of his paper. To my mind, the importance of the paper is the great detail given in the tables and in the formulae therein.

Initial Liability or Subsequent Deficiency.—Mr. Walker writes regarding "The Lialibity at the Commencement of the Fund." As we all know, the great troubles in starting any pension fund are to know how to treat the old employees, and how to provide for their pensions if the pensions are to be granted for services prior to the institution of the system.

In the simplest case, where the pension is merely a deferred annuity at, say, age 65, a consideration of the following premiums (net American 4 per cent.) at various ages at entry shows the difficulty of providing for the older employes. And I have found such a table useful in explaining to the layman the large annual contributions or the large initial reserves which must be put into the fund for existing employees if the pension system is to be retroactive.

Age When Fund Instituted.	Annual Premium Per $100. Pension.
30	$ 7.28
35	9.95
40	13.98
45	20.48
50	32.07
55	56.78
60	135.12
64	782.71

Mr. Walker writes of providing a special fund so as to allow for benefits based on service prior to the institution of the scheme, and shows in the last part of his paper how he estimated the initial liability to cover such past services.

Various methods of providing for this accrued liability at the institution of any fund have been suggested, and a somewhat similar question arises when, on a valuation, a deficiency is ascertained.

Sometimes it is suggested that the future contributions be increased so as to provide for the full benefits.

In other cases the accrued liability, assuming no increase in the normal contributions, is ascertained, and, if it is not possible to set up funds to meet this liability at once, it is decided to write off the deficiency in some definite number of years by means of equal annual instalments consisting partly of principal and partly of interest. Such a treatment of a deficiency ascertained in a valuation has been resorted to in Great Britain in the case of the pensions granted to its employees by the city of Liverpool, and also, if I am not mistaken, by the city of Manchester. Under this system of treating a deficiency the equal annual instalments can be increased or decreased at each valuation to meet a greater or less deficiency than anticipated.

Sometimes a combination of these two methods is adopted, that is, the future contributions are increased, and at the same time the corporation back of the pension scheme undertakes to pay off the balance of the deficiency by equal payments extending over some definite period of years.

Periodical Valuation.—In the absence of data based on the services of the employees to which any pension scheme is to apply, some data, as nearly suitable as possible, has to be chosen, and it is

necessary, as Mr. Walker points out, that statistics be carefully kept after the fund has been started, and that a valuation be made periodically thereafter. The whole scheme must be revised when the statistics gathered from the actual experience indicate that such a revision is necessary. Even after statistics have been kept for some time and the fund has been adjusted, it will often be found necessary to make a readjustment, as the salary scales, rates of withdrawal (with and without pension) and of death—not to mention the benefits—will doubtless change from time to time.

Lack of Uniformity in Schemes.—No two pension schemes provide for the same benefits, and seldom do the rates of withdrawal and disability which are applicable to one body of employees correctly represent the corresponding rates applicable to another body. The subject of pension schemes is, therefore, a very difficult one. In addition, although a particular fund may be solvent at one valuation, a change in conditions may quite well make it insolvent within a very few years. Actuarial advice must, therefore, be sought by those contemplating the institution of any pension fund if they would avoid trouble, and repeated periodical examinations of the fund are absolutely necessary.

In conclusion, Mr. Walker, in giving us the formulae which he applied to the particular fund which he examined, and the voluminous tables required in calculating the contributions, et cetera, necessary for the scheme in question, has made a contribution to the Society's literature of pension funds, and has given us an example of how the necessary pension tables may be drawn up. Our thanks are, therefore, due to Mr. Walker for his interesting and instructive paper.

MR. O. W. PERRIN:

Mr. Walker's paper on "A Staff Pension Fund" is a most interesting application of pension fund theory, as developed chiefly by Mr. Manly and Mr. King, to an actual pension scheme for one of the large railroads of America. As the fund had already been in operation prior to the date of Mr. Walker's calculations, the plan of the fund was settled in advance and, therefore, precluded any consideration of the relative merits of the collective plan as against that of individual savings. His able presentation of the particulars of dealing with an actual pension scheme will be invaluable to actuaries whose advice is sought in overcoming similar difficulties.

The paper falls naturally into two divisions; first, the rates of contribution required from the time of their adoption, and second, the initial liability due to benefits based upon past service. The members pay the entire contributions and accordingly the practical problems involved in the management of the fund as well as the incentives toward continuity and efficiency of service assume different degrees of importance as compared with a fund which puts the total cost upon the company.

The benefits cover the usual contingencies of withdrawal, disability, death and retirement. The return upon withdrawal of simply the contributions without interest accumulations avoids undesirable refinements; and the death benefits, either before or after retirement, take a commendable form in that they provide for the payment of the same income for a period of two years after death as the member receives in event of disablement or retirement, thus serving to bridge the chasm which the sudden stoppage of income, occasioned by death, might create. As the death benefits account roughly for less than 25 per cent. of the contributions, they do not effect a disproportionate reduction in the pension itself, which after all should be the main object of the fund. The amount of the pension is based on the final salary of the member and his years of service. This is somewhat objectionable from the point of view of a majority of the members, especially when the employees pay the entire contributions, as it treats the highly paid portion of the staff more generously than the rank and file. It might lead to increases in salary just prior to retirement and in any event, as high salaried positions are few in number and not usually attained until late in life, it would require the small minority receiving large salaries, to pay correspondingly large contributions only over a short period prior to retirement. The value of such contributions, therefore, would often be less than the value of the pensions payable. Using 80 per cent. of the final salary as suggested by Mr. Walker in the latter part of his paper, is a modification in the right direction.

In deriving the formulas for the foregoing benefits, Mr Walker separates each benefit into two parts; one due to past service and the other to future service. The object of this procedure is to give effect to actual past salaries as past salaries are realized facts and should be taken into account wherever possible instead of the assumed facts represented by the salary scale. The present value of each benefit is then a matter of summing a series of present values, first, for future service, and second, for past service, over each year in which the benefit will emerge. In this connection Mr. Walker's effective use of commutation symbols should be noted; it is one of the most impressive illustrations of the powers of this instrument that has ever appeared in the *Transactions*.

Passing to one or two matters of detail, we note that Mr. Walker's formula for the present value of contributions returned upon withdrawal—due to future service, is based on the contributions of the full salary for the *first* year as well as for the succeeding years. As withdrawals occur on the average in the middle of the year of service, the contribution for the year of withdrawal averages half of the salary for that year. Is the full salary assumed in the interest of safety, or is it intended in the practical operation of the fund to return the full salary less the portion remaining unpaid in the year of withdrawal? The question of the assumed salary scale

is interesting, and the method originated by Mr. King and followed by Mr. Walker is to value future salaries as if increases of salary would occur in proportion to those of the salary scale. Accordingly in cases where the salary of a member is higher than the average salary the assumed future increases will also be higher than the average increases, or, conversely, if his salary is lower, the assumed future increases will also be lower than the average. Mr. Joseph Burn in his recent paper on the formation and valuation of pension funds in the July number of the *Journal of the Institute of Actuaries,* suggests another method which in his own experience has proved to be more suitable. He assumes the future increases of salary to be the actual increases shown by the scale, instead of proportionate increases, and so his valuation, while it would not perhaps differ materially in its results from that of Mr. King, has the advantage of separating the values of the benefits due to known present salaries from the values due to problematical future increases.

Mr. Walker's paper exhibits much skillful labor and research in the choice of the bases of the fund. He has levied upon German experience for disability and mortality rates among active lives; upon English experience for rates of withdrawal; upon American experience for mortality rates among disabled lives, and finally upon the company itself for the relative salary scale. There still remains, the uncertainty, however, as to whether these bases, separately or in combination, will represent with sufficient faithfulness the future experience of this particular fund. No one realized better than Mr. King the difficulties of securing a satisfactory statistical foundation for pension funds, and he has repeatedly issued warnings against relying too confidently upon the experience of one fund to serve as the basis of another. A periodical valuation, say, every five years, as suggested by Mr. Walker, is essential, of course, to correct mistaken assumptions.

Having access, through the courtesy of the officials of the fund, to the pension records of one of the large railroads of America, the writer endeavored as a matter of interest in the discussion of Mr. Walker's fund, to ascertain how the mortality experience among pensioned lives had worked out. This fund began operations January 1, 1900, and the data covering the experience up to and including the year 1913, had already been tabulated by the company according to calendar years. In the effort to make the data as homogenous as possible, the year 1900 (as it contained many old entrants above the pension ages) was omitted from the tabulation, —and also female lives. The experience was then separated into two sections, the first relating to retirements from 65 to 69, the optional ages, and the second to retirements at 70, the compulsory age. This fund, let me say in passing, was financed entirely by the company, included the entire staff—operating and clerical forces and officers—and provided for a pension allowance of 1 per cent.

per year of service of the average pay for the 10 years preceding retirement, conditioned, however, upon at least 30 years of service for the optional ages. The total number of males who entered observation by being pensioned at age 70 was 2,421, of whom 968 died and the balance of 1,453 were existing at the close of observations; there were no withdrawals. The total years of exposure amounted to 11,605 or an average of 4.6 years. An aggregate table was then formed, as being best suited to the purpose, and the resulting rates were roughly graduated by inspection and a graphic process. The graduated rate at age 70 falls about ½ of 1 per cent. below the American Table, which is the basis adopted by Mr. Walker, and thereafter rapidly rises until it equals the American at age 72, and exceeds it by 1.3 per cent. at age 80. The total expected deaths by the American table are 949.8 as compared with 968 actual and 968.5 by the graduated table. A comparison by ages follows:

Age.	Retired at 70.	American q_x.
70	.057	.062
71	.065	.068
72	.074	.074
73	.083	.080
74	.092	.087
75	.101	.094
76	.110	.102
77	.120	.111
78	.131	.121
79	.143	.132
80	.157	.144
81	.174	.159
82	.193	.174

The other section of the experience covering the optional ages 65 to 69 was not as satisfactorily tabulated nor as homogenous as the first. The numbers under observation and the total years of exposure were smaller, being 2,095 and 8,165, respectively; the number of deaths were 743 actual, and 743.7 by the graduated rate. But for the sake of completeness let me say that the graduated rate at age 65, being about 11 per cent., was almost triple the American Experience rate—a result that might have been expected; it decreased rapidly to 8.4 per cent. at age 67 and then remained nearly constant up to age 73.

The mortality of these two sections evidently exhibits widely different characteristics. One important reason for the heavy mortality of the section relating to retirements from 65 to 69, is the company's treatment of disabled lives. When a disabled employee reaches the pension ages he would nominally be retired from service, provided, of course, his term of employment covered the necessary period of years; in actual practice, however, he is not retired so

long as he may be able to perform a different sort of work from that in which he was previously engaged. He may be retained on the pay roll, for example, as a gatekeeper or a watchman. The withholding of such lives from the pensioned class leaves a group of lives for whom no job can be found or whose disablement is such that they cannot perform any work whatever. As soon as the experience of this fund develops to the point where a satisfactory group of homogenous data may be secured the writer hopes to present a more complete analysis to the Society.

MR. D. A. WALKER:

(AUTHOR'S REVIEW OF DISCUSSIONS.)

Mr. Hutcheson states that no two pension schemes provide for the same benefits. Although my experience in handling pension funds is somewhat limited, I have made calculations in connection with three or four such schemes within the last year or two, and I have found that each scheme seemed to vie with the other in elaborateness of the benefits, which tendency is commendable if not carried to extremes, but that the amount of the contributions necessary to provide such benefits was very much underestimated. As a matter of fact, most pension schemes are commenced without actuarial advice and it is only after a number of years that the managers of the schemes begin to scent trouble ahead. An actuary is then called in to assure the persons in charge that the contributions are sufficient. From my experience I should say that the actuary almost invariably assures them that the contributions are decidedly insufficient. Mr. H. W. Manly stated on page 258, Vol. XXXVI, *J. I. A.*, that although he had valued many of these funds he had never formed one; that he had been consulted often, but had never found the employers willing to pay the price, etc. He refers here to funds in which the employers were to assume part of the contributions.

I may confess that, in connection with the fund upon which my paper is based, the original scheme which was in operation provided for benefits much more elaborate than those shown in my paper, while the contributions were considerably less than those shown. The benefits were therefore scaled down in order to reduce the contributions to a reasonable figure. The benefits at death especially were reduced, the object being to continue the pension to dependents for a length of time sufficient to enable them to take some steps to provide for themselves thereafter.

Mr. Perrin states that the basing of pensions on final salary is objectionable and his points of objection are well taken. However, that question was not left for me to decide upon. Further on Mr. Perrin asks: "Is the full salary assumed in the interest of safety, or is it intended in the practical operation of the fund to return the full salary less the portion remaining unpaid in the year of with-

drawal." In reply I would say that q_x^{aw} in Table I has been assumed to represent the probability of withdrawal at end of a year and not on the average at the middle of a year of service. If I had been making up q_x^{aw} from the experience of the company I should have made up the number of those withdrawing at any age $x + n$ by adding together all of those whose nearest service age was $x + n$ so that such number would include some lives who had completed more than n years of service and on the average an equal number with less than n years service. When q_x^{aw} is made up in this way my formula provides for the return of only the actual contributions paid in, on the average. In actual practice more than n contributions would be returned in some cases and less than n in others, but the average return would be n, as provided for in the formula. Mr. Perrin's table of rates of mortality based on the actual experience of a railroad is very interesting, and it would seem to confirm my impression that the values of my pension annuities where retirement occurred after age 64 are high enough. My rates at some ages were higher and at some lower. Mr. Perrin states that he hopes at some future time to present to the Society a more complete analysis. It would be very desirable if he could present rates of mortality amongst railroad employees retired on account of invalidity at all ages prior to 65. In fact I should consider that my paper had been well worth while if it should stimulate the production of such statistics.

BOOK NOTICES.

Mortality Laws and Statistics. Robert Henderson. New York, John Wiley & Sons, Inc. 1915. Pp. vii + 111.

This volume is number fifteen of the Mathematical Monograph Series and is the work of the actuary of the Equitable Life Assurance Society. Mr. Henderson says in his preface "The present work is designed to set forth in concise form the essential facts and theoretical relations with reference to the duration of human life." The first chapter contains a brief description of those mortality tables which have had the greatest influence on the development of the science of life contingencies or on its application in this country, and covers a wide field from the Breslau Table up to the Medico-Actuarial Mortality Table. The four chapters following cover in very concise form much of the ground of the first six chapters of the Institute of Actuaries' Text-Book, Part II, incuding probabilities involving one or more lives and the general mathematical laws which have been proposed to express the facts of human mortality. The construction and graduation of mortality tables form the subject of the two following chapters. The methods of constructing mortality tables from census and death returns and from insurance experience are described. A very important chapter is that on graduation of mortality tables, and it contains a concise but very complete review of the several methods of graduation. The book concludes by illustrating some of the methods of constructing and graduating mortality tables by the construction of a new mortality table for the Northeastern States. The data for this table are the death returns for the five calendar years, 1908 to 1912 inclusive, and census returns as of June 1, 1900, and April 15, 1910, for the New England States and the three Middle Atlantic States—New York, New Jersey, and Pennsylvania. In an appendix the Northeastern States mortality table is given in full and a series of tables of the fundamental columns and other data from various important mortality tables are given. This volume is confined to life contingencies and excludes all monetary applications.

The Mathematical Theory of Probabilities and its Application to Frequency Curves and Statistical Methods. Arne Fisher. Volume I. New York, Macmillan Company. 1915. Pp. xx + 171.

A field which is of prime importance to the actuary and in which the information available to him in book form has been scant, is that of the mathematical theory of probabilities. No modern and comprehensive work on this subject has existed in the English language. In consequence, the need for such a work has been much greater than in most other lines of applied mathematics. It is to meet this need that Mr. Fisher's book has been prepared. The fact that the author's point of view is that of one who is as familiar with the work done on this subject in Continental Europe as well as by the English-speaking nations, adds greatly to its breadth and value.

There are three distinct points to be noted in reference to the method of treatment of the subject in this book.

(1) The theory is developed consistently from a mathematical standpoint;

(2) The assumptions which are made in applying the mathematical theory of probability are not passed over but are duly considered;

(3) The book contains the application of the theory to practical problems.

A part of particular interest is that devoted to a *priori* probabilities.

A second volume will be issued later which will deal with Frequency Curves, Heterograde Statistics and Correlation.

Abstract from the Minutes of the Semi-annual Meeting of the Actuarial Society of America, Held in Philadelphia, on Thursday and Friday, October 28 and 29, 1915, Penn Mutual Building, Philadelphia, Pa.

FIRST DAY.

The meeting was called to order by the President, J. M. Craig, at 11 o'clock A. M.

The following members were present:

FELLOWS.

Alsop,
Blehl,
Cammack,
Carpenter,
Coburn,
Craig, J. D.,
Craig, J. M.,
Dawson,
Dow,
Fackler, D. P.,
Fackler, E. B.,
Gore,
Hardcastle,
Hemperley,
Henderson,
Hunter, A.
Hunter, R. G.,
Hutcheson,
Johnson,
Kilgour,
Kime,
Linton,
McKechnie,
Macaulay,
MacLean, A. T.,
Marshall, E. W.,
Marshall, E. P.,
Mead,
Moir,
Mowbray,
Murphy,
Papps,
Perrin,
Portch,
Rhodes,
Rice,
Rose,
Sanders,
Stephenson,
Strong,
Torrey,
Vineberg,
Walker,
Watt,
Welch,
Wells,
Wing,
Wood, A. B.,
Wood, W. A.

ASSOCIATES.

Allison,
Angell,
Armstrong,
Blackadar, W. L.,

Breiby,
Brown,
Bulkley,
Davenport,
Flanigan,
Forster,
Franks,
Gibb,
Gould,
Griswold,
Hemsing,
Howe,
Kaufman,
King,
Larkin,
Larus,
Lithgow,
Milligan,
Parker,
Phillips,
Smith, V. R.,
Speers,
Tebbetts,
Thomas,
Washburne, A. C.,
Williamson,
Wolfenden.

President Johnson of the Penn Mutual Life Insurance Company was introduced by the Chairman and made a short address of welcome.

After the calling of the roll the President read his address to the Society.

The Minutes of the Annual Meeting held in New York, May, 1915, were approved as printed in the *Transactions*.

The Secretary reported that, at the meeting of the Council, the Committee of seven on the New Mortality Table had presented a report of progress and had been continued.

The reading of papers was then proceeded with.

At 12:30 the Society adjourned.

Afternoon Session.

The Society reassembled at 2:30 P. M.

The reading of papers was resumed.

When the reading of papers had been concluded the discussion of papers presented at the spring meeting was taken up.

On motion the Secretary was instructed to write to Mr. DeBoer, conveying the sympathy of the members in view of his indisposition.

At 4 o'clock the Society adjourned to meet at dinner at 7 o'clock and reassemble for business on the following morning at 10:30 A. M.

SECOND DAY.

Friday, *October* 29, 1915.

The meeting was called to order by the President.

The Secretary reported that, at the meeting of Council just concluded, a delegate and alternate had been appointed to the Second Pan-American Scientific Congress to be held in Washington in December of this year.

The discussion of papers was then proceeded with. At the conclusion of this discussion a vote of thanks was passed to the Officers and others of the Penn Mutual and the Provident Life and Trust for the hospitality which had been extended to the Society.

On motion a message of congratulation was sent to Mr. E. McClintock on the occasion of his seventy-fifth birthday.

At 2:20 the Society finally adjourned.

Examinations.

The following is a list of those who passed the examinations held by the Society on May 26 and 27, 1915.

FELLOWSHIP.

Part I.

Blackadar, W. L.,
Cushing, R. M.,
Hurd, H. G.,
Lithgow, J. H. F.,
McLean, P. S.,
Rydgren, A. A.

Part II.

Dark, T. A.,
Rice, H. I. B.,
Stephenson, H. R.,
Wolfenden, H. H.

ASSOCIATESHIP.

Section A—Part I.

Armstrong, A. O.,
Bassford, H. R.,
Bonynge, W.,
Bosworth, L. A.,
Carr, A. I.,
Craw, J.
†Duckworth, E. E.,
Hart, W. V. B.,
Henderson, E. C.,
Hoskins, J. E.,
Howell, V.,
Kattell, S. C.,
Killam, S. D.,
Kurtz, F. (cum laude),
Lundgren, E. A.,
McLachlin, J. L.,
Mabon, J. B. (cum laude),
Maccabe, T. J.,
†Madrill, J. D.,
Mason, R. L.,
Millyard, R. de L.,
Murray, J. R. C.,
Peirce, H. H.,
Rothschild, S. Z.,
Rowan, H. R.,
Simonson, L. D.,
Strachan, W. B.,
Tribble, J. W.,
†Wheeler, R. A.,
Willows, C. E.

Section A—Part II.

Booth, R. I.,
Bowerman, W. G. (cum laude),
Fassell, E. G.,
Hartwell, G. W.,
Hoskins, J. E.,
Hezlett, E. H.,
Howell, V.,
*Johnston, A. W.,
Langman, H.,
Mabon, J. B. (cum laude),
Reichgott, H.,
Rothschild, S. Z.,
Tribble, J. W.

Section B—Part I.

Bagley, W. N.,
Beecroft, J. D.,
†Birkenshaw, J. H.,
Elston, J. S.,
Hill, E. B.,
Koeppe, A. J.,
McConney, E. M.,
McLean, J. V.,
MacCharles, F. D.,
Stevens, L. L.,
Wheeler, R. P.,
†Williamson, J. D.

Section B—Part II.

Buchanan, J. D.,
*Cann, F. E.,
Gamerdinger, C. W.,
Griswold, R. N.,
Koeppe, A. J.,
Larkin, H. F.,
Larus, J. R.,
Rutherford, C. D.,
Stevens, L. L.,
Williamson, W. R.

* Part I not yet passed.
† Part II previously passed.

Joint Committee on Standards for Graphic Presentation.

Preliminary Report Published for the Purpose of Inviting Suggestions for the Benefit of the Committee

As a result of invitations extended by The American Society of Mechanical Engineers, a number of associations of national scope have appointed representatives on a Joint Committee on Standards for Graphic Presentation. Below are the names of the members of the committee and of the associations which have coöperated in its formation.

Willard C. Brinton, *Chairman,* American Society of Mechanical Engineers, 7 East 42d Street, New York City.

Leonard P. Ayres, *Secretary,* American Statistical Association, 130 East 22d Street, New York City.

N. A. Carle, American Institute of Electrical Engineers.

Robert E. Chaddock, American Association for the Advancement of Science.

Frederick A. Cleveland, American Academy of Political and Social Science.

H. E. Crampton, American Genetic Association.

Walter S. Gifford, American Economic Association.

J. Arthur Harris, American Society of Naturalists.

H. E. Hawkes, American Mathematical Society.

Joseph A. Hill, United States Census Bureau.

Henry D. Hubbard, United States Bureau of Standards.

Robert H. Montgomery, American Association of Public Accountants.

Henry H. Norris, Society for the Promotion of Engineering Education.

Alexander Smith, American Chemical Society.

Judd Stewart, American Institute of Mining Engineers.

Wendell M. Strong, Actuarial Society of America.

Edward L. Thorndike, American Psychological Association.

The committee is making a study of the methods used in different fields of endeavor for presenting statistical and quantitative data

in graphic form. As civilization advances there is being brought to the attention of the average individual a constantly increasing volume of comparative figures and general data of a scientific, technical and statistical nature. The graphic method permits the presentation of such figures and data with a great saving of time and also with more clearness than would otherwise be obtained. If simple and convenient standards can be found and made generally known, there will be possible a more universal use of graphic methods with a consequent gain to mankind because of the greater speed and accuracy with which complex information may be imparted and interpreted.

The Following Are Suggestions Which the Committee Has Thus Far Considered as Representing the More Generally Applicable Principles of Elementary Graphic Presentation

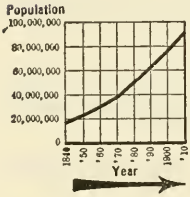


Fig. 1.

1. The general arrangement of a diagram should proceed from left to right.

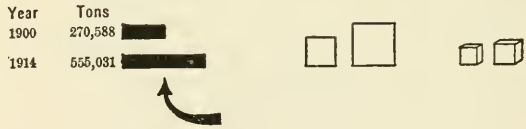


Fig. 2.

2. Where possible represent quantities by linear magnitudes as areas or volumes are more likely to be misinterpreted.

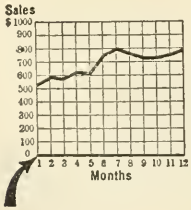


Fig. 3.

3. For a curve the vertical scale, whenever practicable, should be so selected that the zero line will appear on the diagram.

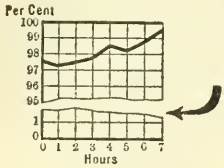


FIG. 4.

4. If the zero line of the vertical scale will not normally appear on the curve diagram, the zero line should be shown by the use of a horizontal break in the diagram.

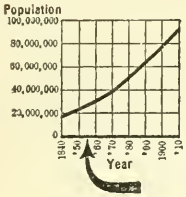


FIG. 5A.

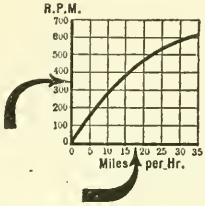


FIG. 5B.

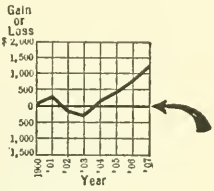


FIG. 5C.

5. The zero lines of the scales for a curve should be sharply distinguished from the other coördinate lines.

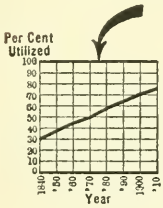


FIG. 6A.

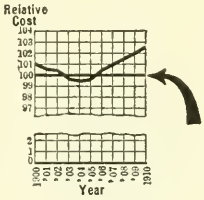


FIG. 6B.

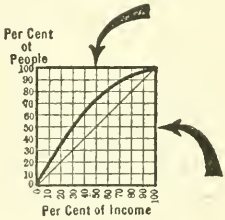


FIG. 6C.

6. For curves having a scale representing percentages, it is usually desirable to emphasize in some distinctive way the 100 per cent. line or other line used as a basis of comparison.

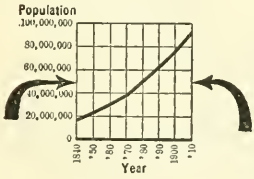


FIG. 7.

7. When the scale of a diagram refers to dates, and the period represented is not a complete unit, it is better not to emphasize the first and last ordinates, since such a diagram does not represent the beginning or end of time.

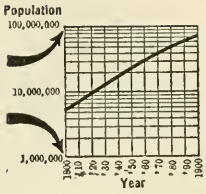


FIG. 8.

8. When curves are drawn on logarithmic coördinates, the limiting lines of the diagram should each be at some power of ten on the logarithmic scales.

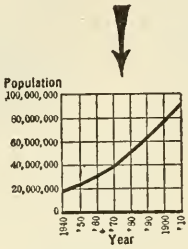


FIG. 9A.

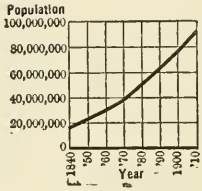


FIG. 9B.

9. It is advisable not to show any more coördinate lines than necessary to guide the eye in reading the diagram.

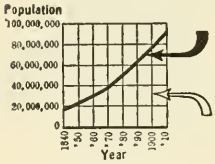


FIG. 10.

10. The curve lines of a diagram should be sharply distinguished from the ruling.

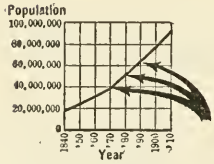


FIG. 11A.

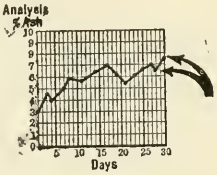


FIG. 11B.

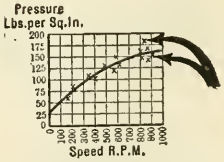


FIG. 11C.

11. In curves representing a series of observations, it is advisable, whenever possible, to indicate clearly on the diagram all the points representing the separate observations.

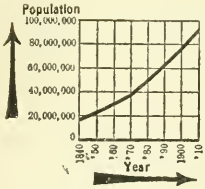


FIG. 12.

12. The horizontal scale for curves should usually read from left to right and the vertical scale from bottom to top.

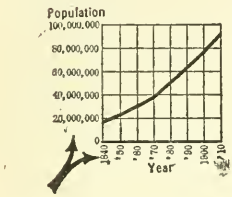


FIG. 13A.

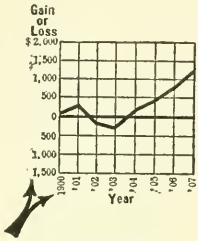


FIG. 13B.

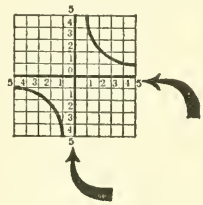


FIG. 13C.

13. Figures for the scales of a diagram should be placed at the left and at the bottom or along the respective axes.

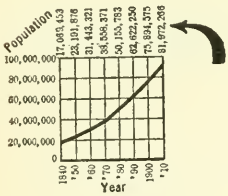


FIG. 14A.

FIG. 14B.

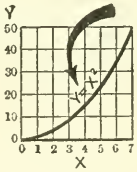
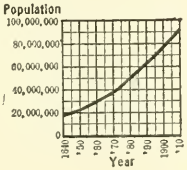


FIG. 14C.

14. It is often desirable to include in the diagram the numerical data or formulæ represented.



Year	Population
1840	17,069,453
1850	23,191,876
1860	31,443,321
1870	38,558,371
1880	50,155,783
1890	62,622,250
1900	75,994,575
1910	91,972,266

FIG. 15.

15. If numerical data are not included in the diagram it is desirable to give the data in tabular form accompanying the diagram.

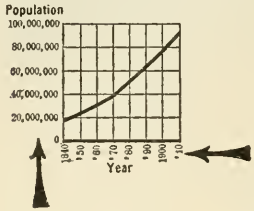
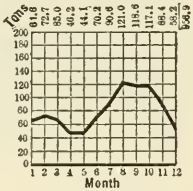


Fig. 16.

16. All lettering and all figures on a diagram should be placed so as to be easily read from the base as the bottom, or from the right-hand edge of the diagram as the bottom.



Aluminum Castings Output of Plant No. 2, by Months, 1914. Output is given in short tons. Sales of scrap aluminum are not included.

Fig. 17.

17. The title of a diagram should be made as clear and complete as possible. Sub-titles or descriptions should be added if necessary to insure clearness.

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